BRILL'S COMPANION TO

THE RECEPTION OF PYTHAGORAS AND PYTHAGOREANISM IN THE MIDDLE AGES AND THE RENAISSANCE

Edited by

Irene Caiazzo,
Constantinos Macris
and Aurélien Robert

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Irene Caiazzo
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Paris, 4th August 2021

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Druzes: Rasāʾil al-Ḥikma, Volumes 1 et 2, Introduction, édition critique et traduction annotée des traités attribués à Ḥamza b. ʿAlī et à Ismāʿīl at-Tamīmī (Louvain, 2007); and *La philosophie ismaélienne: Un ésotérisme chiite entre néoplatonisme et gnose* (Paris, 2012). He has co-edited *Controverses sur les écritures canoniques de l'islam* (Paris, 2014) and *L'ésotérisme shiʿite, ses racines et ses prolongements* (Turnhout, 2016) (both with M.A. Amir-Moezzi); *Noétique et théorie de la connaissance dans la philosophie arabe du IXe au XIIe siècle. Des traductions gréco-arabes aux disciples d'Avicenne* (Paris, 2019; with M. Sebti).

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INTRODUCTION

Pythagoras, from Late Antiquity to Early Modernity. A Multicultural Approach

Irene Caiazzo, Constantinos Macris and Aurélien Robert

In the last twenty years or so, a number of important studies have been published on the reception of Pythagoras in Antiquity, as well as on the transformation of Pythagorean traditions from Late Antiquity to the Renaissance, in both the eastern and western parts of the Mediterranean Sea.[1] However, it is worth noticing that there has been almost no room for the Middle Ages in this recent renewal of the traditional historiography.[2] At best, the existing textbooks devote a single chapter to the Middle Ages.[3] On the other hand, medieval studies have long been busy analysing the reception of Aristotle, which is undoubtedly central, and only in the last decades have they progressively shifted their focus to embrace the Platonic heritage, too – which, though, most of the time is inextricably linked to Pythagoreanism. Hence, the aim of this volume is to provide what seems to be the first general overview of the reception

1 See in particular Kahn 2001; Riedweg 2007[2] [2005[1]]; Bonazzi, Lévy and Steel (ed.) 2007; Zhmud 2012; Cornelli, McKirahan and Macris (ed.) 2013; Huffman (ed.) 2014; Renger and Stavru (ed.) 2016. Cf. also Renger 2015; Riedweg 2013. For the Syriac and Arabic world see Izdebska 2016 and 2018. For the Renaissance, Celenza 1999 and 2001a; Robinson 2013; Allen 2014; Robichaud 2018. See also the useful introductory orientation offered in dictionary and encyclopaedia entries like Renger and Ißler 2013 and Thom 2017. For an overview of the bibliography concerning the reception of Pythagoras, see Macris 2018b, 1087–1174.

2 An exception is Joost Gaugier 2006 and 2009 (cf. the critical reviews of E.A. Browning, *BMCR* 2007.02.43, and St. Gersh, *Speculum* 83/2 [2008]: 444–445, and the more positive one by M.J.B. Allen, *Renaissance Quarterly* 66/1 [2013]: 225–227), and most recently Albertson 2014 (focusing on "mathematical theologies"); Hicks 2017 (on microcosmic and macrocosmic harmonies), 2019a (for a novel comparative exploration in Persian sufism) and 2019b; Robert 2017 and 2018 (on Pythagorean atomism); Macris 2018b, 768 (on pictorial representations of Pythagoras) and 2021, n. 100–105 (on "later ascriptions" of texts to Pythagoras and the Pythagoreans, like the *littera Pythagorae* [Y], various *prognostica* [Guardo 2006], *Sortes* books [Wickersheimer 1914; Iafrate 2014], the *Sortes Astrampsychi*, the *Sphaera of Life and Death* [Liuzza 2005, Chardonnens 2007, 181–221 (*passim*), Edge 2014], etc.). Among earlier thematic studies one may note Hopper 1938 (on number symbolism); Chenu 1961 (on a Pythagorean definition of truth); Münxelhaus 1976 (on music theory); Prelog 1990 (on the biographies of Pythagoras); Maaz 1998, Zander 1999 and Roling 2016 (on metempsychosis).

3 Hicks 2014.

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of Pythagorean and Neo-Pythagorean doctrines during the so-called "long Middle Ages,"[4] conceived in a multicultural perspective.[5]

The present collection of papers constitutes only a first step towards a completely renewed understanding of the place and role of Pythagoreanism – a Pythagoreanism that is sometimes assumed and proclaimed, sometimes attributed to others, sometimes supported or appropriated without any explicit reference to Pythagoras – in the *longue durée*, extending from Late Antiquity to Early Modernity.[6] In the articles that make up this book, the term "Pythagoreanism" is rather broadly conceived (as it was the case already in Antiquity), its focal points being (a) mathematics, number philosophy, music and the quadrivium; (b) a metaphysics that most of the time joins and expands that of the Platonists and neo-Platonists; and (c) a singular ethics, transmitted in various doxographies and gnomologies.[7] The results are self-explanatory: there were, indeed, innumerable debates about Pythagoras and Pythagoreanism during this period, and attitudes ranging from antiquarian interest and encyclopaedic curiosity to a real engagement with the tradition. And there were as many Pythagoras*es* and Pythagoreanism*s* in medieval and early modern times as there were in Antiquity.

Following Gabriele Cornelli's suggestion in his essay *In Search of Pythagoreanism*, this diversity should prompt us to consider Pythagoreanism as a (constructed, but relatively fluid) "historiographical category" rather than an "essentialized" historical reality with a strict and well-defined, "monolithic" identity.[8] Consequently, our purpose here is not to recover the historical Pythagoras and the ancient Pythagoreans between the lines of medieval and early modern manuscripts and printed editions, but to understand the historical construction of a kaleidoscopic image of the philosopher, his disciples and the "tradition" they represent, and to follow its vicissitudes – its continuities and discontinuities – for nearly ten centuries.

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4 On this idea of a "long Middle Ages", see Le Goff 2015, 79–112.

5 A similar perspective is adopted, e.g., on an akin subject by Acevedo 2020 (see also Hicks 2019a and 2019b), and on other subjects by Akasoy, Montgomery and Pormann (ed.) 2007; Lagerlund (ed.) 2011; Brentjes and Renn (ed.) 2016; less explicitly in Arfé, Caiazzo and Sannino (ed.) 2011.

6 On Pythagorean cosmology during the Renaissance see also Heninger 1961, 1965 and 1974.

7 Other subdivisions or topics are possible of course; cf., e.g., Kahn 2001 for a tripartition of the "Pythagorean heritage" into (1) "The Pythagorean tradition of the occult and the supernatural" (pp. 139–146), (2) "Transmigration and vegetarianism" (146–153), and (3) "Mathematics, music, and astronomy" (153–172).

8 On Pythagoreanism as an historiographical category, see Cornelli 2013, with the review of C. Macris, *Revue philosophique de Louvain* 114/1 (2016): 127–129.

To carry out this investigation, the contributions collected here deal with a wide range of topics, authors, and disciplines. Their aim is to shed some light upon a rich and multifarious *Nachleben*, which embraces a wide spectrum of fields and disciplines in intellectual history: mathematics, music and the quadrivium, cosmology, metaphysics, psychology, ethics and way of life, number symbolism and magic. More importantly, this volume is conceived in such a way that the resulting overview is not limited to the Western Latin tradition, but gives equal weight to the Arabic, Hebrew, and Persian cultures.[9]

Not everything could have been surveyed in such a vast panorama. The greatest absence is undoubtedly that of Byzantium – both because some aspects of its early years will be studied soon in a separate *Brill's Companion to the Reception of Pythagoras and Pythagoreanism*, covering the Classical, Hellenistic and Late Antiquity,[10] and because its Middle and Late periods still give the impression of a totally unexplored *terra incognita*: to date there are neither detailed studies nor synoptic overviews dedicated to this aspect of the reception of Pythagoreanism.[11] After having consulted some of the best specialists of this period of Greek history, reluctantly we have decided to leave it out of this volume: as a matter of fact, there is no such a thing as conclusions to be extracted from previous explorations undertaken in this area and to be offered in a companion volume, but an almost virgin field awaiting to be investigated thoroughly in a series of preliminary surveys, in specialized papers focusing on

9 On the presence of Pythagorean doctrines in the Caucasian philosophy of the Middle Ages, esp. in Joane Petritzi's commentaries on Proclus' *Elements of Theology* (12th c.), see Iremadze 2016.

10 The pioneering work on late antique Pythagoreanism has been done by Dominic O'Meara (1989, 2013 and 2020).

11 It is indicative of the situation that in the massive (almost 800 p.) and broadly conceived Kaldellis and Siniossoglou (ed.) 2017, which gathers most of the major byzantinists currently active, Pythagoreanism appears only in passing and no special entry is dedicated to it in the otherwise rich and detailed indexes. Similarly, in Ierodiakonou (ed.) 2002 or in Magdalino and Mavroudi (ed.) 2006, which focuses on the occult sciences in Byzantium, one can find only some passing references to Pythagoreanism (noted in the indexes). – For some sporadic explorations of single authors, texts or thematic *dossiers*, see De Falco 1923; Treadgold 1978; Whittaker 1980; O'Meara 1981, on Psellos (later integrated in Id. 1989); Costanza 2005; Rutherford 2006; Katsiampoura 2010; Marino 2017; Hofstetter 2018; cf. also Macris 2018b, 752–753 (on an anonymous *Life of Pythagoras* epitomized in Photius' *Myriobiblos* [9th c.]) and 760 (on the entry "Pythagoras" of the *Souda* Lexicon [10th c.]); Piccione 2021 (on pseudo-Pythagorean texts in Stobaeus' *Anthologion* [5th c.]), and below, n. 20 and 24. – One may also take into account the circulation of Empedocles' poems in Byzantium, given his close ties with Pythagoreanism; see Rashed 2001 and 2014; Primavesi 2002 and 2006; Ferella 2020. – On the Byzantine reception of Apollonius, the Neopythagorean magus, see Grimm-Stadelmann 2020, 71–75, and below, n. 85.

single authors or topics and/or, ideally, in an ambitious monographic synthesis or a Doctoral Dissertation.

Another aspect that would have probably deserved to be present here is classical Greek and Roman literature indirectly relevant to Pythagoreanism and its reception in the Middle Ages and the Renaissance. To give one example out of many: Book XV of Ovid's *Metamorphoses*, containing the famous speech of Pythagoras[12] – one of the best-sellers in Latin from the 12th century onward – has been subject to many comments and interpretations.[13] The *Moralized Ovid* and other medieval and Renaissance uses of the *Metamorphoses* would probably reveal another Pythagoras.[14] One should also take into account the reception (1) of some emblematic Pythagorean pseudepigrapha such as the cosmo-psychology of pseudo-Timaeus Locrus,[15] the Letter of Lysis to Hipparchus on secrecy and the Pythagorean "mysteries"[16] or Brotinus' *On the Intellect and Discursive Reasoning*,[17] but also (2) of Philostratus' *Life of Apollonius of Tyana*, describing the words and deeds of an influential Neopythagorean sage of the first century CE,[18] and even (3) of the more syncretistic but still Pythagoreanizing *Tablet* of Cebes.[19] The so-called "hexaemeral literature," consisting in commentaries on the creation narratives found in the first two chapters of the book of Genesis, seems also a promising field to explore, given the arithmological speculations it may contain regarding the cosmological and theological implications of the six days of the world's creation.[20] Despite these shortcomings, the reader will find references here and there allowing them to go deeper into this or that issue.

The general assumption of the volume is that it is not always possible to find an undisrupted and mono-linear continuity in the reception of Pythagorean

12 For a bibliographic orientation on this important piece of Latin literature see Macris 2018b, 1123–1125.

13 On the reception of Ovid, see for instance Clark, Coulson and McKinley 2011. On Ovid's Pythagoras, see McGowan 2014.

14 Cf. Tilliette 2009; Coulson 2019.

15 See Macris 2018c, and cf. Jonkers 2017 on the manuscript tradition of Plato's *Timaeus*, which in some cases goes hand-in-hand with that of its pseudepigraphic counterpart.

16 Del Soldato 2020b.

17 Robichaud 2016.

18 See Mandosio 2013 and Porreca 2014 (Middle Ages); Dall'Asta 2008 (Renaissance); Marein 2015 (16th–17th c.).

19 On the Greek original see Macris 2018b, 1144–1145. On its reception see Schleier 1973.

20 Robbins 1912 (not by chance, the author is also a specialist of Nicomachus and of Greek arithmology); Bouteneff 2008; Brown 2014; Elliott 2019, 51–90, 119–158. Of course, not all of the arithmological speculations found there have a Pythagorean origin. At the same time, the prototype of the "*hexaemeral* tradition" is Philo's *De opificio mundi*, a work much indebted to Pythagorean number philosophy and arithmology (cf. Zhmud 2021).

doctrines and patterns, in so far as it obviously depends on the availability of ancient sources. Consequently, a common concern of all chapters of this Companion is to look for the sources of medieval and early modern interpretations of Pythagoreanism, in order to delineate and follow their circulation in different cultural and linguistic areas.[21] For instance, in the case of arithmetic, it is clear that the Neo-Pythagorean mathematician Nicomachus of Gerasa – author of an *Introduction to Arithmetic* written in Greek toward the turn of the 1st and 2nd century C.E., then translated and commented upon in many languages through the centuries – plays a central role, quasi simultaneously, not only in the Greek and Latin, but also in the Syriac, Arabic and Hebrew traditions. The situation seems quite different though for metaphysics and psychology in so far as it is the Platonic dialogues as well Aristotle's treatises that are frequently used to interpret the central tenets of what was perceived as the Pythagorean *Weltanschauung* – without being used exactly in the same way in the Latin- and Arabic-speaking world for instance. In the case of ethics, the translations from Arabic to Latin played a crucial role, but a lot of information collected in Latin encyclopaedias comes from classical sources as well as from the Church Fathers. As a result, such a comparative approach reveals the need to adopt different scenarios for the transmission of Pythagorean texts and ideas. In some cases, there has been a direct transfer from Greek to Latin, whereas in other cases, a crucial mediation occurred through the Syriac, Arabic and/or Hebraic languages. Therefore, unlike, e.g., the reception of Aristotle in the Latin Middle Ages, that of Pythagoras does not always take the paths of the *translatio studiorum*.[22] In this volume an effort has been made to provide a fairly complete and nuanced picture of this multifaceted situation.

Undoubtedly, the most important aspect of the reception of Pythagorean philosophy during the Middle Ages and the Renaissance concerns the organization, teaching and content of the mathematical sciences, and more specifically the nature and role of the quadrivium, i.e., arithmetic, geometry, music and astronomy, which constitute the four "liberal arts."[23] As it appears in the first part of this Companion, this is due mainly to the massive circulation in all

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21 A similar approach has been adopted by Gersh and Hoenen (ed.) 2002 for the study of the Platonic tradition in the Middle Ages.

22 On which see, e.g., Delgado, Méla and Möri (ed.) 2014.

23 On the ancient Pythagorean (Archytean) pedigree of the quadrivium, see Zhmud 2006, 62–64; 2018, 191–192. On its elaboration in Late Antiquity, see I. Hadot [2005^2]. On the role played by Varro in this process, see Larionova 2020.

available languages and cultural areas of Nicomachus of Gerasa's *Introduction to Arithmetic*.[24]

The profound impact of this text has already been noticed by some scholars, but the reader will now be able to find here a more "global" view of his parallel reception in the West as well as in the East. In the West, Boethius' Latin adaptation of the text has been a huge success, the impact of which is not yet adequately apprehended, evaluated and accounted for.[25] In addition to the interest *per se* of such a synthesis, it is expected that it would allow us to nuance certain well-known positions about the role of Pythagoreanism in the modern era. Indeed, the myth of a return of Pythagoreanism during the Renaissance after a long medieval sleep is still prevalent in the "grand narratives" of modernity. Following Alexandre Koyré,[26] some scholars assert that the scientific revolution would have been, among other things, the result of the rehabilitation of Pythagoras by the Humanists. This, in turn, would have enabled modern thinkers to conceptualize a first form of mathematization of reality in general and of physics in particular. For example, sometimes we read about a Pythagorean Galileo Galilei. In his chapter here David Albertson recalls that in 1633 the Jesuit Melchior Inchofer warned anyone who followed Galileo's teachings that he would no longer be considered a Christian but a *Neopythagorean*.[27] However, such a Pythagorean mathematization of reality – which has nothing to do, in fact, either in its principles or in its main theses, with the Euclidean geometrization of space in the 17th century – has never ceased to be revived, transformed and discussed during the Middle Ages. For this reason, it is particularly useful, and important, to present, as far as possible, the most complete overview of the reception of Pythagorean number theory during this period.

24 Greek text and translations: Nicomachus of Gerasa 1866, 1926 and 1978. On the rich tradition of Neoplatonic commentaries on this text see, e.g., Iamblichus 2014; Asclepius 1969; John Philoponus 1999; cf. Westerink 1964; Giardina 2016 and 2020. On Nicomachus and his impact in Late Antiquity, O'Meara 1989, 14–23 and index, *sv*. For the manuscript tradition of the text, which clearly shows its wide circulation in Byzantium, Hofstetter 2018, 2018 [2019] and 2021.

25 For a first survey, see Guillaumin 2012. For medieval commentaries on Boethius' *De arithmetica*, Thierry of Chartres 2015 and Caiazzo 2020a. On the larger context of Boethius' reception in the Middle Ages, Kaylor Jr. and Phillips (ed.) 2012.

26 Koyré 1957.

27 On this, see more extensively Martínez 2018 and 2019.

1 Pythagorean Number Theory and the Quadrivium

Part 1 of the Companion deals with *Pythagorean Number Theory and the Quadrivium*, and its indisputable protagonist is Nicomachus of Gerasa.[28]

In the first chapter, **Cecilia Panti** discusses the role played in Western Europe by Boethius' Latin adaptations of Nicomachus of Gerasa's *De institutione arithmetica* and *De institutione musica* in the formation and transmission of the image of Pythagoras as the founder of the mathematical arts. After reviewing the available evidence from ancient sources, both Greek and Latin (Plato, Aristotle, Varro, Cicero, etc.), she carefully demonstrates the unifying role played here by the Neo-Pythagoreanism of Nicomachus of Gerasa. Indeed, from the first chapters of his treatise on arithmetic to his introduction to music, the Greek mathematician presents the system of the four liberal arts as a Pythagorean doctrine,[29] which would enable us to understand what is quantity – the key concept for mathematical science, given that numbers, geometric figures, musical harmony, and the position and motion of planets and stars have all something to do with quantity. Some Neo-Platonist authors took up this presentation, such as Iamblichus, but also Martianus Capella and Calcidius,[30] who were well known and constantly read throughout the Middle Ages. It is during this period that a great importance was attributed to the quadrivial *mathemata*, with Nicomachus being one of the main authorities in the basic school curriculum.[31] Nevertheless, Panti shows persuasively why and how, in Western Europe, it was Boethius who established Pythagoras as the initiator and main thinker in the fields of arithmetic and music in particular, from the Carolingian time to the 14th century.

In line with this inaugural panorama, **Andrew Hicks** takes a closer look at the reception of Pythagorean music theory and harmonics in the West.[32] Indeed, in his *De institutione musica* Boethius refers several times to a "*scientia*

28 On whom see Centrone 2005; Macris 2018b, 736–737, 743, 1152 (cf. also 1157 and 1159), with bibliography. – On number theory in the Middle Ages see Wedell 2015, 1243–1258.

29 See I. Hadot 2005² [1984¹], 63–69.

30 On Iamblichus' Pythagoreanizing "decalogue", structured with the quadrivium in mind, see O'Meara 1989, 30–105 and Macris 2009. On Martianus Capella, I. Hadot 2005² [1984¹], 137–155, 391–410 (with the bibliographical *addenda* suggested in Macris 2018b, 1160). On Calcidius, Bakhouche in Calcidius 2011; Reydams-Schils 2020.

31 On the teaching of the quadrivium as part of the school curriculum, see Caiazzo 2020b, who focuses on the 12th century.

32 See more extensively his monograph: Hicks 2017, as well as Dyer 2009; Rico 2005; Mews 2012 (the latter two focusing on the "music or harmony of the spheres" – on the Renaissance and Early Modern developments on the same subject, see Prins 2015; Prins and Vanhaelen [ed.] 2018).

pythagorica". What is this about? Hicks shows that it is essentially about the science of arithmetic, which serves as a foundation for musical knowledge, and which is based on the elementary concepts of equality and inequality, and above all on a very elaborate theory of numerical ratios. Thus, between the 12th and 14th centuries, many medieval harmonic and acoustic theories were developed on the basis of these Neo-Pythagorean concepts elaborated by Nicomachus and re-interpreted by Boethius. Moreover, these musical theories reflect a much broader worldview, a metaphysics, one might say, which is largely dominated by number theory. According to Hicks, however, as far as music is concerned there is no medieval Neo-Pythagoreanism or even Neo-Pythagoreanism*s*, but only repetitions, often distorted and amplified, of Nicomachus' central intuitions, filtered through Boethius, Martianus Capella or Cassiodorus. Notwithstanding, this legacy literally colonized medieval imagination, and the medieval authors themselves rarely felt the need to make explicit or claim this affiliation with Pythagoreanism, even though some of them did so.

The same phenomenon can be observed in the East. After a brief presentation of the Arabic translations of Nicomachus' *Introduction to Arithmetic*, **Sonja Brentjes** describes the different types of appropriation of this text in the Arabic and Persian world over a long period of time, between the 9th and 14th centuries. Her analysis follows the chronology of this reception and also considers the different literary genres and groups or schools in which certain elements of this Neo-Pythagorean handbook of arithmetic can be found. Indeed, if traces of Pythagoras and Nicomachus are obviously found in mathematical treatises – from the 10th century onwards, mathematicians used to write commentaries on the *Introduction to Arithmetic* in Arabic[33] –, one can also find them in various other contexts. To give an example, in the 9th century, al-Kindī used elements of the *Introduction to Arithmetic* in his medical works in order to analyze the notion of "crisis" or to describe the degrees of qualities in medicines. As for the famous *Brethren of Purity* (end of the 9th-first half of the 10th century; exact dates disputed), the mathematical developments in their epistles are clearly influenced by the content of Nicomachean arithmetic.[34] Later, Ibn Sīnā (Avicenna) (dates: 980–1037) includes elements from

33 Exactly as their Byzantine Greek colleagues did before them; see above, n. 24.

34 On this point see also the chapter by Carmela Baffioni in the present volume, and more extensively Ead. 1997. – Recognizing the extent of Nicomachus' influence on the Brethren's *Epistles*, de Vaulx d'Arcy 2019b goes so far as to suggest that their redactor was a single scholar, Aḥmad b. al-Ṭayyib al-Saraḫsī, who was also a reviser of the *Introduction to Arithmetic* in Arabic. But this is far from certain.

Nicomachus – sometimes to criticize them[35] – in his great encyclopedia of the *Shifā* (*The Book of Healing*). The *Introduction to Arithmetic* was read not only by mathematicians and philosophers, but also by theologians of the *Kalām* (i.e., theoretical theology), teachers of the Madrasas, jurists, and physicians. Its basic concepts can be found almost everywhere, and circulated widely in many textbooks as it is shown by Sonja Brentjes. From a mathematical point of view, it emerges from this survey that most readers of Nicomachus tended to read his work in parallel with Euclid's *Elements*, and that, beyond the technical discussions, some of them were interested in philosophical reflections on the status of unity and the one.[36] What is striking is that most of the authors studied in this chapter were fully aware of the Pythagorean background of their reflections. For example, Thābit ibn Qurra wrote an entire treatise on amicable numbers in the tradition of "Pythagoras and the ancient philosophers of his school"[37] because, as he puts it, neither Euclid nor Nicomachus dealt with this aspect in detail. Therefore he saw it as his task to develop the Pythagorean theory of numbers.

The *Introduction to Arithmetic* was successful not only in the Islamic world, but also in Jewish thought, expressed both in Arabic and Hebrew. In order to understand what may have been received from the Greek original in Hebrew, **Gad Freudenthal** examines the extremely complex history of the translations of the Nicomachean text from Greek to Syriac, from Syriac and Greek to Arabic, and from Arabic to Hebrew. The result of this process can be found in the Hebrew translation made by Qalonymos ben Qalonymos of Arles (France, 1286-after 1328). This version, published in 1317, is particularly interesting because it contains many extrapolations from the Arabic tradition – including passages from a commentary of Iamblichus added by Ibn Bahrīz, glosses from al-Kindi, and anonymous additions – and also because, on the whole, it is more an adaptation than a translation in the strict sense.[38] The most interesting feature is the addition of drawings and diagrams that are not found in the Greek original or in Syriac or Arabic manuscripts. More surprisingly, some

35 On the generally critical attitude of Avicenna toward Pythagoreanism (influenced by Aristotle's *Metaphysics*), see Izdebska 2014, 38; La Sala 2016, 423–428.

36 This kind of reflections are studied more thoroughly in the chapter by Daniel De Smet.

37 See the French translation of this text, with an introduction, in R. Rashed and Houzel 2009. For the reception of Thābit's approach on amicable numbers in medieval *Hebrew* literature, see Lévy 1996. On Thābit more generally, R. Rashed (ed.) 2009. On his relation to the Pythagorean tradition, M. Rashed 2009, 702–704, 711–712.

38 By so doing Qalonymos somehow follows in the footsteps of Iamblichus 2014, whose *In Nicomachi Arithmeticam introductionem* is in fact a "new edition" of Nicomachus' treatise, revised and augmented, as has been rightly stressed by F.E. Robbins in Nicomachus of Gerasa 1926, 124–137; see also Mansfeld 1998, 87.

of these drawings are found in the Latin adaptation by Boethius, which perhaps suggests the existence of a common source, probably of Alexandrian origin according to Gad Freudenthal. Thus, when one speaks of the reception of Nicomachus' Neo-Pythagorean arithmetic, one must bear in mind that it is in fact a multiform text, some versions of which contain many layers of additions and interpretations.

Qalonymos ben Qalonymos' version enjoyed some success, since it is preserved in eight manuscripts and has been the subject of several commentaries.[39] However, in his paper **Tzvi Langermann** examines two other cases of Jewish Pythagoreanism, in which the sources are multiple and sometimes hidden.[40] The first is Moses Maimonides (d. 1204). Some chapters of his *Guide of the Perplexed* make it possible to understand the impact of some points of Pythagorean doctrine in medieval Jewish philosophy, although Maimonides himself never claims to adhere to the teachings of Pythagoras. Of course, Maimonides was not really a Pythagorean philosopher, and he borrowed more from Aristotle than from Pythagoras and his school. However, as Tzvi Langermann shows, many elements of Pythagoreanism had penetrated the *Guide of the Perplexed*. The second case is a less well known text of the already mentioned Qalonymos ben Qalonymos, entitled *Sefer Melakhim* (*The Book of Kings*), which he addressed to King Robert of Anjou when he was official translator at the court. The *Sefer Melakhim* is even more interesting than the *Guide of the Perplexed*, since it is the work of a translator of Nicomachus. Indeed, this testimony is particularly intriguing insofar as it begins by severely criticizing one of the great theses attributed to Pythagoras and his school, according to which all reality, including sensible reality, is composed of numbers.[41] Like Maimonides, Qalonymos rather agrees with the criticism of this theory formulated in Aristotle's *Metaphysics*. But in spite of this first attack, the rest of the treatise, devoted to numbers as well as to arithmology, borrows a lot of elements from the Pythagorean tradition. The *Sefer Melakhim* is still unpublished, but future studies will perhaps show the impact of Nicomachus and other sources on a text which apparently disguises the author's Pythagoreanism behind the allegiance to Aristotle.

39 Langermann 2001.

40 For a third, still earlier case, namely the *Sefer Yeṣirah*, see Langermann 2002, who dates it to the early 9th century CE.

41 It is interesting that the same criticism is found in the Greek tradition in a fragment attributed to Pythagoras' wife, Theano; on this *pseudo-pythagoricum* see Macris 2016, 834.

2 Pythagorean Way(s) of Life, East and West

Already from the time of Plato, Aristoxenus and the Middle Comedy, it is the distinctiveness of the Pythagorean way – or rather, ways – of life that, along with number philosophy, made the "Italian school" most famous in the ancient Greek world[42] (and later among the Romans, too[43]). Part 2, entitled *Pythagorean Way(s) of Life, East and West* shows that the ethical dimension of the Pythagorean tradition remained alive for centuries, attracting the attention of Christians and Muslims alike, thus having a pervasive influence in both the Arabic Middle East and Latin Europe. The main vehicles for its diffusion are two distinct but complementary kinds of sources, namely biographies and collections of maxims or precepts, the so-called gnomologies (which often ended up mixed together). The two following chapters are devoted to the study of the parallel reception and re-elaboration of this material in the Arabic and Latin sources.

Anna Izdebska opens the diptych by showing how, since the 10th century, Arabic intellectuals popularized the figure of Pythagoras and his teaching by translating and adapting – for a readership going beyond the limits of tiny philosophical conventicles to embrace educated people, aristocrats, court officials, rulers, etc. – the Pythagorean biographies and gnomologies inherited from the Greek Late Antiquity, sometimes through Syriac intermediaries. These two genres of "popular" philosophical writing[44] are extremely important, not only due to their wide diffusion, but also because they were not merely informative, but also *formative*, i.e., intended to have a real influence on the reader's life.[45] This is particularly true for the Pythagorean *Golden Verses*, a relatively late poem taken to contain in condensed form the quintessence of Pythagoras' teaching. Differently from the Latin West, where the presence of this poem was sporadic, partial and indirect before the Renaissance,[46] in the Arabic Middle Ages it was considered a moral and philosophical testament bequeathed by Pythagoras to his students, providing spiritual guidance to many generations due to its "universal" character. Moreover, two commentaries on it preserved

42 See generally Macris 2013, esp. 59–60. On Plato, *Republic* X, 600 a–b, ibid., 60–61; on Aristoxenus' *Pythagorean Precepts*, see most recently Huffman 2019; on the "Pythagorists" of Middle Comedy, Macris 2018b, 1037–1038, 1046–1047.

43 Macris 2018b, 1115–1129 (*passim*).

44 On the notion of "popular philosophy" / *Popularphilosophie*, see Thom 2012.

45 This distinction, derived from a formulation used by Victor Goldschmidt *à propos* of the Platonic dialogues, is an idea cherished by Pierre Hadot (e.g., 2002) and applied by him to other kinds of philosophical literature.

46 Celenza 2001a.

in Arabic, one attributed to Iamblichus and another one to Proclus, supposedly translated by Ibn al-Ṭayyib (neither surviving in Greek), put the text in a markedly Neoplatonic perspective thus confirming the affinities of the late antique and medieval Eastern Mediterranean not only with the philosophy of Plotinus (as attested in the so-called *Plotiniana Arabica*, or *Theology of Aristotle*) but also with post-Iamblichean Neoplatonism.

It is still another Neoplatonist, Porphyry, who, as a historian of philosophy, offered to the Greek- and Syriac-speaking Christians, and later to the Arabo-Islamic world, the only continuous narrative of some length on the life of Pythagoras (on which are otherwise preserved in Arabic only some scraps from Iamblichus' *Pythagorean Way of Life*) – although "it is probable that most of the Arabic writers in the Classical period had almost no clue who Pythagoras really was." A. Izdebska examines the two abridged versions of Porphyry's *Life of Pythagoras* (part of his *Philosophical History*) extant in Arabic. They are contained, first, in the ethically oriented *Book of the Choicest Maxims and Best Sayings* of the 11th-century Egyptian scholar and historian al-Mubashshir ibn Fātik, and, two centuries later, in the biographical history of Greek, Arabic, Persian and Indian physicians, scholars and philosophers written by the Syrian physician Ibn Abī Uṣaybiʿa. The latter used al-Mubashshir as his main source – not without adding a precious extra piece of information coming from Porphyry but apparently through another channel (and not extant in Greek), concerning the 280 genuine Pythagorean books preserved in Italy by Archytas and others, which are emphatically distinguished from pseudo-Pythagorean forgeries.[47] In both works the chapter on Pythagoras is stripped of almost all the doctrinal philosophical *exposés* and the legendary-miraculous elements that can still be read in the Greek original, and has a twofold structure, combining biographical information drawn from Porphyry with an extensive collection of wise sayings and moral prescriptions endowed with an ascetic flavour, mainly deriving from gnomological collections (the *Pythagorean Sentences*). To them are added *symbola* reported by Porphyry, selected lines from the *Golden Verses*, and even the wise words allegedly engraved on Pythagoras' ring and belt.[48]

Interestingly, the works of al-Mubashshir and Ibn Abī Uṣaybiʿa were translated into Latin and into various vernacular languages, reaching Western Europe from the 13th century onwards.[49] However, the profile of what we may

47 On this striking passage see Izdebska 2018, 861 (with bibliography) and Horky 2021 (forthcoming).

48 Cf. Zakeri 2020.

49 This constitutes a useful reminder of the significant role played by Arabic thinkers (and sources) in the advancement of both science and philosophy during two formative

call the *Pythagoras Arabus "par excellence"* drew its main lines from other, much more influential sources, dating back to the second half of the 10th and the beginning of the 11th century. On the biographical level, the Muslim theologian and philosopher al-ʿĀmirī, in all likelihood under the influence of Christian historiography, asserted that, while in Egypt, Pythagoras "learned the physical and metaphysical sciences from the companions of Solomon," i.e., the Jews who came to Egypt from Syria, and that "he claimed that he had acquired them from 'the niche of prophecy'," thus rooting the Samian's philosophy in the Abrahamic (monotheistic) tradition and integrating him into the Islamic past. But the Arabic Pythagoras was above all an exceptional moral teacher and a source of wisdom, reflected in his wise words, and in order to have access to those one may have more probably consulted the most widely copied of the Arabic gnomologies: a (now lost) collection of sentences and anecdotes known as the *Ṣiwān al-ḥikma* (*The Cabinet of Wisdom*), of which three abridgements or selections survive. Significantly, Pythagoras is respectfully called *zāhid* (ascetic) here – a term reserved for the Muslim saints of the first centuries of the Hijra.

A similar appropriation can be observed in the Latin *dossier* studied by **Aurélien Robert**, especially in the collections of *exempla* and the *Lives* of ancient philosophers dating from the 12th to the 15th centuries, out of which, by a process of selection operating in the sources, a "proto-Christian" Pythagoras emerges, a kind of epitome of the medieval monk, embodying a rigorous moral stance based on strict obedience to divine law. Such a transformation, which would not have been possible without the – quite exceptional – previous approval of Pythagoras by some Church Fathers,[50] nay his "Christianization," had the paradoxical consequence of making him acceptable when, from the Renaissance onwards, he returned to the philosophical scene in a fully secularized, "heterodox" form. Robert suggests a number of reasons why late antique and medieval Christians could have considered that Pythagoras, although a pagan himself, deserved to be "saved":[51] the sincerity of his virtuous, ascetic way of life; the proximity of his thought to Christian "truths" like the immortality

periods of the West: the Middle Ages and the Renaissance, as has been shown in the masterful survey of Hasse 2016 (cf. also Bevilacqua 2018 for the period of the Enlightenment).

50 Most notably by Clement of Alexandria (Macris 2018b, 1161; Id. 2019), but also by (the young) Augustine (Feichtinger 2015; Macris 2018b, 1163). For other Christian thinkers, see Thom 2017; Macris 2018b, 1160–1165.

51 Contrary, e.g., to Epicurus (Robert 2021). Aristotle's post-mortem fate remained undecided in the Middle Ages, fluctuating between damnation (Imbach 1994) and salvation (Imbach 1995; von Moos 2013). On the problem of the salvation of the pagans more generally, see Capéran 1934²; von Moos 2014; Marenbon 2015.

of the soul and the unicity of God; his summons to "Follow God"; his humble self-presentation as a simple lover or seeker of wisdom (*philosophos*), and not as a wise man (*sophos*). So, despite a fundamental divergence of perspective, one can observe an unsuspected continuity in terms of sources: "when the first humanists are laudatory, if not hagiographic, regarding Pythagoras, when they extol his virtues, his discipline and his rectitude, they are but repeating, in a new language, backed by hitherto undetected sources, a message that was already highly structured in the Middle Ages."

One of the novel results of the *Quellenforschung* undertaken in Robert's chapter is to uncover the medieval sources which, in Western Europe, transmitted the precepts encapsulating the Pythagorean ethics or presented Pythagoras' life as a model of virtue for the Christians. The Latin and Christian environment here explains why in both cases (i.e., in the gnomologies and the biographies) the Latin Christian, or at least Roman, authors were almost exclusively favored (differently from the ultimate Greco-centrism observed in the Arabic and Middle Eastern reception of Pythagoreanism), with the exception of the gnomological collections translated from Arabic.

The precepts, maxims, aphorisms or sayings attributed to Pythagoras have been transmitted by means of three different types of collections: (1) the *symbola* or *aenigmata*, drawn from Porphyry, who, in turn, used (directly or, most probably, indirectly) Aristotle's lost monograph *On the Pythagoreans*: first included in Jerome's *Apology against Rufinus*, from the 9th century onwards these authentic "pearls" of Pythagorean ethics have been circulating independently under the misleading title *Aenigmata Aristotelis*, later provided with a moral and theological commentary mobilizing Biblical references as parallels,[52] and renamed *Aenigmata moralizata*; (2) Latin collections of *sententiae* such as the ones by Publilius Syrus or Ps.-Caecilius Balbus; and (3) the gnomological collections translated from Arabic (al-Mubashshir and Ibn Abī Uṣaybīʿa). By contrast, the *Sentences of Sextus*, although known and copied in several manuscripts produced between the 9th and 15th centuries, were not widely used, undoubtedly due to the negative verdict expressed on them by Jerome.

As regards the biographical sources on Pythagoras, in reality Western Europe would have to wait for the Latin translation of Diogenes Laertius' *Lives and Opinions of Eminent Philosophers* by Ambrogio Traversari in 1433,[53] and

52 As did Clement of Alexandria with the Pythagorean *symbola* in his *Stromateis* (book V); see the commentary of Le Boulluec 2009[2] [1981[1]].

53 One should note here the total absence of traces of Diogenes Laertius' *Lives* in the Syriac and Arabic tradition. – Traversari's translation of Diogenes Laertius will soon become also

the rediscovery of Porphyry's and Iamblichus' Lives of Pythagoras during the Renaissance[54] in order to have access to continuous narratives and substantial, systematic treatments. Until then, one had to content oneself with erratic glances or short biographical sketches inserted in encyclopedias and universal chronicles, piecemeal compilations in the context of collective biographies of illustrious men, or, at best, rich patchworks of quotations drawn from classical Latin literature like the ones found in Helinandus of Froidmont[55] or Vincent of Beauvais (13th c.), and a bit later also with collections exclusively dedicated to philosophers, such as the ones compiled by John of Wales, Benzo of Alessandria or pseudo-Walter Burley (early 14th c.).[56] What is remarkable about these sources, especially those dating from the 13th and 14th centuries, is that they are unanimous in drawing an unequivocally positive portrait of Pythagoras, elevating him to a true model of virtue – the secular equivalent of the saints of Christian hagiography –, as well as of the Pythagoreans, who appear as ideal monks.[57] In sum, the medieval collections of Pythagorean *exempla* and *dicta* not only served the purpose of supplying material for scholarly works, but also "provided clerics with a pagan mirror in which to contemplate, with shame, their own shortcomings."[58]

a source for *bons mots* of ancient philosophers (although not of Pythagoras in particular), as is shown by Martina 2019 in the case of Guillaume Tardif (*ca* 1436–*ante* 1495).

54 This happened mainly after the migration of the Greek intellectuals (and of their books) from Byzantium to Italy in the aftermath of the fall of Constantinople in 1453, but it is probable that the humanist Giovanni Aurispa had already brought to Italy a manuscript of Iamblichus' *Pythagorean summa* a few years earlier (on this, see D. Robichaud's chapter). – On *Laurentianus Plut.* 86,3, the most ancient manuscript of Iamblichus' *De vita Pythagorica*, see Giacomelli 2014.

55 Cf. Smits 1995, 14–15, according to whom the chapter on Pythagoras in Helinandus' *Chronicon*, where the data concerning the philosopher's life are gathered "from a variety of sources," well illustrates how "[n]ew lives were drawn up by scrutinizing the texts from Antiquity for biographical data."

56 On pseudo-Walter Burley and "the roads of transmission that led to the making of *De vita et moribus philosophorum*," see Copeland 2016.

57 On the "monastic" interpretation of both Pythagoras and the communities founded by him in the ancient Greek sources, see Philo's *De vita contemplativa*, describing the way of life of the Pythagoreanizing Jewish Therapeutae (where we also have the first occurrence of the term *monastērion*), Athanasius' *Life of Anthony* (the first extant Life of a Christian monk), inspired by Porphyry's *Life of Pythagoras*, or the *Sentences* of Sextus and their reception by Evagrius Ponticus. See Taylor 2003; Rubenson 2006; Bremmer 2016; Macris 2018b, 1138–1139, 1162; Prochenko 2018 and forthcoming; Taylor and Hay (ed.) 2020.

58 On the presence of Pythagorean *symbola* later, in Erasmus (1466–1536) and Rabelais (*ca* 1483/1494–*ante* 1553), see Heninger 1968 and Traninger 2012 respectively. On the adage "Friends hold all things in common," see Eden 1998 and 2001. On Rabelais see also Roose 2015, on the famous five years of silence allegedly imposed by Pythagoras on neophytes.

3 Theology, Metaphysics and the Soul

From Pythagoreanism as a lived and life-transforming philosophy we then turn to what constitutes at the same time its doctrinal foundation and its speculative apex: Part 3 deals with *Theology, Metaphysics and the Soul*.

In continuity with the preceding part, the first chapter by **Daniel De Smet** offers a comprehensive survey of the third great category of sources pertaining to the reception of Pythagorean ideas during the Middle Ages, namely doxographies (i.e., collections of philosophical opinions or doctrines). The focus here is on the doxographic reports written in Arabic, which, "with the exception of the *Placita philosophorum* attributed to Plutarch or to Aetius, are generally not translations of known Greek texts, but rather genuine compilations, based on ancient sources, readapting and transforming the borrowed material in a Neoplatonic and, above all, Islamic perspective." The main subject developed in these numerous texts dating from the 10th–12th centuries – [1] the *Book of Ammonius Concerning the Opinions of the Philosophers and Their Different Doctrines about the Principles and the Creator* ("Pseudo-Ammonius"); [2] the *Placita philosophorum* translated by Qusṭā b. Lūqā ("Aetius Arabus"); [3] the alchemical treatise *Turba philosophorum*;[59] [4] an anonymous "philosophy reader" compiled in the circle of Miskawayh; [5] the equally anonymous *Most Precious Words of the Philosophers Professing God's Oneness and of the Authorities of the Past* recently discovered in a Tehran manuscript; [6] al-ʿĀmirī's *Book on the Afterlife*; and [7] al-Shahrastānī's *Book of Religions and Sects* – is the presentation of the Greek philosophers as true monotheists who share two fundamental tenets of Islam: the unity and uniqueness of God (*tawḥīd*) and the creation of the world *ex nihilo* (*ibdāʿ*).[60] In this way doxography was put at the service of (specifically Islamic) theology, and ancient Greek philosophy was fully legitimized and perfectly integrated in an Islamic context. De Smet shows how, within this wider project, Pythagoras holds a most prominent position, especially thanks to the Ps.-Ammonius, whose influence is felt in most of the other sources. This is not surprising, if we consider the long tradition of Pythagorean(izing) speculation about the numbers of the decade (often referred back to the founder himself), in which the monad, qua first number and source of the entire series of existing numbers,

59 On the *Turba* as a "Pythagorean congress on the art of Hermes", see Lacaze 2018, with the reviews of D. Moenaert, *Kritikon Litterarum* 47/3–4 (2020): 205–216, and C. Macris, *Revue philosophique de Louvain* (forthcoming).

60 On this general trend see Wakelnig 2015 and De Smet 2019.

is especially praised.[61] Given the homology between the system of derivation of numbers from the one-monad, and the creation of the world by the One God, Pythagoras was clearly more susceptible to 'monotheization', and in a sense the perfect candidate for it. However, the Arabic authors favored religious explanations for the provenance of such a sound conception of God's unity, oneness and uniqueness: they saw it as the result of spiritual contemplation, experienced at the very end of Pythagoras' celestial ascent (which is strongly reminiscent of the Prophet Muhammad's *miʿrāj*) (Ps.-Ammonius), they attributed a prophetic-Abrahamic origin to it, via Pythagoras' contact with the "companions of Solomon" in Egypt (al-ʿĀmirī), or they explained it by transforming Pythagoras into "a monotheist sage, one of the people of Ḥarrān in Syria"[62] (Ikhwān).

In a way that echoes David Albertson's chapter on "Latin Christian Neopythagorean Theology",[63] De Smet is sensitive to the *continuity* with the Greek sources (both direct and indirect, Christian and Neopythagorean or Neoplatonic). He stresses more particularly (1) the influence of the Christian [Hippolytus'?] *Elenchos* (or *Refutatio*), either directly or through the Ps.-Ammonius, as well as of an overall Neoplatonic background, and (2) in matter of content, the authentic tension – inherited from conflicting strands already attested within Greek Neopythagoreanism[64] – between dualistic (Aetius Arabus) and monistic views. More importantly, he points to another tension, between [a] what we may call a "soft" or "immanentist" version of monism, where the ultimate unity and first principle is identical with the first of numbers, the number one or monad (Ikhwān, Aetius, commentary on the *Golden Verses* attributed to Proclus), and [b] a "hard" or "transcendentalist" one, where the ultimate One, qua Principle and uncreated Creator identified

61 It is notable (and significant) that the famous Pythagorean *tetractys* (i.e., the sequence of the first four integers), which is equally if not more venerated than the One in the Greek sources, arithmological, philosophical and other (see Pieri 2005, Macris 2018b, 831–832, Zhmud 2021 [forthcoming]), does not occur at all in the texts discussed here, apparently because its plural character seemed unfit (in the Pythagorean oath, also included in the *Golden Verses*, the *tetractys* is bound to the material world as "fount and root of the ever-flowing nature"). For the presence of the *tetractys* in other contexts in Arabic literature, see Izdebska 2020.

62 To the explanations provided for this surprising statement by A. Izdebska, D. De Smet and C. Baffioni in their respective chapters one may add that already in some of the Greek sources (cited by Porphyry) Pythagoras was presented as being of Syrian origin, or in some way connected with Syria (Iamblichus); see Macris 2018b, 781.

63 See also Albertson 2014, 21–89.

64 To the pertinent references given by De Smet in his notes 36 and 48–50, one may add Staab 2009, Bonazzi 2013 and Zimmerman (forthcoming). See also Thomassen 2000.

with the Most High God/*Allāh* of Abrahamic-Islamic monotheism, transcends numerical unity, i.e., the created unity of the monad from which all other numbers and beings are derived (Ps.-Ammonius, *Turba*, Oxford manuscript, Tehran doxography, al-ʿĀmirī, al-Shahrastānī) – for this reason the absolutely transcendent One is sometimes presented apophatically, as hidden in His essence (Ps.-Ammonius, al-Shahrastānī), not perceivable by the intellect nor by the soul (Ps.-Ammonius, Tehran manuscript).

At the same time, De Smet well brings out how, among the "variations on a theme" offered in the Arabic sources, each author feels free to reformulate, modify and adapt the received ideas concerning Pythagoras' philosophy of unity, to use new arguments or to make his own original contributions, often inspired by the Qurʾān and its interpretation by the Islamic *auctoritates*. A slow shift is thus observable, leading from Neopythagorean "number theology" to a properly Islamic "philosophy of oneness," which culminates with the particularly complex and sophisticated, but systematic, sui generis theory on the kinds of unity attributed to Pythagoras by al-Shahrastānī. But one can also trace an inverse or, rather, specular development. According to De Smet, the philosophy of unity professed by the Arabic Pythagoras really influenced more than one Muslim philosopher, mainly in the Shiʿi tradition, in their endeavor to give a philosophical foundation to Qurʾānic monotheism: for example, the Ikhwān al-Ṣafāʾ, who explicitly claimed to be the heirs of Pythagoras, or the Ismaʿili theologian al-Kirmānī who, without mentioning the Samian sage, elaborated in his *Brilliant Epistle on the Meaning of Unification, the Unifier and the Unified* a complex reflection about the concept of "one."

After this broad survey, a single but very famous *dossier* is revisited by **Carmela Baffioni**, who attempts to take a fresh and more attentive look at the relation of the Ikhwān al-Ṣafāʾ (the "Brethren of Purity") with Pythagoreanism. Going beyond a mere synthesis on the *status quaestionis* typical of a Companion volume, the author discusses Yves Marquet's hypothesis, according to which the Brethren of Purity would be the "*Pythagoriciens de l'Islam*."[65] Baffioni analyses

65 Marquet 2006. – Most recently, de Vaulx d'Arcy 2018 and 2019a stresses the "arithmetism" and "complementarism" (as he calls it) animating the Brethren's Encyclopaedia, which both, in his view, enabled the Brethren to claim a Pythagorean identity; see the (perhaps a bit excessive) criticisms of G. de Callataÿ in his review (*Studia graeco-arabica* 9 [2019]: 310–312). For earlier studies focusing on the Brethren's Pythagoreanism, see Nasr 1978², 23–104 (*passim*); Netton 1982, 9–15 (otherwise speaking of "Muslim Neoplatonists"); Straface 1987 (emphasizing the elements of the Pythagorean metaphysics of numbers in the *Rasāʾil*); cf. also Sharma 2001, 51f. ("Pythagorean philosophy"). One may add to the *dossier* the great importance attached by the Brethren to friendship (see Ep. 45, with Bonmariage 2009), as well as their high degree of sympathy for animals (Ep. 22: Goodman and McGregor [ed.] 2010, with de Callataÿ 2018, who also draws attention to

in detail the Epistles 32 and 33 of the Encyclopaedia (*Rasāʾil*) of the Ikhwān al-Ṣafāʾ, of which Paul E. Walker and his collaborators provided recently a new critical edition.[66] The two epistles are transmitted in multiple versions, so that only a philological and philosophical analysis would make it possible to investigate in more depth the link between the Brethren and Pythagoreanism. Baffioni checked Ep. 32 and 33 on manuscripts and she concludes that the text of these two epistles should be divided differently from previous printed editions. She then describes the doctrinal content of the epistles according to her own reconstruction of the text. Baffioni meticulously indicates what is specifically Pythagorean in the Ikhwān's Encyclopaedia and what belongs to other ancient Greek philosophical schools, while at the same time indicating the ideas that are proper to Islamic thought. She also quotes from other epistles (Ep. 3, 44, 45, 48, and more), and concludes that the links with Pythagoras and Pythagoreanism are in fact relatively weak and generic. In sum, the sources of Ep. 32 and 33 should be sought in the Neoplatonic tradition. Moreover, Baffioni discovered an unpublished article by the Arabist Albino Nagy (1866–1901) entitled "Archytas in the Arabic tradition." Nagy had analysed in it the chapter devoted to Pythagoras by al-Shahrastānī (d. 1153) in his *Book of Religions and Sects*, and definitely linked it to Proclus' *Elements of Theology* and to Ibn Gabirol's *Fons vitae*. Finally, Nagy claimed that the Pythagorean doctrines known to the Arabs are in fact Neo-Pythagorean or Neoplatonic. Baffioni agrees with him on this point and concludes that "the core of the Pythagoreanism of the Ikhwān al-Ṣafāʾ is the conception of the number one as the root of numbers, which makes it wholly different from all other numbers," and its use in order

the arithmological aspects of this epistle). Izdebska 2018, 882–883 aptly summarizes the situation: "There is no doubt […] that the Ikhwān's attitude towards Pythagoras was very positive, and that they presented themselves as his heirs and continuators. Yet, as has been demonstrated by De Smet 2007, their actual knowledge about Pythagoras and his philosophy remains unclear. It is also difficult to ascertain whether their Pythagoreanism contains any traces of the ancient Pythagorean tradition, since the system they present as Pythagorean is indeed genuinely Neoplatonic, and it is actually more characteristic of the Arabic Neoplatonism rather than of its late antique version." In this perspective, it is interesting to recall that al-Ghazālī (d. 1111) criticised the Ikhwān (most prominent, in his eyes, among the Baṭīniyya, i.e., the Ismailis) for their love of the "worthless" philosophy of Pythagoras; see Izdebska 2014, 39–40; La Sala 2016, 423, 428–434. On the Brethren see also the brief synthesis of de Callataÿ 2005.

66 Walker *et al.* (ed.) 2015. – For the shortcomings of the previous, Beirut edition of the Encyclopaedia, see Poonawala 2008; Hamdani 2008. – De Vaulx d'Arcy 2019c, on the contrary, is very critical of the new edition provided by Oxford University Press in association with the Institute of Ismaili Studies.

to "clarify the difference between God and created things." She also points to several textual quotations from Nicomachus in the Brethren's Ep. 1 and 6.[67]

In the next chapter, **Irene Caiazzo** deals with one of the major doctrines of ancient Pythagorean philosophy, the transmigration of souls, whose presence among medieval Latin authors has been understudied so far.[68] She begins with Nicola Baldelli, a Jesuit from the early 17th century, who asserts that a rational soul cannot be present in several human beings, either simultaneously or successively. Baldelli has in mind both Averroes, with his doctrine of the unity of the intellect, and the tenants of the transmigration of the soul, viz. Pythagoras and Plato, as well as certain heretics, such as the Albigensians and the Cathars. Baldelli relies on the *Conimbricenses* – i.e., the Coimbra Jesuit Aristotelian course –, which in turn used medieval scholastic philosophy. In fact, the doctrine of the transmigration of souls has been criticized by almost all medieval authors.[69] The sources that mention transmigration are highly scattered and heterogeneous, but this doctrine was mainly known through Calcidius' *Commentary on the Timaeus* and Macrobius' *Commentary on the Dream of Scipio*.[70]

The philosophical arguments used in order to prove Pythagoras' error and to refute him are presented by Caiazzo in some detail. At the end of the

67 On arithmology and "the occult in numbers" in the Brethren's Encyclopedia, see El-Bizri 2018.

68 This is also the case with the Byzantine authors; see Alexakis 2001.

69 The degree to which this most central and notorious doctrine of ancient Pythagoreanism caused embarrassment and was 'stuck like a fish bone in the throat' even later, during the Renaissance, can be better measured if one considers the reaction to it of an authentically Pythagorean thinker like Reuchlin: "Reuchlin […] is very concerned to protect Pythagoras from the charge of transmigration of souls. So he finds a way – counter to all ancient, late ancient, medieval, and Renaissance knowledge of the Pythagorean tradition – to argue that Pythagoras did not, in fact, believe in transmigration. […] As the real backbone of his response, Reuchlin offers the sort of argument familiar enough to early modernity: the argument from integrity. How *could* such a great man have written such nonsense? How could someone who had so carefully distinguished the forms of things – the *exemplaria* – from their transitory outward appearances – the *species* – have thought that the human essence, which is the same as its form, would have anything in common with a beast? What Pythagoras meant was that only those living beings whose reason shows that they can distinguish differences in things are the divine genus, the genus that does not suffer change. […] Pythagoras, when speaking of transmigration, was actually speaking, not about souls, but about matter, and was referring to the fact that matter always has a desire to take on new forms" (Celenza 2001b, 126–127) – On Ficino's reaction, see Gersh 2021, 234–235.

70 On these two important texts see Calcidius 2011 (with Reydams-Schils 2020) and Macrobius 2003. On the medieval reception of Calcidius' harmonic diagrams, see Huglo 2005. On the transmission of Cicero's *Somnium Scipionis* during the Middle Ages, see Caldini Montanari 2002.

11th century, Manegold of Lautenbach refutes Pythagoras' metempsychosis because it would be impossible for the human soul to deviate from rationality and become irrational once it enters the body of a brute animal. For his part, the 12th century philosopher William of Conches considers the doctrine to be a mere metaphor (*similitudo*) used for moral purposes, aimed at encouraging virtuous behavior. The moral interpretation is also adopted in the anonymous *Glosae Colonienses super Macrobium* and in the *Commentary on the Timaeus* attributed to Bernard of Chartres. From the 13th century onward, the diffusion of the Latin translation of Aristotle's *De anima* marked a turning point in the reception of the Pythagorean doctrine of the transmigration of the soul. The most articulated discussion is to be found in the *De Universo* of William of Auvergne, who, however, used mainly logical arguments to reject the Pythagorean doctrine. The first author to explicitly refer to Aristotle's *De anima* in order to counter Pythagoras is Roland of Cremona in his *Summa theologiae*. The quotation from *De anima* on "the soul which cannot enter just any body"[71] becomes almost a *Leitmotiv* in the 13th century: it is discussed in the anonymous commentaries on the *De anima* from the Faculty of Arts, by Albert the Great, Bonaventure, Siger of Brabant, and Thomas Aquinas. Nicolas Oresme is one of the very few authors from the 14th century to mention the doctrine of transmigration of souls which will not appear any more among the subjects dealt with by the commentators on the *De anima*. In contrast, it is in a non-university intellectual that we encounter Pythagoras once again. In the *De sui ipsius et multorum ignorantia*, Petrarch (1304–1374) sketches in a few lines the life and philosophy of Pythagoras, mentioning and rejecting the doctrine of metempsychosis, which he sets forth following the *Divinae Institutiones* of Lactantius (III, 18, 15–17).

Staying within the confines of Medieval commentarism on Aristotle, **Marta Borgo** and **Iacopo Costa** deliver the very first study on the presence of Pythagoras and Pythagoreanism in Thomas Aquinas' Aristotelian commentaries. Aquinas was acquainted with Pythagorean physics, cosmology, and ontology through the Latin translations of Aristotle's *libri naturales*, especially the *Metaphysics* and *On the Heavens*, where Pythagorean doctrines are discussed in detail and very often criticized.[72] Aquinas is led to discuss them, too, thanks to Aristotle. Borgo and Costa point out at the outset that the presence of Pythagoras is rather weak in the theological works of Thomas Aquinas. That is why they chose to focus on his Aristotelian commentaries. In line with the

71 *De anima* I, 3, 407b20–23. On this Aristotelian passage see Cornelli 2016.

72 On the critical discussions of Pythagoreanism by Aristotle, see Macris 2018b, 1054–1057; Álvarez Salas 2021.

doxographical tradition, Aquinas presents Pythagoras as the first philosopher and gives him a special place in the history of philosophy. He points out several times Pythagoras' pioneering attention to the immaterial causes of material things, and he connects him and his followers more closely to Plato than to any other Presocratics. In Aquinas' view, the Pythagoreans are more interested in metaphysics than any other pre-Platonic thinker. Here Aquinas proposes a teleological history of philosophy, in which the doctrine of Pythagoras follows that of the Presocratics and precedes that of Plato. But he also proposes a more unexpected, long-term connection by bringing together Pythagoras and the 11th-century Andalusian philosopher Ibn Gabirol (Avicebron). Four doctrines are consistently attributed by Aquinas to the Pythagoreans: (i) the transmigration of souls; (ii) number (and ultimately the unity) as a principle which is intrinsic to sensible things; (iii) the table of opposites that allows an overall approach to reality; and (iv) heavenly harmonies.[73] Borgo and Costa conclude that although Pythagoras and the Pythagoreans are not the main sources of inspiration in the work of Thomas Aquinas, they nevertheless feed his reflection on physics and metaphysics.

The last chapter of the third section delves into "Latin Christian Neopythagorean theology." **David Albertson** starts with the analysis of the *Centheologicon* (the Hundred-fold Theology) of Heymeric de Campo (d. 1460). Elaborating the idea of a *theologia humana*, of which he intends to follow developments and outline trends, the Flemish master speaks of *theologia pythagorica* and *theologia platonica*. For him, "Pythagorean" means precisely the theological discourse inherent in the speculative number theory, as sketched in Boethius' *De arithmetica* and further developed by medieval commentators. Heymeric writes that "the principles of numbers are the true principles of all things, and the disciplines of the quadrivium teach mathematical but also metaphysical truths. In arithmetic, unity is the root of identity, equality, and concord, and the binary is the root of alterity, inequality, and difference; in geometry, the highest equality is thought as a square, incommensurate with the circle." (D. Albertson's translation) Heymeric invents an inheritance that has never been explicitly formalized as *theologia pythagorica* or *theologia geometrica*. He could have discovered it in twelfth-century thinkers like Thierry of Chartres (d. 1157) or Achard of St. Victor (d. 1171), or very likely in Nicholas of Cusa's works. Heymeric explicitly defines the cardinal's theology as "Pythagorean." In Albertson's view, Nicholas of Cusa (1401–1464) shaped a Christianized Neopythagoreanism that

73 On these four doctrines of ancient Pythagoreanism discussed by Aristotle, see the bibliographical orientation provided in Macris 2018b: [i] 826, 833–836 and *passim*; [ii] 1054–1057, 1075–1077 (see also 831–832, 842–843); [iii] 1063–1064; [iv] 843, 1086, 1166.

had few precedents in the Latin West.[74] In the following section of his chapter, Albertson offers a survey of some highlights of the reception of Pythagoreanism within medieval Christian theology, from Augustine to Cusanus.[75] He calls this tradition "mathematical theology," which he defines as "a species of Neopythagorean henology, oriented around the coeval mediation of *Logos* and *Arithmos*, which assumes a universal *mathesis* grounded in the quadrivium." This tradition maintains that alongside the mediation of God through the Word, there is a concomitant mediation by number, since the mathematical structure of the cosmos has its origin in the Trinity itself. Albertson identifies three founding doctrines of medieval Christian Neopythagoreanism: the essence of the Trinity, the providence of the Creator, and the way to Wisdom. Each of these doctrines germinates with Boethius, grows in Thierry of Chartres, and blooms in Cusanus. Other authors are mentioned in this chapter, including Achard of St. Victor, Alan of Lille and Robert Grosseteste. Finally, Albertson puts forward several starting points for reflection on the relationship between early modern and medieval thinkers. Specifically, he suggests that the *mathesis universalis* of which Descartes speaks in his *Rules for the Direction of the Mind* (1628), a universal mathematical method, is reminiscent of Thierry of Chartres' system of four modes of being: necessity (*necessitas*) and possibility (*possibilitas*); enfolding (*complicatio*) and unfolding (*explicatio*). Other tracks have been sketched by Albertson that will arouse the curiosity and interest of readers.

4 New Trends in Early Modern Pythagoreanism

With the last section of the volume we move to the *New Trends in Early Modern Pythagoreanism*, which focuses on the Latin West.[76] **Denis Robichaud** provides an overview of Pythagoreanism in the Renaissance, broadly conceived: from Pico della Mirandola to Johann Jakob Brucker. In his 900 *Conclusiones* Pico asserts: "what the Kabbalists say, that the light deposited sevenfold shines greater than the remaining light, agrees miraculously with Pythagorean arithmetic." (D. Robichaud's translation) According to Pico, the wedding between

74 On Nicholas of Cusa's "Pythagorean theology" and on his influence on later thinkers, see also Blum 2010, 21–42, Albertson 2014, 286, n. 66 (bibliography), and the chapter of Jean-Pierre Brach in this volume.

75 For more details, see Albertson 2014.

76 See the brief but useful overview of Neumann 2016. – For some recent explorations in the Eastern reception of Pythagoreanism during the same period, especially in Persia, see Melvin-Koushki 2017 and 2020; Terrier 2019. For the Ottoman empire, Melvin-Koushki 2021.

the Kabbalah and Pythagoreanism allows the human mind to mystically ascend to the truth of the divine mysteries of the Bible and the nature of the cosmos.[77] Despite the ecclesiastical condemnation, several of Pico's theses were quite successful among the northern Humanists. In the *Augenspiegel* (1511), Johann Reuchlin approves of Pico's Pythagorean Kabbalah, quoting directly from the 900 *Conclusiones*.[78] Likewise Jacques Lefèvre d'Étaples, in his *De magia naturali* (1492–1495), keeps trying to delve into Pythagoreanism with the Kabbalah.[79] In 1496 Lefèvre is also the editor of a volume including Boethius' *De arithmetica*, Jordanus de Nemore's *Elementa arithmetica*, and a *Rithmomachia* (which in fact corresponds to the medieval lore of arithmetic[80]). For his part, Peter Ramus (1515–1572) intends to dissociate ancient Pythagorean mathematics from the Kabbalah, which is why he undertakes the analysis of the sources of Pythagoreanism with the intention of rewriting its history. Ramus wants to put mathematics in university curricula, relying on the historical figure of Pythagoras, the mathematics teacher[81] and school reformer. Robichaud points to the multiplication of Pythagoreanism*s* in the Renaissance and Early Modern period, just as it had happened to ancient Pythagoreanism.[82]

The three principal lives of Pythagoras – Diogenes Laertius', Porphyry's and Iamblichus' – were rediscovered in the Latin West during the Renaissance. Although excerpts from the *Life of Pythagoras* by Diogenes Laertius had been translated in the 12th century, it is Ambrogio Traversari's translation that became popular and had a wide circulation.[83] A number of translations followed between the 15th and 17th centuries, meticulously listed by Robichaud.

In order to make more palpable the profound change of attitude noticeable from the second half of the 16th century onwards, Robichaud takes over from A. Robert's chapter to study how modern scholars, progressively adopting the critical methods of historical-philological analysis, take a much more skeptical

77 On the *scientia cabalae* in Pico see now Buzzetta 2019. On the role played by the Italian Jewish humanist scholar and Christian convert Flavius Mithridates in the "wedding" between the Kabbalah and Pythagoreanism, see Martini 2019.

78 On the mixture of Kabbalah and Pythagorean philosophy in Reuchlin see Spitz 1956; Idel 2014.

79 On Lefèvre see also the chapter of Jean-Pierre Brach.

80 On this mathematical game see Folkerts 2003.

81 On Pythagoras' real involvement in mathematics see Zhmud 2012, 270–274.

82 For such a pluralistic understanding of ancient Pythagoreanism see Zhmud 2012, Macris 2018b, 1025–1089 (*passim*), as well as the numerous Pythagorean entries of the *Dictionnaire des philosophes antiques* (Goulet [ed.] 1989–2018), mostly due to B. Centrone and C. Macris.

83 On the medieval *Lives* of Pythagoras see more extensively Aurélien Robert's chapter in this volume.

stand *vis-à-vis* the authority of the ancient sources. Joseph Scaliger (1540–1609) studies Iamblichus' *Life* in Arcerius' 1598 edition and adds critical notes to it (see Robichaud's transcriptions). The historical truth of Iamblichus' account is seriously questioned, and one easily arrives at the conclusion that there are too many inconsistencies in the Neoplatonist's chronology of Pythagoras' life.[84] The English scholars Henry Dodwell (1641–1711), Bishop William Lloyd (1627–1717), and Richard Bentley (1662–1742) go in the same direction by trying to discover contradictions in the ancient biographies of Pythagoras. Finally, it is the traces of these three innovative scholars that Jakob Brucker will follow when, in his *Historia critica philosophiae* (1767), he intends to detect authentic Pythagoreanism behind the multiple ancient and Renaissance sources. Brucker is particularly interested in Diogenes Laertius, Porphyry, and Iamblichus. According to him, Porphyry and Iamblichus are pitiful historians, who, moreover, have added stories to make Pythagoras a kind of wise miracle-worker rival to Jesus Christ.[85] Brucker finds discrepancies in the chronology of Pythagoras' life, especially in Iamblichus. He doubts the many journeys of Pythagoras and the teachings he would have received from Egyptian, Babylonian, Hebrew, and Chaldean sages. His purpose is to purge ancient and modern Pythagoreanism from "oriental" religions and wisdom. That is the trajectory followed by the early modern reception of Pythagoreanism, from the enthusiasm of the Renaissance thinkers to the source criticism of the historians of philosophy in the 18th century.

A similar development is put forward in the chapter by **Jean-Pierre Brach**, which closes the volume, and which offers an overview of Pythagorean number mysticism during the Renaissance. In this way, in a kind of ring composition, we return to the subject matter to which Part 1 of the volume is devoted. In the "guided visit" offered by Brach to a "portrait gallery" containing more than twenty sketches of thinkers involved in speculation about numbers, we move on from a bold rehabilitation of the number symbolism inherited from Greek Neoplatonism (Ficino), and its "wedding" with the Jewish kabbalah and natural magic (Pico, Reuchlin) during the Early Renaissance to the "disenchantment"

84 On these serious contradictions see also, from a modern perspective, Brisson and Segonds in Iamblichus 2011^2, xxviii–xxxii.

85 Brucker's criticism recalls the way the Neopythagorean Apollonius of Tyana was viewed by ancient Christian authors like Eusebius (in his *Against Hierocles*); see Baur 1832; Dzielska 1986; Hahn 2003; Hägg 2004, and for the perpetuation of this view during the Renaissance, Dall'Asta 2008.

of the late 16th and 17th centuries[86] (not without some backward steps to a medieval understanding of numbers as, e.g., in the case of Clichtove).

Already in Marsilio Ficino's seminal view, number acts as both a cosmogonic and an ontologically productive principle,[87] and numbers are conceived as parts of a more general worldview, based on correspondences between the different levels of reality. But it is Pico della Mirandola who really revolutionized the field of arithmology: indeed, his claim to have rediscovered number symbolism as a "new way of philosophizing by numbers" does not seem exaggerated at all. (1) Arithmology, with him, ceased to be mainly a hermeneutical tool in Biblical exegesis, to become an almost autonomous "art of numbers" (thus giving later, e.g., to Bovelles, the possibility to have Scripture and Christian theology conspicuously absent from his work). (2) The combination of arithmology with the kabbalah opened a "new" and long-lived tradition in number mysticism, from Reuchlin to Francesco Zorzi and Alessandro Farra. (3) The concept of "formal number" was introduced, whereby mathematical entities possess a specific (intermediary) ontological status (in a similar vein, J. Lefèvre d'Étaples will later assimilate numbers to "ideas"). Reuchlin, who in his *De arte cabalistica* explicitly stated his aspiration to "bring out the reborn Pythagoras," ingeniously pointed to the correspondences between (a) the Pythagorean *tetractys* and the Jewish *tetragram* (containing the unpronounceable name of Yahweh), (b) the Decad and the ten *sefirot*, as well as (c) between the pentagram and the name of Jesus, thus concluding that both Pythagoras and the kabbalah ultimately drew from the same Mosaic source.[88] Less familiar is perhaps the so-called "French school" of arithmology (16th century), known for its speculative approach to numbers (and to mathematics), in which one can find tendencies ranging from natural magic (Lefèvre, Bovelles) to Christian mysticism (Clichtove, Roussel). It is of crucial importance that in Lefèvre's hitherto unpublished *De magia naturali* the numbers play a practical role in a natural magic of an astral kind, which tries to "draw down" the celestial influences and benefit from them, while at the same time the *scala numerorum* paves the way

86 On the transformation of Pythagoras the number philosopher "from pious to polite," see Oosterhoff 2013, whose focus is on French Renaissance mathematics.

87 On Ficino's cosmology see most recently Rutkin and Robichaud (ed.) 2020. On Ficino's and Patrizi's theories on music and the cosmic order, see Prins 2015.

88 In a similar vein, ancient Jewish and Christian sources, as well as al-ʿĀmirī *apud Arabes*, brought Pythagoras in contact with "the companions of Solomon" and "the niche of prophecy" on the biographical level; see the chapter by A. Izdebska. The dependency theme (on which see Macris 2019, 770–771 with n. 17) also recalls Plato's ambiguous designation as an "Atticizing Moses" by the pagan philosopher Numenius in the 2nd century CE.

(as in Bovelles) to the anagogical movement of the spirit towards the supreme unity of the Divine.

From a literary point of view, number speculation (differently from late Antiquity when literature on the Decad or on arithmological *theologoumena* flourished[89]) began its life rather discreetly, in the guise of scattered thoughts in Ficino or sporadic developments in Pico, before undergoing a process of systematization. This happened, first, in the context of a wider project, as in Reuchlin's three books on the art of the kabbalah, Pacioli's *On Divine Proportion* (*sc.* the "golden ratio"), Roussel's number symbolical commentary on Boethius' *On arithmetic*, Zorzi's vast *exposé* of the mysteries of creation (*De harmonia mundi*), Agrippa's treatise on magic (*On Occult Philosophy*) and John Dee's preface to the first English translation of Euclid's *Elements*.[90] On the other hand, systematicity is characteristic of works entirely devoted to number speculation, which often displayed encyclopedic ambitions: this is the case, e.g., of Bovelles' *Book of the Twelve Numbers*, Clichtove's *Treatise on the Mystical Meaning of Numbers in the Bible*, Bruno's *On the Monad, Number and Figure*, Bongo's *Mysteries of Numbers*, Borromeo's treatise *On the Pythagorean Numbers*, Jan van Meurs the Elder's *Pythagorean Decad* and Kircher's *Arithmology*. Some of these works may even study a given symbolic or religious theme by means of a monograph devoted to a single number of the decad, usually "seven": here belong Farra's *Septenary of the Human Conversion* and Paolini's *Hebdomads, or Seven Books on the Septenary*.

Lively and long-standing though it used to be, with the advent of Modernity this Pythagorean tradition of number symbolism gradually became scientifically irrelevant, as "number found itself reduced to the status of a mere logical operator, devoid of any reference to a living, inner essence of things," thus transforming Pythagoras from an exponent par excellence of *prisca theologia*[91] – endowed with a prophetic status, and at the same time incarnating the ideal of the *homo univeralis* – to a "stranger in a strange land." But in the meantime the sage of Samos gave extraordinary inspiration to the fascinating quest for the mystical harmonies of numbers and of geometrical proportions that lie hidden behind the "veil of Isis" enfolding the natural world,[92] thus providing support both to the scientific mind and to the mystic in his effort to ascend.[93]

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89 See the references given in the notes 8–9 of J.-P. Brach's chapter.

90 On his *Monas hieroglyphica* see Clucas 2010.

91 On this notion, see, e.g., Walker 1954; Vasoli 2005; Gentile 2012; Toussaint 2014.

92 Cf. P. Hadot 2006.

93 The Pythagorean inspiration of the 'New Science', from Copernicus and Bruno to Galileo, Kepler and Campanella – a huge and fascinating subject deserving, in our view, a separate treatment – has been emphasized times and again in modern literature. See (in

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alphabetical order) Biliński 1977; Casini 1994a, 1994b, 1996 and 1998; Ernst 2015; Field 1988; Gatti 1999 and 2011; Gatti (ed.) 2002; Giovannozzi 2012; Heninger 1965; Macris 2018a, 665–666; Martínez 2011, 2014, 2018 and 2019; Matteoli 2016; Montano 2004 and 2013; Omodeo 2014; Peruzzi 2014; Redondi 1983 [= 2009, 294–304]; Rossini 2018; Rothman 2017; Schaffer 2004; Tessicini 2007.

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PART 1

Pythagorean Number Theory and the Quadrivium

CHAPTER 1

Pythagoras and the Quadrivium from Late Antiquity to the Middle Ages

Cecilia Panti

The present paper has two main goals. The first is to illustrate the two most influential portraits of Pythagoras as a philosopher, described in such a way *because of* his research on the four mathematical disciplines. These portraits are by Nicomachus and Boethius, and special attention is paid to how, for Boethius, Pythagoras "invented" music. The second aim is to consider how the idea of Pythagoras as the father of the quadrivium was transmitted from Antiquity to the Middle Ages up to the thirteenth century, when the impact of the Aristotelian natural philosophy fostered an interpretation of Pythagoras' theory of numbers as one which was autonomous from the quadrivial frame. This paper deals only with references concerning Pythagoras, not "the Pythagoreans"; nonetheless, it does not enter into the complex question of whether the historical Pythagoras was really a mathematician.[1]

1 Introduction

It is well known that the history of Pythagoras and of his thought is essentially that of his legend.[2] All portrayals of Pythagoras coming from the past, however, dwell on a sort of twofold dimension, the scientific one and the religious one, which nourished his representations as philosopher and shaman, rational mathematician and "inspired mystic."[3] This dualism, united to the *ipse dixit*

1 The view that associates the historical Pythagoras with mathematics has been defended in recent times by Khan 2001, and Zhmud 2012 with various arguments, among which Plato's statement (*Rep.* 7, 530d) that harmonics and astronomy are "sister sciences, as the Pythagoreans assert," which recalls Archytas (fragment 1), who describes astronomy, geometry, arithmetic, and music precisely as "sister studies" (see Khan 2001, 13). Other scholars, including Huffman (2018) refute the arguments about Pythagoras' real association with mathematics. See also Netz 2014, and for more bibliography Macris 2018, 842–843 (cf. also 1078–1086). I wish to thank Constantinos Macris for his precious improvements of the first version of this paper and of its bibliography.

2 See, for instance, O'Meara 1989; Macris 2018, 768–850, esp. 810–818 (with bibliography).

3 See Cornelli 2016, 51, note 3, referring to E. Frank's view of Pythagoras.

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that justified everything the Pythagoreans attributed to their master, impeded his concrete association with any specific philosophical content.[4] This is true also with regard to the link between Pythagoras and the liberal arts of arithmetic, geometry, music and astronomy, namely the four disciplines of mathematics commonly known, after Boethius, as the quadrivium. Cicero (*Resp.* I, 16) refers that in many passages of Plato's works Socrates, following the methods of Pythagoras, tries to combine the consideration of arithmetic, geometry and harmony with other subjects; moreover, a link between Pythagoras and each of the four *mathemata* was established by Varro and others, as we will see hereafter. In the second century AD, however, the mathematician and Neopythagorean philosopher Nicomachus of Gerasa turned Pythagoras' reputation as a mathematician into his eminent role in the quadrivium. The Nicomachean view of the sage of Samos, in turn, reached the Latin Middle Ages mainly thanks to Boethius' *De institutione arithmetica* and *De institutione musica*.

Rediscovered and studied in the Carolingian age, these two Boethian works contributed to associating the figure of Pythagoras to the quadrivium as such, and particularly to the "invention" of music, a tenet that was regarded as being true up until the late Renaissance. Conversely, Pythagoras' special connection to the invention and/or development of arithmetic, astronomy and geometry, taken individually, grew in importance in different periods:[5] while arithmetic was the foundation stone of Nicomachus' view of Pythagoras, his link with geometry played a pivotal role in Proclus and in ancient doxography, as well as in Vitruvius' *De architectura*, in which Pythagoras is associated to a demonstration based on theorem 47 of Euclid's *Elements*, book 1, now known as Pythagoras' theorem. Astronomy, in turn, was connected to Pythagoras mainly because of his alleged discovery of the ratios of the music of the spheres, which had popularity in the High Middle Ages thanks, chiefly, to the astronomical section of Pliny's *Historia naturalis*.[6] From the thirteenth-century, however, the introduction of the Aristotelian writings of natural philosophy in the Latin world suggested a new interpretation of Pythagoras' mathematics, which supplanted its quadrivial frame with a sort of "atomistic" ontology of numbers, regardless of the fact that Aristotle did not attribute the idea that the substance of reality is number to Pythagoras himself, but to "the so-called Pythagoreans."

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4 The *ipse dixit* principle was strongly criticized by Cicero 1967 (*De natura deorum* I, 10–11).
5 For earlier, non-systematic attributions, see Zhmud 2012, 270–274, 285–293.
6 See Burkert 1961, 28–43 ("Ein System der Sphärenharmonie"), *passim*; Richter 2006, 555–564.

2 The Ancient Doxography Concerning Pythagoras as a Mathematician

Pythagoras is famous for being a mathematician mainly because of the theorem named after him, his discovery of the ratios of consonances and their application to the music of the spheres. However, neither Plato nor Aristotle, who are among the most ancient sources for Pythagoras, referred directly to him as a mathematician, but rather as the founder of "a way of life," as stated in Plato's *Republic* (600a). Scholars are still debating whether the first successors of Plato were or were not responsible for tracing back to Pythagoras the late teaching of their master and its mathematical foundation.[7] In *Metaphysics* book 1, Aristotle connects "the so-called Pythagoreans" to the doctrine of the limit and the unlimited, and suggests that Plato followed this theory, to which he added the principle of the dyad (*Metaphysics* 987b26). However, the view that the harmony of human beings and of the universe could be instantiated in the physical expressions of symbolic numbers, such as the dyad, the tetractys and so on, gradually emerged from the ancient records about Pythagoras. Perhaps, the earliest indirect evidence is by Aristoxenus of Tarentum (4th c. BC), the first Greek authority for music theory, who came into contact with the Southern-Italian Pythagoreans before becoming a disciple of Aristotle. According to the *Elenchos* (*Refutation of All Heresies*) attributed to Hippolytus of Rome (170–235 AD), Aristoxenus, in agreement with Diodorus of Eretria, said that Pythagoras accepted a "dual" feminine/masculine explanation of reality:

> Diodorus of Eretria and Aristoxenus the musician say that Pythagoras traveled to Zaratas the Chaldean [i.e., Zarathustra]. He expounded to Pythagoras that, at the beginning, there were two causes of existing things: Father and Mother. Now the Father is light, and the Mother is darkness. The constituents of light are heat, dryness, lightness and quickness; those of darkness are cold, moisture, heaviness and slowness. From these the entire cosmos came together, namely, from female and male principles.[8]

7 For this debated question, see Huffman 2018 and his careful historiographical reconstruction.

8 See fr. 13 in Wehrli 1945, 11; English translation by Litwa in [Hippolytus of Rome] 2016, 19; see also Pasquinelli 1958, 74; on the unknown Diodorus, see Flamand 1994; Zhmud 2012, 88–89; Macris 2018, 742. On Aristoxenus as a source here, see Huffman 2012, 132–133 and Id. 2014a, 289 n. 18, *contra* Zhmud 2012, 88–91; cf. Macris 2018, 719–722, 1060–1061 (on Aristoxenus and Hippolytus respectively).

In addition to this, Aristoxenus was credited also with believing that Pythagoras elevated arithmetic "above the needs of traders by likening all things to numbers."[9]

The Roman doxography and encyclopedic literature, which strongly influenced the early medieval scholars, deepened the nexus between Pythagoras and numbers.[10] Varro (116–27 BC) attributed to Pythagoras the medical, musical and arithmetical theory of the two types of pregnancy, of seven and ten months respectively, which caused changes in the fetus and, if compared to the numerical series associated to them, determined the ratios called "voices" or "harmonies" – as reported in Censorinus' *De die natali* (238 AD).[11] Also Cicero's *De natura deorum* was probably influenced by Varro in referring to Pythagoras' doctrines, including that of the world harmony.[12] Surprisingly, Pythagoras is not mentioned in the *Somnium Scipionis*, in which the mathematical harmony of the cosmos is presented on the basis of Plato's *Timaeus* and *Republic*. Cicero, however, was well aware of a link between Pythagoras and mathematics, as he reports that "Pythagoras made a new discovery in geometry, for which he sacrificed a cow to the Muses."[13] Cicero does not specify which discovery it was, but the *De architectura* by Vitruvius (*ca* 80 BC–15 AD) connects it to a demonstration of the theorem that, in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Vitruvius does not refer to Euclid (*Elements*, book 1, theorem 47) as the source of the theorem, but to Plato's *Meno*, and tells how Pythagoras used this theorem for proving that a right angle can be formed without the aid of the approximate squares of the carpenters, but from "reasoning and method" (*rationibus et methodis*).[14] In agreement with Cicero's source, Vitruvius also mentions Pythagoras' sacrifice of the cow to the Muses, who had guided him in this exceptional discovery.[15]

Other statements of Pythagoras' proficiency in mathematics are to be found in the *Historia naturalis* by Pliny the Elder (23–79 AD). Here, the philosopher

9 See fr. 23, from Stobaeus, in Wehrli 1945, 14; for discussion of its meaning, see also Zhmud 2012, 261; Huffman 2018.

10 See for instance Kahn 2001, 88–93.

11 For a wide discussion about this passage, see Restani 2007, with reference to *De die natali*, IX, 331. See also Delatte 1930, and most recently Barker 2016.

12 Cicero 1967, 312 (*De natura deorum* III, 27); see also *De re publica* I, 16.

13 Cicero 1967, 372–374 (*De natura deorum* III, 88). For Diogenes Laertius' reference to the episode of the discovery see below, at note 38.

14 Vitruvius 1912, 196 (*De architectura* IX, 4). For Vitruvius, Pythagoras used three rules of three, four and five feet each, which necessarily form a square triangle, since the sum of the areas of the squares on the sides of three and of four feet equals that of the square on the side of five feet.

15 Ibid., 197 (*De architectura* IX, 6 and 7).

of Samos is linked to music and astronomy for several reasons, including the identification of the diapason/octave as the basis of cosmic harmony and the individuation of two series of planetary distances, the one expressed in terms of spatial measures, and the other by means of tones and semitones performing an octave. Pliny states that Pythagoras established the distance from the Earth to the Moon as being 126,000 *stadia*, from the Earth to the Sun the double, and from the Sun to the twelve signs three times this same distance.[16] Then, he asserts that Pythagoras named "tone" the Earth-Moon distance, "half-tone" Moon-Mercury, and so on, for a sum of seven tones (the octave), called "diapason harmony."[17] The tone, therefore, was the unitary value equal to 126,000 *stadia* which was assumed as the measure of both the acoustic and the spatial/astronomical distances. This is also reported later by Martianus Capella, Censorinus, Hyginus and, after them, by several other medieval scholars. Pythagoras is associated by Pliny with number symbolism, too, according to which an odd number of vowels in the name given to infants caused them infirmities on the right side of their body, while an even number of vowels caused them the same on the left side.[18]

Other computations of Pythagoras are reported by Plutarch (46–125 AD), followed by Aulus Gellius (125–180 AD), such as the height of Hercules and the course of the Moon.[19] Pythagoras' arithmology can be also found in the *Institutio Oratoria* by Quintilian (40–96 AD), who mentions, *inter alia*, that for Pythagoras and his followers "the universe is constructed on the same principles which were afterwards imitated in the construction of the lyre" and "out of the concord of discordant elements which they name harmony."[20] Quintilian also recalls the musical expertise of Pythagoras in calming young men who "were led astray by their passions" by submitting them to proper melodies,

16 Pliny the Elder 1906, 83 (*Historia naturalis* II, 19): "Pythagoras vero, vir sagacis animi, a terra ad lunam CXXVI stadiorum esse collegit, ad solem ab ea duplum, inde ad duodecim signa triplicatum." Pliny, however, in the subsequent chapter states that the Moon is equally distant from the Sun and the Earth. See Richter 2006, 557–567.

17 Pliny the Elder 1906, 84 (*Historia naturalis* II, 20): "Sed Pythagoras interdum et musica ratione appellat tonum quantum absit a terra luna, ab ea ad Mercurium dimidium spatii et ab eo ad Veneris, a quo ad solem sescuplum, a sole ad Martem tonum [id est quantum ad lunam a terra], ab eo ad Iovem dimidium et ab eo ad Saturni, et inde sescuplum ad signiferum; ita septem tonis effici quam διὰ πασῶν ἁρμονίαν vocant hoc est universitatem concentus." *Sescuplum* is a tone and a half.

18 See *Historia naturalis* 28, 6. Pliny refers several times (for instance ibid., 25, 5) also to Pythagoras' knowledge of botany, and ascribes to him a treatise on the properties of the plants.

19 Gellius 1903 (*Noctes atticae*, I, 1 and 20, 1).

20 Quintilian 1854, vol. 1, 47–48 (*Institutio oratoria* I, 10.12–13).

which, still in Quintilian's account, should be of spondaic measure.[21] Another relevant assertion by Quintilian is that Pythagoras did not desire to be called "a wise man," like the sages who preceded him, but rather "a student of wisdom" (*studiosum sapientiae*), i.e., a philosopher, a term which he was reputed to be the inventor of.[22] Now, it was precisely the link between the "invention" of philosophy and its foundation on mathematics which grounded the portrayal of Pythagoras by Nicomachus of Gerasa.

3 Nicomachus' Pythagoras: The Father of Philosophy and His "Four Ways" (*Tessares Methodoi*)

In the writings of eminent philosophers of Late Antiquity, such as Iamblichus and Proclus, the mathematician Nicomachus of Gerasa (*ca* 60–120 AD) had the reputation of being a "true" Pythagorean. This assured the popularity of his works in Neoplatonic schools and of his understanding of the place of Pythagoras in the development of science and philosophy.[23] It is exactly the relation between mathematics and philosophy which turned Pythagoras' reputation as a mathematician into his eminent role in the quadrivium.

Nicomachus' *Introduction to Arithmetic* opens with a sort of preface (chapters 1 to 6), which explains the relation between the four mathematical sciences (arithmetic, music, geometry, astronomy) and philosophy, as well as the role of number in cosmogony. Nicomachus states that Pythagoras defined philosophy "as a desire for knowledge (*sophia*) of being." The science of being is about "true being," i.e., "those things which always continue uniformly and the same in the universe and never depart even briefly from their existence."[24] This statement is supported by evoking Plato's *Timaeus*:

> What is that which always is, and has no birth, and what is that which is always becoming but never is? The one is apprehended by the mental processes, with reasoning, and is ever the same; the other can be guessed at by opinion in company with unreasoning sense, a thing which becomes and passes away, but never really is.[25]

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21 Ibid., vol. 2, 128 (*Institutio oratoria* IX, 4.12–13).

22 Ibid., vol. 2, 240 (*Institutio oratoria* XII, 1.19).

23 For what follows, see in particular O'Meara 1989; Huffman 2018, and, for a wide discussion of the ancient sources, Zeller-Mondolfo 1950.

24 Nicomachus 1866, 2 (*Introductio arithmetica* I, 1.2).

25 English translation by Hicks 2017, 76–77; the passage is *Timaeus*, 27d6–28a4.

According to Nicomachus, therefore, Pythagoras was the true philosopher because he discovered "what always is" by shifting from the realm of what "never really is" up to the higher truths of the mind. This shift did not occur by revelation or intuition, but thanks to an intermediate discovery, namely that of the physical existence of special objects which are immutable and fixed for the mind, but not in their physical instantiation. These are magnitudes, qualities, relations and so on, their status being that of "immanent forms of being":[26]

> The bodiless things, however, of which we conceive in connection with or together with matter, such as qualities, quantities, configurations, largeness, smallness, equality, relations, actualities, dispositions, places, times, all those things, in a word, whereby the qualities found in each body are comprehended – all these are of themselves immovable and unchangeable, but accidentally they share in and partake of the affections of the body to which they belong. Now it is with such things that "wisdom" is particularly concerned, but accidentally also with things that share in them, that is, bodies.[27]

These unchangeable forms or categories are the indispensable premise for selecting the one that pertains at most to mathematics, namely quantity. Following Aristotle's *Categoriae*, chapter 6, Nicomachus divides quantity into magnitude and multitude: magnitude is continuous with its constitutive parts, as, for instance, in a tree or a stone, and is equivalent to a geometrical figure, while multitude consists of many parts, as in a flock or a wood, and is equivalent to number. Thanks to this division, the forms of quantity eventually give birth to an exact correspondence to the four mathematical sciences: arithmetic studies multitude, or number in itself; music studies number in relation to something else; geometry deals with immovable magnitude; and astronomy with movable magnitude. These four branches of learning are, in fact, "four ways" (*tessares methodoi*), later translated by Boethius as *quadruvium* ("four paths"), which were fundamental in the educational system by Plato, who

26 Various solutions have been advanced to delineate the relation between Forms and numbers in Nicomachus. Here, I follow Helmig 2007, 130: "My main thesis is that Nicomachus does not make a clear-cut distinction between Forms and numbers and that he does not anticipate later Neoplatonic discussions on the issue. [...] on a cosmic level (in the demiurgic paradigm) Forms are replaced by number. However, the objects of philosophy/arithmetic, as described in the *Introduction*, are universals in things or immanent Forms."

27 Nicomachus 1866, 2 (*Introductio arithmetica* I, 1.3–4); translation from Helmig 2007, 132.

refers to them in the *Protagoras*, *Republic*, the *Laws*.[28] This conclusion was essential to the connection between Pythagoras' search for philosophical wisdom and the quadrivium.

Nicomachus not only led us to consider Pythagoras as the father of the quadrivium as such, but offered a concrete example of how he discovered the immutable forms of being starting from the instable perception of mutable things. This example, presented in his *Handbook of Harmonics* (*Harmonikon Encheiridion*), had an everlasting fortune in late Antiquity and the Middle Ages, particularly after it was considered by Boethius to be the main proof for the foundation of the *scientia musicae*. It is the story of Pythagoras at the blacksmith's shop, which narrates how he established the numerical quantity of the intervals *diatessaron*, *diapente*, *diapason* and *epogdoon* (i.e., the fourth, fifth, octave, and tone) and how they determine the two tetrachords forming the octave-system. Pythagoras is portrayed as the philosopher in search of a reliable instrument to assist the sense of hearing, thanks to which he might move from the perceptive realm of sounds towards their immutable being, i.e., quantity "in relation." Nicomachus describes at first how physical music is nothing but imitation of the ratios of planetary measures, whose concord contains "all *harmonia*";[29] then, by means of the story he narrates, he assigns the invention of the complete octave system to Pythagoras in order to ground the sensible musical concord as an imitation of the Timaean harmony of the universe. In so doing, Nicomachus offers "an attempt to underwrite the credentials of the 'Pythagorean diatonic octachord,' first from the point of view of mathematics and metaphysics, and secondly from the facts of musical history."[30]

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28 On the origin of the quadrivium, see Merlan 1975, 88–95. In Plato's *Republic* (522c–617b) the sequence of the mathematical disciplines is: arithmetic, namely the study of the unit (defined as "a point without position"), numbers and calculation; geometry, which deals with one-dimensional lines and two-dimensional surfaces, up to solid geometry (stereometry); astronomy, in which the solids are in motion, i.e., in the circular revolutions of the heavens; and harmonics, which studies these motions according to numerical ratios. The decisive mention of the four branches as a unity – already mentioned in Archytas' fragment 1 (see above, note 1) – and its identification with philosophy is in the late, perhaps spurious, *Epinomis* (991 D–E), from which Nicomachus likely took inspiration. On this see Alesse and Ferrari (ed.) 2012, and especially, in the same volume, Giardina 2012 and Petrucci 2012.

29 Nicomachus 1895, 241–242 (*Enchiridion*, 3); English translation in Barker 1989, 250–253.

30 Barker 1989, 246. In this regard, it can be mentioned that the tetractys plays a relevant role, being the basis of the number series in the first four numbers (1 + 2 + 3 + 4 = 10), which display the principal concordant ratios in sounds and in the cosmos. On the tetractys see ibid., 30, n. 3.

In chapter 6, Nicomachus reports these "facts."[31] Pythagoras was in search of a reliable tool "for the hearing which would be consistent and not prone to error." One day, "by divine providence," he passed a blacksmith's shop. There, he listened to hammers beating the iron and producing sounds, which were partly consonant to each other, and partly dissonant. The wise man immediately realized that the consonances corresponded to the intervals of octave, fifth and fourth, while the dissonances to the tone. He experienced how these combinations were due to the weight of the hammers by asking the blacksmiths to change hammers with one another, and he thereby realized that the consonances did not change. Once at home, Pythagoras assembled different instruments, such as bells, strings etc., to reproduce the intervals, up to the final assessment of the *chordotonon*, a device for regulating the tension of the strings, thanks to which he eventually fixed the mathematical ratios of musical intervals already represented by means of the weight values 12, 9, 8 and 6. In fact, the combinations of these values, reduced within the first four natural numbers, express the consonances of octave (12:6 = 2:1), fifth, (12:8 = 3:2), and fourth, (12:9 = 4:3), plus the dissonant tone (9:8). For Nicomachus, Pythagoras' interest in these experiments was to prove that the mind knows the stable property of sounds, namely the "form" of quantity, by starting with experiencing it in natural things: while musical sounds are in themselves subject to change, the form of quantity expressing them, in fact, remains the same.

4 Pythagoras' Quadrivium in Later Neoplatonic Accounts

Among the Neoplatonic philosophers who were inspired by Nicomachus, the Syrian Iamblichus (*ca* 250–330 AD) gave Pythagoras and the quadrivium a pivotal role.[32] In agreement with Nicomachus, his *On the Pythagorean Way of Life* presents the Samian sage as the symbol of the philosophical life and the founder of the educational method based on the four mathematical disciplines, which disclose the primary forms and causes of reality. For Iamblichus, the task of mathematics is to know these forms, while the last step of learning is to connect mathematics with theology.[33] Iamblichus follows Nicomachus also for the story of the blacksmiths' hammers (in chapter 26 of *On the Pythagorean Way of Life*); nonetheless, he went beyond his source with the intention to achieve a great work, which would exemplify the intrinsic relation between

31 Nicomachus 1895, 245–248; English translation in Barker 1989, 256–258.

32 O'Meara, 1989, 44–46; Romano 1995, 38–41.

33 O'Meara 1989 and 2007.

Pythagoras and Plato. In so doing, Iamblichus follows a plan of ten books which, after a presentation of the Pythagorean school and an exhortation to philosophy (both extant), is entirely centered on the quadrivial sciences, dealt with in books the majority of which did not reach us:[34]

Introduction to Pythagoreanism:
- Pythagoras and his School: *The Life of Pythagoras* (book I)
- Exhortation to practice the Pythagorean philosophy in general: *Protreptic* (book II)

Introduction to Pythagorean mathematics:
- *On General mathematical Science* (book III)
- Arithmetic
- in itself: *Introduction to Nicomachus' Arithmetic* (book IV)
- in relation to physics (book V)
- in relation to ethics (book VI)
- in relation to theology (book VII): *Theological Principles of Arithmetic* (perhaps spurious)
- Geometry (book VIII)
- Music (book IX)
- Astronomy (book X)

Arithmetic, geometry, music and astronomy constitute mathematics, whereas physics, ethics, and theology, which correspond to the three parts of philosophy, are related to arithmetic alone, the first and highest of the quadrivial disciplines. Arithmetic is the basis of the other three disciplines, though it is the most speculative of them. Music is fundamental from the first stage of the system of education, because appropriate songs and rhythms harmonize human temperaments and heal passions and diseases. Astronomy and geometry, in turn, are connected by Iamblichus to Pythagoras' introduction to mysteries, during his Egyptian stay.[35]

34 Romano 1995; see also Brisson 2012, 38 and Macris 2009.

35 Contrary to Iamblichus, Proclus favored geometry over arithmetic as the privileged discipline, since "geometry better fulfills the anagogical potential of mathematical beings" (Albertson 2014, 67). Regarding music, the correspondences between melodies and human moods (*ēthē*) such as anger or courage, as well as the use of music as allopathic therapy are attested in the episode of the drunken man excited by a Phrygian melody by *aulos* and appeased by music played by Pythagoras. This story, which likely originated with Aristoxenus (cf. Provenza 2012), was narrated, among others, by Quintilian, Iamblichus' *On the Life of Pythagoras*, pseudo-Plutarch's *De musica*, and, among the

This consideration of the human capacity to harmonize soul and body reveals the affinity of the human being with the macrocosm and of music with astronomy, nourishing a long-lasting Pythagorean and Platonic tradition, and expanding the authority of Pythagoras to medicine, cosmology, and theology. For instance, the music theorist Aristides Quintilianus, who lived in the late third or early fourth century AD, linked music and astronomy to Pythagoras by presenting a seasonal harmony correlated with an elemental harmony derived in turn from the Platonic solids, so that "spring stands to autumn in the ratio of the fourth, to winter in that of the fifth, and to summer in that of the octave."[36] Moreover, he says that Pythagoras, on his deathbed, recommended his pupils to study the monochord, "as it was necessary to excel in music mentally more through numbers than sensibly through hearing."[37] In this crescendo of consideration for Pythagoras' mathematics, the influential *Lives of philosophers* by Diogenes Laertius (*ca* 180–240) can be considered a sort of "doxographic apex," given that this work associates the Greek sage with the highest developments of mathematics, including the discovery of the celebrated geometrical theorem,[38] the invention of the monochord, the introduction of weights and measures, and eventually the elaboration of a sort of holistic world view based on the monad as the principle of everything, from which mathematical entities (dyad, numbers, points, lines, surfaces, volumes), then the four elements, physical bodies and eventually the entire cosmos, animated and provided with intellect, are successively derived.[39] In later doxography, therefore, Pythagoras' medical, cosmological and psychological knowledge is fully interrelated to the Pythagoreanizing Platonism that connected doctrines of the Pythagoreans to the Platonic view of the world.[40]

Latins, by Boethius' *De institutione musica*, which, in turn, follows Cicero. All these reports agree on the elements that determined such effects, namely the melodic-interval (*harmonia*), the rhythm (*rhythmos*) and the instrument used.

36 Plato, *Timaeus* 55d–56b. See Hicks 2017, 23. The correspondences are: fire = pyramid = summer = 4; earth = cube = autumn = 6; air = octahedron = spring = 8; water = icosahedron = winter = 12.

37 Aristides Quintilianus 1983, 162 (*On Music*, 3).

38 Diogenes refers to the testimony of the mathematician Apollodorus, but the attribution of theorem I 47 remained doubtful also for Proclus, who celebrated the great service by Pythagoras to geometry, although he supported the Euclidian paternity of the theorem. For an English translation of the passage, see Heath 1921, 145 and 141–169 for a detailed summary of all mathematical discoveries attributed to Pythagoras and/or the Pythagoreans.

39 See Diogenes Laertius' *Vitae philosophorum* 8, 1–47, at chap. 11, 12, 15, and 25. Cf. Laks 2014.

40 For this, see, *inter alia*, O'Meara 1989 and Hicks 2014.

Up until now, we have considered the Greek and Latin pagan portrayals of Pythagoras with reference to his link to the quadrivium, but the early Christian philosophers and Church Fathers also left relevant indications about this nexus. Augustine was by far the most influential for medieval scholars. He probably knew about Pythagoras from Varro's and Cicero's accounts, and, may be, also from Porphyry.[41] In the early *De ordine* Augustine commends the *divus* Pythagoras for having considered the numerical basis of the seven liberal arts and the role of the mathematical sciences in leading the soul to philosophy, although in his later *Retractationes* he repents for having praised the philosopher of Samos too much.[42] Pythagoras favored "physics," namely the quadrivial arts, as the key to the knowledge of the work of creation, an idea that the contemporary Martianus Capella presents us in a fully Neoplatonic background and in a cosmic wide-ranging numerical symbolism:[43]

> For the number four with its parts makes up the whole power of the decade itself [i.e., 1 + 2 + 3 + 4 = 10] and is therefore perfect and is called quadrate, as is the Cyllenian himself [i.e., the god Mercury], with whom are associated the four seasons of the year, the regions of heaven, the elements of earth. That celebrated oath of old <Pythagoras>, who did not refrain from swearing by the tetrad (*ma ten tetrada*) – what does that signify except the number of perfect ratio?[44]

Also the most typical feature of the Neoplatonic Pythagoras, i.e., his close link with Plato's *Timaeus* and its mathematical/cosmological theories, entered the Middle Ages, and the most influential source for this transmission was Calcidius' *Commentary to the Timaeus*, in which it is stated that Timaeus of

41 Gersh 2002, 26, in which the hypotheses of I. Hadot and A. Solignac are confuted. See also Feichtinger 2015; further bibliography in Macris 2018, 1163.

42 See in particular *De Ordine* 2, 20.53 and 10.28.

43 See McGowan 2014, 201–204.

44 Translation in McGowan 2014, 202, from *De nuptiis* 2.106–07: "nam quaternarius suis partibus complet decadis ipsius poetestatem, ideoque perfectus est et habetur quadratus, ut ipse Cyllenius, cui anni tempora, caeli climata mundique elementa conveniunt. an aliud illa senis deieratio, qui *ma ten tetrada* non tacuit, confitetur nisi perfectae rationis numerum?" The expression *ma ten tetrada* ("in the name of the tetrad") was used in the Middle Ages, for instance in the interesting sequence *Vite dator, omnifactor* (written about 1000), which presents an articulated view of Pythagoras. See Dronke 2007, 98–104, also for further bibliography. This expression was possibly at the origin of the alleged book by Pythagoras named *Matentetrade* by Hugh of St. Victor. See below, note 82.

Locri was "a follower of Pythagoras' teaching."[45] Following Numenius, Calcidius states that Pythagoras taught two principles, i.e., "unity, naming God, and the binary, denoting matter,"[46] and accepts a direct influence of Pythagoras on Plato's view of the world:

> He [*sc*. Plato] traces, then, an image or shape of the world which strives to approximate this sketch for his depiction of the soul, and he establishes the seven planetary orbits and sets them apart from one another by musical intervals such that the stars, as according to Pythagoras, in harmonized movement produce musical modes as they rotate in their spinning; this is consistent with his claim in the *Republic* that the Sirens, each positioned on its own orbit, set a single mellifluous song into motion, [...] and that from the eight different sounds one harmonious consonance is stirred up.[47]

The same indication is found in Macrobius' commentary on the *Somnium Scipionis*, which also names Pythagoras in referring to the numerical and musical structure of the soul.[48] Yet, the most interesting feature of Macrobius' Pythagoras emerges from the account of the discovery of the consonances, probably an adaptation from Nicomachus, which, however, differs considerably from that narrated by Boethius, as we will see below. For Macrobius, Pythagoras wanted to understand the "reason" (*ratio*) governing the mundane harmony; thus, when he passed "by chance" (*casu*) at the blacksmith's shop, he happened to understand "by means of eyes and hands" how the musical ratios were organized, so to prove "by observation" what the mind grasps by simply contemplating the harmony of the universe.[49] Macrobius neither insists on numbers nor mentions the weights of the hammers, though he does remark that consonance entirely depends upon the "laws of weights." He, in conclusion, outlines a picture of Pythagoras as an experimentalist, more than a mathematician. This is the reverse of Boethius' portrait.

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45 Calcidius 2016, 202 (*Commentarius in Timaeum* 50): "iste enim Timaeus, qui in hoc libro tractat, ex Pythagorae magisterio fuit, quem rationabiliter inducit Plato domesticis et familiaribus sibi probationibus utentem docere animae naturam congruere numeris, concinere etiam modulationibus musicae" (transl. J. Magee).

46 See Albertson 2014, 123, who notes that Thierry of Chartres derives from Calcidius his references to Pythagoras.

47 Calcidius 2016, 283–285 (*Commentarius in Timaeum* 95). See also Hicks 2014, 200. Flamant (1977, 373) argues that the celestial harmony of Calcidius and Macrobius derives ultimately from Adrastus, but Macrobius reveals also Porphyry's interventions.

48 Macrobius 1981 (*Commentarii* 1.14.19 and 2.2.1); see Hicks 2017, 120–121.

49 Macrobius 1981, 248–251 (*Commentarii* 2.1.8–14).

5 Boethius' Pythagoras

The Roman Severinus Boethius offered to the Latin Middle Ages the most detailed and long-lasting representation of Pythagoras as father of the quadrivium. Boethius derived this directly from Nicomachus, but interpreted differently the special convergence of Aristotelian logic and Platonic theology, which was the specific feature of Nicomachus' Pythagoras. As we have seen above, Nicomachus had stated that the disciplines of arithmetic, music, geometry and astronomy were "methods" (*methodoi*) for progressing upwards from the senses towards speculative knowledge, and Boethius, in his early *De institutione arithmetica*, follows the same methodology, openly summarizing Nicomachus' *Introductio arithmetica*.[50] Boethius asserts that all ancient sages starting with Pythagoras (*Pythagora duce*) reached perfect knowledge by investigating along the fourfold crossroad of the philosophical disciplines (*in philosophie disciplinis ad cumulum perfectionis evadere* […] *quasi quadruvio vestigatur*). Then, he turns to the objects of such a wisdom (*sapientia*), which are "what exists" (*esse*) and "has immutable substance," i.e., what is not subject to quantitative, qualitative, or substantial change.[51] Then, following Nicomachus' idea of forms as immanent universals, Boethius states that of this kind are "qualities, quantities, forms, greatnesses (*magnitudines*), smallnesses (*parvitates*), equalities (*aequalitates*), habitudes, acts, dispositions, places, times, and anything that is found in some way united to bodies." These are incorporeal and immutable *essentiae*, although they are immersed in the flux of changeable things.[52] Thus, for taking the mind "from the knowledge offered by the senses to these certain things of the intellect" it is necessary to go deeper into the nature of quantity, divided into (1) discrete quantity or multitude, based on the indivisible unity that increases into infinite number, and (2) continuous quantity or magnitude, endlessly divisible though delimited with respect to increasing.[53] Each of them is then subdivided into two sub-species of multitude in itself and in relation, magnitude as such and in motion, corresponding in turn to the subjects of arithmetic, music, geometry and astronomy. Thus, the quadrivium is fully defined by the *genus*-quantity and its four *species*, which opens the door to the highest philosophical knowledge.[54]

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50 Boethius 1999, 5 (*De arithmetica, Prologus*). Boethius wrote this treatise early in his career, about 500, when he was also writing his first *Isagoge* commentary.

51 Ibid., 9 (I, 1).

52 Ibid.

53 Ibid., 10–11.

54 Panti 2018, 71–72, and 2020, 452. On Boethius' adaptation of Nicomachus' passage see also Merlan 1975, 92–93, and Pizzani 1981, 214–216; on Boethius' theory of number see Crialesi 2020.

A more succinct abridgment of this same passage about the quadrivium is found in the subsequent *De institutione musica*, in which the references to Pythagoras are, conversely, much more elaborated. Here, Boethius is still greatly indebted to Nicomachus' Pythagorean methodology, to which Cicero's Pythagoras is added for introducing the topic of music therapy.[55] But Boethius' Pythagoras has also an original feature, emerging in the intimate association with a purely speculative mathematical approach to musical intervals. This is surely one of the major and long-lasting legacies of the music theory transmitted by Boethius, who adopts the figure of Pythagoras not only in order to revive his philosophical methodology, but also to investigate each quadrivial science by means of it. Boethius, in fact, compares what he himself considers Pythagoras' method of inquiry to be with that of other major ancient *musici*, particularly Claudius Ptolemy (100–170 AD) and Aristoxenus.[56] While the latter is deemed as the leading figure of an empirical approach to music based upon sense perception, Ptolemy is representative of a balanced methodology, which takes into due account both reason and the senses. In his *Harmonics*, which Boethius quotes in book 5 of the *De institutione musica*, Ptolemy refuted the conclusion of the Pythagoreans that only the three intervals of octave, fifth and fourth are accepted by perception as consonant inasmuch as they are expressed by means of the first four natural numbers. Ptolemy notes that hearing also accepts the octave and a fifth (3:1), the double octave (4:1), and the octave and a fourth (8:3), notwithstanding the fact that the Pythagoreans ruled out the latter, for its not being entirely expressed by the first four numbers. For Ptolemy, this means that the Pythagoreans "did not follow the impressions of hearing even in those things where it is necessary for everyone to do so."[57] Ptolemy deemed the scientist's task as being that of accounting for the rightness of sense perception not by means of the perfection of numerical relations as such, separated from the phenomena.[58] Now, Boethius juxtaposed Ptolemy's scientific attitude grounded on the right balance between sense and reason with the purely speculative method attributed to Pythagoras.

The juxtaposition between Pythagoras and Ptolemy is emphatically represented by Boethius, who uses similar phraseology in book 1 and book 5 of *De institutione musica* respectively for stressing their diverging attitude to music, as shown in the following table:[59]

55 Boethius 1867, 184–185 (*De institutione musica* I, 1).
56 For a wide discussion of this topic, see Panti 2017.
57 Barker 2004, 70 (translation from Ptolemy's *Harmonics*, 6.1–2).
58 Ibid., 27.
59 See also Panti 2017, 28–30.

Pythagoras is a *musicus* (*Inst. mus.* I) because:	**Ptolemy is an *armonicus* (*Inst. mus.* V) because:**
(1) has a pure rational method, which leaves aside the judgment of the ear (I, 9, 196.4–5: *non auribus, quarum sunt obtusa iudicia, sed regulis rationique*)	(1) believes that there is nothing contrary to reason and hearing (V, 3, 355.6–7: *nihil auribus rationique possit esse contrarium*)
(2) trusts only reason (I, 9, 196.7: […] *iudex vero atque imperans ratio*)	(2) favors the cooperation of reason and hearing (V, 2, 352.5–6: *sensus enim ac ratio* […] *facultatis armonicae instrumenta sunt*)
(3) requires no instrument in musical investigation (I, 10, 196.22: *nullis etiam deditus instrumentis*)	(3) considers the eight string *kanon* to be the indispensable instrument of harmonics (V, 1, 351.20–22: *non unus tantummodo nervus adsumitur, qui positis proportionibus dividatur, verum octo, atque huiusmodi fiat chitara*)
(4) his ruling principle is the gaze of the mind (I, 11, 198.23–27: *Invenit regulam* […] <*quae*> *sit inspectio fixa firmaque*)	(4) his ruling principle is the *kanon/ chitara*, which demonstrates the ratio of proportions to the eyes and the ears (V, 1, 351.22–24: *chitara, ut* […] *in pluribus nervis tota proportionum ratio quasi oculis subiecta cernatur*)
(5) follows a “firm doctrine” driven by reason (I, 33, 223.7–8: *animus firmiore doctrina roboratus*)	(5) introduces to an “intermediate doctrine” (V, 1, 351.18–19: *mediocris doctrinae dispensatione*)
(6) follows the path of speculation thanks to the judging of reason (I, 34, 224.19–20: *qui ratione perpensa* […] *imperio speculationis adsumpsit;* 225.11–15: *cui adest facultas secundum speculationem rationemque* […] *iudicandi*)	(6) harmonizes the judgment of hearing with that of reason (V, 3, 355.9–10: *id quod sensus indicat, ratio quoque perpendat* […] *ratio proportiones inveniat, ut ne sensus reclamet*)

These juxtapositions do not mean that Boethius rejects or limits the importance of the *musicus* over the *armonicus* or vice versa, but that the latter operates in a different area of musical investigation, for which Ptolemy's *Harmonics* is a reliable methodological guide. As I have tried to show in a recent study, the ultimate goal of this distinction seems to be the assignment of each of these two authorities to a different *genus* of music. Boethius divides music into the famous tripartition: "of instruments," "of the human being," and "of the cosmos" (*in instrumentis*, *humana* and *mundana*).[60] Now, Pythagoras' purely mathematical, i.e., "disciplinary," approach is, for Boethius, the foundation of the *musica in instrumentis*, namely, of the correct knowledge of music according to the proportions of the consonances, while the *humana*, which is totally introspective and associated to the "harmonic power" (*vis harmonica*) of every human being, can be fruitfully cultivated thanks to the Ptolemaic "intermediate" approach.[61] Although the unfinished *De institutione musica* does not deal with the *musica mundana*, Boethius' references to the *Timaeus* and *Republic* in relation to this division of music in book 1 chapter 2 let us foresee that Plato might have incarnated the right methodological approach to this higher kind of music. "Instrumental" music, therefore, which is the science of sounds and voices according to the purely mathematical method of Pythagoras, is the science of quantity "in relation to something" (*ad aliquid*), namely of the numerical ratios defining intervals between two sounds in terms of consonance or dissonance. The anecdote of Pythagoras at the blacksmith shop must, therefore, account for such a methodology.

From what has been said, it becomes clear why Boethius stresses the "purely mathematical discovery" made by the philosopher of Samos much more emphatically than Nicomachus.[62] The Boethian Pythagoras deemed all instruments and tools as unreliable, as he wished to acquire full knowledge of consonances by means of "reason alone." By a "divine will," he passed the blacksmith's shop and heard the consonances. Then, "firmly guided by reason," he concluded that the octave was generated by two hammers in the ratio of

60 Boethius 1867, 187–189 (*De institutione musica* 1, 2). The first kind of music is commonly referred to as *instrumentalis* ("instrumental"), but this adjective is never used by Boethius. In fact, he defines it as the music "which rests in certain instruments (*quae in quibusdam constituta est instrumentis*), such as the *chitara* or the *aulos* or other instruments which serve melody" (ibid., 187; transl. in Boethius 1989, 9). Accordingly, the music of instruments deals with the mathematical proportions of instrumental and vocal consonances (*consonantiae*).

61 See, again, Panti 2017. It is also relevant to note that the link of music to ethics envisaged by Boethius is closely related to the human music.

62 Boethius 1867, 197–198 (*De institutione musica* 1, 10–11); for an English translation, see Boethius 1989, 18–19.

the double, the fifth by two hammers in the ratio of the sesquitertian, and so on. Consequently, Pythagoras' request to the blacksmiths to change hammers was done only "to confirm" his initial rational intuition, while for Nicomachus it was the first of a series of practical "experiments" made for proving the same intuition. The reference to the numbers 12, 9, 8, 6 is quickly introduced by Boethius only "for the sake of illustration"; on the contrary, for Nicomachus this was the pivotal assertion for justifying how Pythagoras found the mathematical foundation of music in all experiments implying the "law of weights." Boethius, moreover, recalls the experiments made by Pythagoras at home with strings and bells only as a means to enjoy a most complete validation of the mathematical foundation of consonances, and remarks that the monochord or "rule" (*regula*), which consists of a string stretched over a case and divided into proportional sections by a movable bridge, was invented by Pythagoras for fixing the already gained results, so that "no researcher would be misled by dubious evidence."[63]

The Boethian Pythagoras, therefore, represents the true *musicus*, who relies upon reason, disregards sense perception and considers the experiments with bells and strings only as a confirmation of what the mind has already grasped. If, in Nicomachus' story, Pythagoras' experimental proofs are a strategical part of his method of inquiry, in Boethius' portrait of Pythagoras, they are nothing more than a mere accidental support. Boethius' Pythagoras is a speculative philosopher: mathematics, in other words, is intended by him as one of the Aristotelian theoretical divisions of philosophy, firmly grounded on syllogistic demonstration. Macrobius, for his part, portrayed Pythagoras as a pure "experimentalist." In conclusion, medieval scholars knew two opposite interpretations of Nicomachus' story. None of them, as well as none of the subsequent reports, let emerge that the "experiments" by Pythagoras are totally fictional, because the pitch of sounds produced by hammers is not directly proportional

63 Ibid. On the invention of the monochord by Pythagoras, see the discussion in Creese 2010, 90–93, 102–103. On the implications of the dissonance "of tone," corresponding to the ratio 9:8, in the Pythagorean world-view see Heller-Roazen 2011, 11–40. In these pages, however, the author does not distinguish the Boethian story of Pythagoras' discovery of the ratios of consonances from the harmonic problems discussed later by the Pythagoreans. In fact, such a distinction emerges from Boethius' examination of the Pythagorean (i.e., Nicomachean) theory of the consonances (*De institutione musica* II, 18–20 and 27) and of Philolaus' division of the tone (ibid., III, 1 and 5). Moreover, according to Heller-Roazen (2011, 28), in the Pythagorean view the "tone interval" would correspond to the unity in number; this, in fact, is contradicted by Boethius, who states in *De institutione musica* (II, 20) that Nicomachus equated the unity to the diapason interval (Boethius 1867, 251).

to their weight![64] Notwithstanding its erroneous experimental basis, it was thanks to this story that the "venerable" Pythagoras was unanimously regarded as the inventor of music in medieval culture, and well beyond.

In MS Cambridge, University Library, Ii.3.12, datable to *ca* 1130 and transmitting Boethius' *De institutione musica* and *De institutione arithmetica*, the suggestive illustration at fol. 61v stands as a vivid testimony of how the science of music was grounded on Pythagoras' experiments, to which the greatest philosophers from the past, namely Plato, Nicomachus and, finally, Boethius, were connected in a special way. Here, in fact, Pythagoras "the physicist" (*physicus*) is depicted in the upper left side of the folio, while experiencing the laws of music with bells and weights; in the lower section of the folio, the *summus* Plato sitting enthroned over the world is discussing music with Nicomachus, each with a book open in their hand. Plato indicates Pythagoras, while Nicomachus points in the direction of Boethius, who, in the upper right side, is in turn empirically judging the laws of consonances by means of the monochord. This sort of visual crossroad offers a vivid representation of music as an historically founded discipline as well as a fully technical and self-sufficient science, with the philosophical base of a mathematical discipline (the books by Plato and Nicomachus) and the valid experimental proofs by Pythagoras with hammers and bells and by Boethius' monochord. It is accompanied by eloquent *didascaliae*:

> [*captions*:] Consul and eximious investigator of philosophy, [Boethius] judges by ear the sound while moving the string with the finger, in order to grasp the differences of the voices by means of the monochord. Pythagoras the physicist and friend of the hidden nature distinguishes the weights by pondering them and disregards the dissonances, proving how every proportion is a certain number. The greatest of the same philosophers, Plato, teaches how a single concord sounds from different sounds, and Nicomachus replies to him with analogous reason.[65]

64 Marin Mersenne (1588–1648) proved the falsity of these "experiments" in his 1634 *Questions harmoniques*.

65 "[Boethius:] Consul et eximiae scrutator phylosophyae / Vt uideat uocum discrimina per monochordum / Iudicat aure sonum percurrens indice neruuum. [Pythagoras:] Pythagoras physicus physicaeque latentis amicus / Pondera discernit trutinans et dissona spernit. / Pulsans aera probat quota quaeque proportio constat. [Plato and Nicomachus:] Edocet ipsorum summus Plato phylosophorum / Quomodo disparium paritas sonat una sonorum. / Obuiat instanti ratione Nichomacus illi." See Gibson and Smith 1995, n. 6 (42) and 54 (84–85). My translation.

FIGURE 1.1 MS Cambridge, University Library, Ii.3.12, fol. 61v
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6 Pythagoras and the Medieval Quadrivium

In his extant quadrivial treatises, Boethius had linked Greek mathematics to the Roman educational system, as well as to a new Christian search for wisdom already pursued by Augustine[66] and testified by Cassiodorus in a famous letter to Boethius himself:

> For according to your translations, Pythagoras the musician and Ptolemy the astronomer are read as native Italians; Nicomachus the arithmetician and Euclid the geometrist are heard as native Ausonians; Plato and Aristotle, masters of theology and logic respectively, debate in the language of Romulus; and you have given back the mechanician Archimedes to his own Sicilian countrymen, who now speak Latin.[67]

Leaving aside the debated problem of Boethius' completion of this huge programme of translations, for the purpose of the present study the main interest of this passage is the definition of a "father" for each art of the quadrivium, plus mechanics, as well as for theology and dialectics. If we consider that, on the basis of Augustine, the quadrivium was generally considered as coincident with the "physics" of the Stoic tripartite division of philosophy into physics, logic and ethics,[68] we might read the passage by Cassiodorus as a means to dignify the entire Roman educational system through the patronage of Greek authorities, whose translation/transmission into Latin culture was successfully conducted thanks to Boethius. As we can see, Pythagoras' aegis extends to music alone, and his special patronage is pointed out by Cassiodorus also in another occurrence.[69] From now onwards, Pythagoras was basically the *inventor* of music, namely, the authority who first established the corpus of knowledge

66 See, for instance, Chadwick 1981.

67 My translation, partly taken from McGowan 2014, 189–190 (see also Chadwick 1981, 103), of Cassiodorus, *Variae* I, 45.4: "Translationibus enim tuis Pythagoras musicus, Ptolemaeus astronomus leguntur Itali: Nicomachus arithmeticus, geometricus Euclides audiuntur Ausonii: Plato theologus, Aristoteles logicus Quirinali voce disceptant: mechanicum etiam Archimedem Latialem Siculis reddidisti." It is interesting to note that, for Cassiodorus, Nicomachus is the authority for arithmetic as such and that Cassiodorus knew Ptolemy's astronomical work and Euclid's geometry, whose knowledge disappears in the Latin world by the late 11th century. For Cassiodorus' mention of *Pythagoras musicus* as referring to Boethius' *De institutione musica* see Pizzani 1981, 224–225.

68 See for instance Hicks 2017, 70.

69 Ibid., 87, with reference to *Institutiones* (PL 70, 1209A): "coelum quoque terram, uel omnia quae in eis dispensatione superna peraguntur, non sunt sine musica disciplina, cum Pythagoras hunc mundum per musicam conditum et gubernari posse testetur."

conveyed by Boethius and which, in the ninth century, will be applied by the Carolingian scholars to the theory of the ecclesiastical plain chant.[70]

This can be easily exemplified by referring to an early Carolingian treatise written after the rediscovery of Boethius' musical treatise: the *Epistola de harmonica institutione* by the Benedictine monk Regino of Prüm (died 915). In chapter 11, Regino narrates the story of Pythagoras at the blacksmith's in order to prove that the musical consonances are "firmly and invariably" (*firmiter et constanter*) established in intervals, so that all melodies consist "in measurements" (*in dimensionibus*). For Regino, the harmonic discipline itself concerns how the consonances discovered by Pythagoras "by a divine will" are known "to the mind and to the ears" (*animo atque auribus*).[71] Regino insists on the fact that Pythagoras' discovery allowed for *melodies* of the plain chant to be inscribed within the realm of mathematics, given that they are grounded on the consonances. Thus, the mathematical science of music "invented" by Pythagoras and transmitted by Boethius becomes the theoretical foundation of a concrete musical repertory. This transformation was an extraordinary achievement in the Carolingian age, and we may have an idea of how relevant the Pythagorean foundation of music was by referring to the *Musica disciplina* (*ca* 840) by the monk Aurelian of Réôme. Following, again, the story narrated by Boethius, Aurelian associates each of the consonances produced by the four hammers to a specific *Introitus* of the plain chant in its own *tonus*:

> The already mentioned Pythagoras was the first to discover how different proportions are related to consonances in sounds. For instance, let they be four hammers having the weight expressed by numbers 12, 9, 8, 6. The hammers of, respectively, weight 12 and 6 will sound the octave in the ratio of the double, as for instance <it happens> in the antiphon *Inclina*

70 For a brief exposition and further bibliography on the importance of Boethius' musical theory for the definition of the Gregorian modal system and music notation, see Bernhard 2007 and Panti 2008, 113–118.

71 Regino of Prüm 1963, 240 (*De harmonica institutione*, 11). Pythagoras' characterization as the inventor of music was also popularized thanks to some widespread high-medieval sources, such as Isidore's *Etymologiae* (III, 16), which refers to Pythagoras' experiments with hammers and chords. The extraordinary success of this story produced, however, a contamination with the Biblical figures of Jubal and Tubalcain. Jubal, who took the place of Pythagoras as the inventor of music, discovered consonances while hearing the sounds generated by his brother Tubalcain, the blacksmith. The two stories – Jubal and Pythagoras – continued to coexist for centuries, feeding off one another. On this see, in particular, Daolmi 2017.

> *Domine aurem tuam*, and in all the others included in the first <church> tone [...].[72]

Aurelian associates antiphons and *toni* of the plain chant, so that the octave-based Pythagorean scale is exemplified by means of the liturgical chant,[73] with the outstanding consequence that church music becomes the sounding representation of the harmony of the spheres and of human beings. This clearly emerges also from the late ninth century anonymous treatise *Alia musica*, an important and controversial work, which has strong links with Aurelian's work and was assembled in different phases from the ninth to the tenth century. This text adapted the diatonic scale to yield the liturgical *modi* (i.e., Aurelian's *toni*), here named with the Greek ethnic names Dorian, Phrygian, etc.[74] A striking passage from this text connects Pythagoras to their discovery, which in turn derived from his observation of the eight-part structure of winds, waves, thunders and of the heavenly harmony. This passage – certainly worthy of further investigation – refers also to Vitruvius and states that Pythagoras discovered the *toni* on "Mount Atlas" (*in monte Atlante*), because of its closeness to heaven:

> As Vitruvius said in the book *On Architecture*, according to scientists (*secundum physicos*) there are no more than eight winds [...] For, in the waves of the sea and of the rivers always the first wave sounds more than the seven following. [...] Similarly, there are eight <differences> in thunders, thanks to which Pythagoras the philosopher discovered these eight consonances of the *toni*, foreseen (*praefatas*) according to the harmony of heaven, <while he was> on Mount Atlas, which is close to heaven, the proportions of which <mountain> covered (*perficit*) the measure of the five zones of heaven, by the climates of which <zones> this world is governed.[75]

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72 Aurelian of Réôme 1975, 62 (*Musica disciplina* 2, 10–21): "Primus hoc modo iam dictus Pithagoras repperit, qualiter proportionum varietas sonorum iungeretur concordie. Sint verbi causa quattuor mallei, qui subter insertos contineant numeros: XII, VIIII, VIII, VI. Hi igitur mallei qui XII et VI ponderibus vergebant diapason in duplo consonantiam concinebant ut hic: Antiphona *Inclina Domine aurem tuam*, et omnia quae in primo inveniuntur tono [...]." See Bernhard 2007; Panti 2008, 143. For a critical study of Aurelian's *Musica disciplina*, see Morelli 2007.

73 The *De harmonica institutione* by Hucbald of St. Amand associated a specific diatonic extension to each of the eight *modi*, corresponding, in turn, to each of the ancient Greek scales/*toni* of transpositions listed by Boethius. See Bernhard 2007, 90–92.

74 Atkinson 2009, 171–201.

75 My translation from Chailley 1965, 95–97: "Ut Vitruvius dixit in libro de Architectura, secundum physicos non plus sunt quam octo venti. [...] Nam in undis maris et fluminum

When shifting from music to "physics," this passage makes us see how Pythagoras' consonances become the expression of the worldly harmony rooted in the Biblical verse of *Wisdom* 11.21: "You [*sc*. God] ordered all things in measure, number and weight." This hidden allusion is the manifesto of the philosophical masterpiece of the High Middle Ages, Eriugena's *Periphyseon*, which proposes a direct connection between the quadrivium, and arithmetic as its highest discipline, and Pythagoras' doctrine of intellectual numbers as the substance of everything:

> In that, not only it [i.e., arithmetic] is the immobile foundation as well as the primordial cause and principle of the other three mathematical disciplines – namely geometry, music and astrology – which follow it, but, as well, the infinite multitude of all the visible and invisible things receives substance because of the numerical rules pondered by arithmetic, as attested by Pythagoras, the greatest philosopher and inventor (*repertor*) of this art, who asserts with right reasons that the intellectual numbers are substances of all visible and invisible things. And this is not refuted by the Sacred Scripture, which says "all things were made in measure, number and weight."[76]

For Eriugena, Pythagoras presides over the four disciplines: on the basis of Pliny, he is deemed as the authority in astronomy, for his calculation of the planetary distances;[77] while in the *Annotationes in Marcianum* he is the *inves-*

semper magis sonat prima unda, quam septem sequentes. [...] Similiter in tonitruis octo discrimina, a quibus Pythagoras philosophus has octo tonorum consonantias praefatas ad harmoniam coeli in Atlante monte, qui est proximus coelo, adinvenit, cujus proportiones instar quinque zonarum coeli perfecit, quarum temperie mundus iste moderator." Perhaps, the reference to Pythagoras on Mount Atlas comes from the Ovidian "speech of Pythagoras" (*Metamorphoses* XV, 147–152), in which the Samian alludes to his delight "to stand on Atlas' shoulders." On Atlas and astronomy see Diodorus Siculus (4.27.5 and 3.61.1) and note 81 below. Moreover, Ovid's portrayal of Pythagoras includes his search for the causes of all things, such as the thunder, earthquakes, star movements "and whatever else is hidden" (ibid., 62–72).

76 Eriugena 1999, 49 (*Periphyseon* III, 11, 651D–652A): "Siquidem non solo aliarum trium matheseos sequentium se partium (hoc est geometriae, musicae, astrologiae) immobile subsistit [*i.e.* arithmetica] fundamentum primordialisque causa atque principium, verum etiam omnium rerum visibilium et invisibilium infinita multitudo iuxta regulas numerorum, quas arithmetica contemplatur, substantia accipit, teste primo ipsius artis repertore Pithagora summo philosopho, qui intellectuales numeros substantias rerum omnium visibilium et invisibilium esse certis rationibus adfirmat. Nec hoc scriptura sancta denegat, quae ait *omnia in mensura et numero et pondere facta esse*."

77 Ibid., III, 32, 715C.

tigator and *inventor* of both arithmetic and music, thanks to which he emerges *prae ceteris* in the investigation of the musical ratios forming the harmony of the heavens; eventually, the symbolism of the tetractys is by itself an implicit reference to the quadrivium, the *matentetrade* as *doctrina quaternaria*.[78] This last annotation had a suggestive elaboration in the later *Glosses to Martianus* by Remigius of Auxerre:

> He [*sc.* Pythagoras] did not refrain from swearing "by the tetrad," that is the quaternary teaching (*doctrina*). In fact, every perfection of teaching is contained in the four arts: arithmetic, geometry, music, astronomy. This is the quadrivium, without which nobody can master philosophy.[79]

If, therefore, Boethius' *De institutione musica* nourished the long-lasting idea of Pythagoras as *inventor musicae*, the philosopher's links with the quadrivial arts as such continued also to be cultivated in the High Middle Ages, and we can verify the weight of this tenet by considering one of the most famous encyclopedic works of the early twelfth century, the *Didascalicon* by Hugh of Saint Victor. Here, Pythagoras is considered as the first philosopher and the first to give the definition of philosophy as the study "of those things which are really true" (*earum rerum, quae vere essent*), a definition taken from Boethius' *De institutione musica*.[80] Hugh also refers to Pythagoras' eminence in the quadrivium: he considers him as the inventor of arithmetic and as the main Greek authority

78 Eriugena 1939, 19 (*Annotationes in Marcianum* 11, 8): "SUPERUM CARMEN […] prae ceteris Pytagoras arithmeticarum et musicarum proportionum sagacissimus fuit investigator et inventor"; Ibid., 57 (44, 21): "SENIS DEIERATIO de Phitagora dicit. MA THN TETPAΔA doctrinam quaternarium, tetras quattuor."

79 Latin text from Remigius' *Glosses on Martianus* in Paris, BnF, lat. 14754, fol. 18r, for which see Taylor 1961, 208, note 8: "Qui [Pythagoras] non tacuit *mathen tetradem*, id est doctrinam quaternariam. Omnis enim doctrinae perfectio in iiii artibus continetur, Arithmetica, Geometria, musica, astronomia. Hoc est illud quadruvium [*sic*] sine quo nulli proponitur philosophandum." This is a gloss to *De nuptiis*, II, 107, for which see the note 78 above. For the expression *ma ten tetrada*, see also note 44.

80 Hugh of Saint Victor 1939, 6 (*Didascalicon*, I, 2): "*Primus omnium Pythagoras studium sapientiae philosophiam nuncupavit* (*De institutione musica* II, 1) maluitque philosophos dici, nam antea sophos, id est, sapientes dicebantur […] <Pythagoras> philosophiam autem earum rerum, quae *vere* essent suique immutabilem substantiam sortirentur, *disciplinam* constituit." The connection of Pythagoras with number seven is also recalled in the following passage, with reference to the seven liberal arts: "Pythagoras quoque hanc in studiis suis consuetudinem servasse legitur, ut usque ad septenium, secundum numerum videlicet septem liberalium artium, nullus discipulorum suorum de his quae ab ipso dicebantur rationem poscere auderet, sed fidem dare verbis magistri quousque omnia audivisset, sicque iam per semetipsum rationem eorum posset invenire."

for music. But, more importantly, Hugh refers to the *Matentetrade* as if it were a book written by Pythagoras on the very disciplines of the quadrivium:

> Concerning the authors of the Arts […] Pythagoras of Samos was the inventor of arithmetic, and Nicomachus wrote on it. Among the Latins, first Apuleius and then Boethius translated this work. The same Pythagoras also wrote the *Matentetrade*, that is, a book concerning the teaching (*doctrina*) of the quadrivium, and he found in the figure "Y" a likeness to human life. Moses declares that the inventor of music was Tubal, who was of the seed of Cain; the Greeks say it was Pythagoras, others that it was Mercury, who first introduced the tetrachord; others that it was Linus, or Zethus, or Amphio.[81]

The alleged Pythagorean book *Matentetrade* is clearly Hugh's misinterpretation of the "tetradic oath" referred to in the commentary by Remigius; however, a book by this title is also mentioned in the *Liber Hermetis De sex rerum principiis*, which, too, bestows great authority upon Pythagoras. In a marginal gloss in the *Liber Hermetis* the same work by Pythagoras is called *Liber de doctrina quadruuii*. This suggests either that the *Liber Hermetis*, tentatively dated between 1135 and 1147, borrowed from Hugh or that both used the same source.[82] It is possible that the anonymous author of this work is drawing on Hugh's statement, or vice versa, but what is really relevant is the fact that Pythagoras is so tightly linked to the quadrivium to be credited as the author of a book by this name.

81 Translation Taylor 1961, 84 (*Didascalicon*, III, 2). On Pythagoras and the letter Y, see *Etymologiae*, I, 3, 7. The passage goes on with the other arts and authorities: "Geometry, they say, was first discovered in Egypt: its author among the Greeks was the great Euclid. Boethius passed this man's art on to us. Eratosthenes too was most learned in geometry. […] Certain ones say that Cham, son of Noah, first discovered astronomy. The Chaldeans first taught astrology as connected with the observance of birth, but Josephus asserts that Abraham first instructed the Egyptians in astrology. Ptolemy, king of Egypt, revived astronomy; he also drew up the *Canones* by which the courses of the stars are found. Some say that Nemroth the Giant was the greatest astrologer, and to his name astronomy too is ascribed. The Greeks say that this art was first thought out by Atlas, and because of this it is also said that he held up the very heaven."

82 Ibid., 189–190. See Silverstein 1955, 268: "In the *Matentetrade*, mathematicians call the superlunary world "time" because of the course and motion of the heavenly bodies; but they call the sublunary world "temporal" because the mutability of things below is directed according to the perpetual order of the things above." See also Albertson 2014, 345, n. 27. From the notes 44, 78 and 79 above, it is clear that the expression *Matentetrade* was known in the High Middle Ages.

As a final remark concerning the everlasting connection between Pythagoras and the quadrivium in the Middle Ages, we may add a quick mention of the mathematical games and tools bearing his name, which illustrated such a link. This is the case of the most famous monastic game, the *Rithmomachia*, or "Pythagoras' game," which was played with a numbered board and two dice and its goal was to illustrate the number theory of Boethius to students of monastic schools. The oldest written evidence about it is by the monk Asilo of Würzburg and dates around 1030, while Hermannus Contractus of Reichenau is held to have fixed the rules and added music theoretical remarks to them. The game remained very popular up to the Renaissance, when it was played for courtly recreation.[83] Another tool associated with the name of Pythagoras was the medieval abacus, called *mensa Pythagorea*.[84] Eventually, "Pythagoras' sphere" was a sort of soothsaying monastic entertainment consisting of a circle drawn on a sheet and divided into four parts by two crossing lines in the shape of human bones, flanked by a column of Roman numbers with the respective letters of the alphabet.[85] The prognostication was made by adding the numbers of the letters forming the name of a sick man together with the day of the synodic month on which the illness occurred and the day of the week, and the sum divided by 30. If the remainder was in the upper half of the crossbones, the patient would recover; if below, he would perish. Other tools, such as the famous ten-based square used as a multiplication board, were associated with Pythagoras in more recent times.

7 A Conclusive Note: Pythagoras from the Quadrivium to Physics in the Late Middle Ages

In the early 13th century, the Western Latin belief on nature was fully renovated by the introduction of the Aristotelian and Arab philosophy, and the growing interest in natural philosophy in the Faculty of Arts was consequently accompanied by a decreasing importance attributed to the quadrivium as

83 See Folkerts 2003, 1–23.

84 See the pseudo-Boethian *Geometria altera*, XX, 5 (Folkerts 1970, 139). See also Caiazzo 2020, 192–193. This is not the linear abacus used in the Roman era, but an abacus with vertical columns and counters (*apices*) on which the numbers were engraved in various forms.

85 A vivid representation of it is in MS Pal. lat. 176, at fol. 162v (http://microcosmographia.com/2012/12/16/the-half-redacted-sphere-of-pythagoras/), above which a censorious hand wrote *anathema* and crossed out the circle and the instructions with strokes of red ink.

such. As a consequence of these novelties, Pythagoras as father of the quadrivium was supplanted by a new image, largely inspired from Aristotle's assertions concerning the Pythagoreans to whom he attributed the idea that "all is number." Although Aristotle proposes at least three different versions of such a view, namely that numbers are identified with "material indivisibles" (a sort of "numerical atomism"), or with the principles (*archai*) of the real things, or also with what is imitated by real things (i.e., objects are *mímesis* of the numbers), what is certain is that, for Aristotle, Pythagorean mathematics actually concerned the physical world.[86] These ideas were promptly adapted by medieval scholars to shape a new image of Pythagoras, which clearly emerges, for instance, in Albert the Great, who openly refers to the *Metaphysics* as if Aristotle were speaking about Pythagoras himself:

> And his sect descends from the most ancient Pythagorean philosophers, because Pythagoras, as Aristotle reports in *Metaphysics*, book 1, being used to and nourished in arithmetic, posited a single principle for all things and the unity (*unum*) to be all things.[87]

The move between the Pythagoreans and Pythagoras himself with reference to Aristotle's accounts is found also in Roger Bacon, who asserts, for instance:

> But Aristotle proves in *On the Heavens*, book 2, that in no way a sound can be generated in the heavens, and that Pythagoras was wrong.[88]

The popular mid-14th-century *Lives and Customs of the Philosophers* by Pseudo-Walter Burley presents a familiar long portrayal of Pythagoras as a sage of numbers and ethics, a "typical" account of the Samian sage issued by collecting

86 Merlan 1967, 48: "It is striking that Aristotle refused even to recognize that, when Pythagoreans and Platonists spoke of derivation of numbers (mathematicals) from supreme principles, they did so only *didaskalias charin* (or *theôrêsai heneken*). He insisted that they could not have meant anything but genesis in the temporal sense of the word – because they, after all, spoke of the origin of the cosmos and *qua* physicists." On Aristotle's view on the Pythagoreans' opinion about numbers as principle of things see Cleary 1995, 352–356.

87 Albert the Great 1896, 70 (*De homine*, q. 5 a. 2): "Et descendit haec secta a vetustissimis philosophis Pythagoricis, quia Pythagoras, ut refert Aristoteles in primo metaphysicae, propter hoc quod assuetus erat et nutritus in arithmeticis, posuit unum principium esse omnium et unum esse omnia."

88 Roger Bacon 1859, 229 (*Opus tertium*, ch. 59): "Sed Aristoteles probat, secundo Coeli et Mundi (*De caelo* II, 9), quod nullo modo potest sonus generari in coelestibus, et quod Pythagoras erravit."

various sources such as Cicero, Augustine, Isidore, Hugh of Saint Victor and several others, likely also from Laertius' *Vitae*.[89] Yet, apart from quick mentions of Boethius' episode at the blacksmith and of Pythagoras' interest for astronomy, no special attention is given to mathematics.

But the most interesting reshaping of the figure of Pythagoras comes from the early fourteenth-century discussion concerning indivisibles as being that which "compose" continuous quantities, inspired by Aristotle's arguments against indivisibilism. In his *Tractatus de continuo*, Thomas Bradwardine defines Pythagoras as the father of the sect which included Plato and the *modernus* Henry of Harclay, all holding the continuum to be composed out of a finite number of indivisible points.[90] This new view of Pythagoras clearly left out his connection with the quadrivium, but, in a certain sense, it had an impact upon it. In particular, it stimulated scholars into thinking of the philosopher of Samos as a natural philosopher, and this, in turn, helped to reconsider the mathematical foundation of the quadrivium.

In the case of music theory, in particular, this emerges from the interpretation in experimental terms of Boethius' report of Pythagoras at the blacksmith's. For instance, the *Tractatus de musica* by Jerome of Moravia, a Dominican friar of the Parisian convent at the time of Thomas Aquinas, labels Pythagoras as *inventor* of music on the ground "of the authority of Boethius" and repeats verbatim the story of the hammers; nonetheless, Jerome also asserts that for Pythagoras "sense perception is at the base of the art of music, and hearing is its principle."[91] Pythagoras is now depicted as the leader of scholars who held music to be based both on sense perception and on reason, a method that, as mentioned above, Boethius had attributed to Ptolemy, not to Pythagoras!

Another example comes from the very influential *Musica speculativa secundum Boetium* by John of Murs, a Master of Arts and mathematician in Paris at the beginning of the 14th century. Quoting directly from the *Metaphysics*, John declares in his fourth "statement" (*petitio*) on the art of music, that "experience of sensible things makes the art," and consequently, that music is entirely based

89 See Prelog 1990.

90 For the late medieval theories on indivisibilism, see Murdoch 1982, who quotes also (at 576) from Bradwardine's *De continuo*: "Alii autem dicunt ipsum <continuum> componi ex indivisibilibus dupliciter variantes, quoniam Democritus ponit continuum componi ex corporibus indivisibilibus; alii autem ex punctis, et hii dupliciter, quia Pythagoras, pater huius secte, et Plato et Waltherus modernus, ponunt ipsum componi ex finitis indivisibilibus. Alii autem ex infinitis, et sunt bipartiti, quia quidam eorum, ut Henricus modernus, dicit ipsum componi ex infinitis indivisibilibus inmediate coniunctis; alii autem, ut Lyncul<niensis>, ex infinitis ad invicem mediates."

91 Jerome of Moravia, 1935, 12–16.

on experience and on the fundamental role of hearing on which “Pythagoras established the art of melodies.”[92] The shift from pure reason to the realm of experience was transmitted to other works of music theory, such as the anonymous *Quaestiones musicae* wrongly attributed to Blasius of Parma and which, in turn, were a fundamental source for Ugolino of Orvieto’s *Declaratio musicae disciplinae*, which opened the humanistic “return to Boethius” of the 15th century. In asserting that Pythagoras’ invention of music was neither due to demonstration nor to persuasion, but “to experience and reason” (*per experientiam et rationem*) or “to chance and experience” (*casu et experientia*), these works definitely projected music in the field of human acquisitions based on senses and practical skills.[93] The quadrivium as a system of knowledge of “what always is” comes to its end, and, with it, its link with Pythagoras.

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92 John of Murs 1992, 90 and 292–293.

93 For the *questiones* and their influence on Ugolino, see Panti 1992 (quotation from *questio* 1: *Utrum musica sit scientia*, at 300, l. 461); see also Ugolino of Orvieto 1962, 185–188.

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CHAPTER 2

Music and the Pythagorean Tradition from Late Antiquity to the Early Middle Ages

Andrew Hicks

A man walks into a bar … This is an unlikely setting, perhaps, for a chapter attuned to the musical reverberations of Pythagoras and Pythagoreanism in late antiquity and the early middle ages. But Sextus Amarcius' vivid eleventh-century depiction of the musical divertissements demanded by a gluttonous patron at an inn demonstrates the degree to which Pythagoras had captured the popular imagination:[1]

> "Tell me, do you know some lyric poet or skillful cithara player, or one who harmonizes chaste timbrels with a hollow plectrum?" […] And so when a minstrel arrives after his salary has been arranged, and starts to remove his lyre from its ox-hide case, people rush forth from all the neighborhoods and streets, and with eyes fixed on him and with low murmuring observe the minstrel with the tips of his fingers stroke the strings which he furnished from the moist entrails of wethers, and now they put forth a gentle sound, now a harsh one. […] Frequently harmonizing the melodious strings at the fifth, that minstrel [tells] how the shepherd's sling once laid out mighty Goliath, how the sly little Sabian with similar skill tricked his wife, how shrewd Pythagoras laid bare the eight notes of song, and how pure the call of the nightingale echoes forth.

Already by the middle of the nineteenth century, scholars had conjectured that the closely contemporary Cambridge Songs (Cambridge, University Library Gg.5.35, fols. 432r–443v) preserve the thus not entirely fictional playlist that Amarcius provides.[2] Accordingly, the twelfth Cambridge song (*Vite dator, omnifactor*) sings of clever Pythagoras. Across its five stanzas, entitled *Conductus Pitagoricus* in another source (MS Schaffhausen, Stadtbibliothek, min 108, fols. 140v–141r), the verses summarily recount the transmigration of Pythagoras' soul (the obligatory reference to Euphorbus included; cf. Ovid, *Meta*. 15.160);

1 Sextus Amarcius 2011, 1, 397–421 (translation modified). This chapter updates and expands upon Hicks 2014.

2 Ziolkowski 1998, XLIV–LIII.

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his chance discovery of musical harmony in the ping of differently weighted hammers at the blacksmith's forge; his own mental forging of the numerical ratios that generate musical concords (the three *symphoniae* of the octave, fifth, and fourth); the relevance of these ratios to arithmetic, geometry, and astronomy; and finally the ethical power of the Greek hupsilon (Υ), whose two forking branches encapsulate the ethical choice we all must make, whether to follow the wide path to vice or the narrow path to virtue.[3] In the fourth century, Macrobius had related the rebuke of the second-century Neopythagorean Numenius, who had ventured to give an open account of the Eleusinian mysteries; in a dream, Numenius saw the Eleusinian goddesses dressed as prostitutes, ripped from their sacred shrine, now open and available to all. This, Macrobius tells us, is a cautionary tale against publically denuding *natura* of the careful, fabulous veils woven by Pythagoras, Empedocles, Parmenides and Heraclitus.[4] By the eleventh century, Numenius' nightmare had become a reality: the mysteries of Pythagoras, even his sacred oath *ma ten tetraden* ("by the tetrad!"),[5] had become the stuff of popular legend, sung and jauntily rhymed in taverns by itinerant jongleurs to undeserving nobles and eavesdropping commoners.

1 Pythagoras in the Medieval Imagination

"Pythagoras, i.e., one who needs no questioning or the culmination of questioning. For ΠΥΘΟΣ means questioning, ΑΓΟΡΑ means culmination."[6] This creative etymology, variations of which were employed by Carolingian scholars to annotate (or "gloss") mentions of Pythagoras in both Boethius' *Fundamentals of Music* (*De institutione musica*) and Martianus Capella's *On the Marriage of Philology and Mercury* (*De nuptiis Philologiae et Mercurii*),[7] neatly encapsulates the medieval use of Pythagoras. In line with a tradition long established in the late ancient Latin texts that provided exclusive access to "Pythagorean" teachings, the medieval Pythagoras functioned as a "first principle": to invoke his authority was to invoke *the* icon of Greek wisdom traditions.

3 For text, translation, and commentary, see Ziolkowski 1998, 4–59 and 202–206. See also Kranz 1959.

4 Macrobius 1963, 7.23–8.12 (*In Som. Scip.* 1.2.19–21).

5 Martianus Capella 1983, 30.15 (*De nuptiis* 2.107).

6 Bernhard and Bower 1993–2011, 1, 61 (ad *Inst. mus.* 1.1): "Pitagoras, id est non indigens interrogationis vel interrogationis cumulus. ΠΥΘΟΣ enim interrogatio, ΑΓΟΡΑ cumulus."

7 E.g., John Scottus Eriugena 1939, 203, 10 (ad *De nuptiis* 9.923); Remigius of Auxerre 1962, 323.25 (ad *De nuptiis* 9.923).

This gloss' etymological assertion of Pythagorean authority resonates with late ancient and medieval accounts of Pythagoras and the Pythagoreans on multiple levels. First, it well accords with the method of instruction ascribed to the Pythagoreans. According to Boethius, "it was customary for the Pythagoreans, whenever Master Pythagoras said something, that no one thereafter dare to challenge his reasoning; rather the reasoning of the teacher was their authority."[8] By the twelfth century, this account of the Pythagorean method of instruction (*Pythagorica doctrina*) had codified into a supposed imposition of seven years of silence "according to the number of the seven liberal arts," as Hugh of St. Victor explains, during which Pythagorean disciples were allowed only to listen and believe; William of Conches concurs that not until the eighth year were they allowed to broach questions.[9] Second, it echoes the common view of Pythagoras as both the inaugurator of the Greek philosophical tradition and its consummate practitioner, a view exemplified by Boethius' equation of "Pythagorean knowledge" (*scientia Pythagorica*) with "perfect teaching" (*perfecta doctrina*), a loftier mode of philosophizing suited only to those who have already mastered the beginning and intermediate stages of philosophy.[10]

Closely connected to Boethius' invocation of *scientia Pythagorica* as the highest mode of philosophizing is a third level of Pythagorean authority, namely the philosophical (and theological) method ascribed to Pythagoras by (*inter alios*) Proclus: the mathematical and analogic mode. Although Proclus, as Dominic O'Meara has highlighted, parts ways with Iamblichus' Pythagorean Platonism by subordinating Pythagorean "analogic" theology to the more scientific or "dialectical" theology of Plato, he nonetheless valorizes a tradition that became closely linked to the Pythagoreans throughout Late Antiquity and the Middle Ages: the veiling of sacred doctrine through the use of symbolic and analogic discourse.[11] This close connection between Pythagorean science and a veiled presentation of its truths is elaborated upon several times by Proclus in his Platonic commentaries (e.g., *In Tim.* 1.29, 31ff; *In Remp.* 1.73.11ff.) and presented to the Latin West through Macrobius' famous account of the philosophical utility of

8 Boethius 1867, 223.4–7 (*Inst. mus.* 1.33); cf. Valerius Maximus, 8.15.ext.1.

9 Hugh of St. Victor 1939, 53 (*Didascalicon* 3.3); William of Conches 1997, 4 (*Dragmaticon* 1.1.3). In ancient sources, the period of silence imposed upon Pythagorean neophytes ranges from "no less than two years" (Aulus Gellius, *Noctes Atticae* 1.9.4) to the widely reported five years (e.g., Diogenes Laertius 8.10).

10 Boethius 1847 (*In Categorias Aristotelis libri quatuor* 160B); see Ebbesen 1990, 387–391 and Asztalos 1993, 379–388.

11 O'Meara 1989, 148–149. See also Gritti 2007 and Steel 2007.

mythical narrative (*narratio fabulosa*), similitudes (*similitudines*) and analogies (*exempla*) in his *Commentary on the Dream of Scipio* (1.2.9–21). According to Macrobius, philosophers employ a mythic narrative to discuss the soul, airy or ethereal powers, or the other gods, but when they turn their pen upon the Platonic triad – τἀγαθόν, νοῦς, and ἰδέαι (the Good, Mind, and Ideas) – myth is inappropriate, and they can take recourse only in similitudes and analogies, this last being a direct echo of Proclus' "Pythagorean" ὅμοια (similitudes) and εἰκόνες (images), to which Proclus adds σύμβολα (symbols).[12] When Macrobius draws his comments to a close by enumerating the ancient philosophers that employed an analogic mode of theologizing, Pythagoras heads the list (which includes Empedocles, Parmenides, Heraclitus, and Timaeus). In the twelfth century, the Macrobian teaching of *fabulae* and *similitudines* coalesced into a flexible doctrine of philosophical myth and allegory, known as "veils" or "concealments" (*integumenta* or *involucra*), which at least one medieval commentator on the *Timaeus* ascribes to Pythagoras: "But theologians speak with *integumenta* when they discuss the airy and ethereal powers and the world soul. Pythagoras is said by many to have been the leader of this view."[13]

Christiane Joost-Gaugier has deemed the Middle Ages an age of "New Pythagoreanism," but this is a misleading characterization.[14] If anything, medieval Pythagoreanism arises from a reification and simultaneous iconization of a particular strain of late-ancient (Neo)Pythagorean speculation. For it was the "mathematical Pythagoreanism" of Nicomachus (and, indirectly, Iamblichus and Proclus) that most captured the medieval imagination and thereby overshadowed (though did not eliminate entirely) some of the more diffuse Pythagorean contributions in ethics and ontology.[15] Late-ancient and medieval authors themselves inherited and embellished upon a Pythagoras credited with an increasingly illustrious list of discoveries and inventions, most of which were closely connected to the late-ancient "mathematicalized"

12 On which see Dillon 1990.

13 Hicks 2016, 13.300–302. On *integumenta*, see Bezner 2009.

14 Joost-Gaugier 2006, 116.

15 Within Pythagorean ethics, the maxims of "Sextus Pythagoreus," translated in the fourth century by Rufinus of Aquileia, enjoyed a wide circulation in Christian circles (despite Jerome's protestations against the work's presumed Christianity: *Ep*. 133.3); one of these maxims found its way into the Rule of St. Benedict (no. 145: "a wise man is distinguished by the brevity of his words"). See Chadwick 1959. For a résumé of other suggestive but tenuous connections between Pythagoreanism and Christian monastic practices, see Jordan 1961.

Pythagoreanism: the science of music,[16] the monochord,[17] arithmetic and the science of numbers,[18] the abacus,[19] Venus and the computation of planetary distances,[20] even the discipline of philosophy, for which Pythagoras was said to have coined the very term *philosophia*.[21]

The mathematical speculation of the Pythagoreans, according to whom "all is likened unto number" (ἀριθμῷ δέ τε πάντ' ἐπέοικεν: Sextus Empiricus, *Adversus mathematicos* 7.94; cf. Aristotle *Met.* 1.5, 985b24ff.), was transmitted to the Middle Ages through the "Pythagorean" arithmologies of Calcidius (*In Tim.* 35–38), Macrobius (*In Som. Scip.* 1.5–1.6) and Martianus Capella (*De nupt.* 7.732–742), and eventually found laconic expression in William of Conches' formulation: "it was the custom of the Pythagoreans to attribute to number the characteristics of things (*quod erat rerum*) and to attribute to things the characteristics of number (*quod erat numerorum*)," and Thierry of Chartres aphoristic summation: "the creation of number is the creation of things."[22] This mathematical speculation played a central role in the formulation of a Christian Platonism consonant with the oft-cited scriptural testimony that God "hath ordered all things in measure, number, and weight" (*omnia mensura et numero et pondere disposuisti*: Sap. 11:30).[23]

Other elements of mathematical Pythagoreanism found a more circuitous route to their ultimate Christianization. Calcidius' fourth-century partial Latin translation of Plato's *Timaeus* (through 53c) accompanied by his own formidable commentary was the primary conduit through which Plato (and Platonic Pythagoreanism) reached the Latin West. Near the conclusion of his doxographical account of primordial matter (*silva*, *hyle*), he asserts, on the

16 E.g., Boethius 1867 (*Inst. mus.* 1.10–11); Cassiodorus, *Institutiones* 2.5.1; Isidore of Seville 1911 (*Etymologiae* 3.16.1). See Münxelhaus 1976, McKinnon 1978.

17 E.g., Diogenes Laertius 8.12; Gaudentius 341.12; Aristides Quintilianus 3.2; Boethius 1867 (*Inst. mus.* 1.11); Guido of Arezzo, *Micrologus* 20. See Creese 2010.

18 E.g., Isidore of Seville 1911 (*Etymologiae* 3.2.1); Eriugena 1996–2003 (*Periphyseon* 3.652A); Hugh of St. Victor 1939, 49 (*Didascalicon* 3.2). See Heath 1921, 1.65–117.

19 The so-called "table of Pythagoras" (*mensa Pythagorea*) of the Ps.-Boethian *Ars geometrica* (Boethius 1867, 396.7–16), which is held aloft by a personification of Arithmetic in Alan of Lille's *Anticlaudianus* (3.288). On the Ps.-Boethian *Geometry*, see Folkerts 1981.

20 E.g., Pliny, *Naturalis historia* 2.37, 2.83; Martianus, *De nuptiis* 8.882–883; Eriugena 1996–2003 (*Periphyseon* 3.715BC).

21 E.g., Augustine, *De ciuitate dei* 8.2; Boethius 1867 (*Inst. mus.* 2.2); Isidore of Seville 1911 (*Etymologiae* 8.6.2); etc. See Riedweg 2005, 90–97.

22 William of Conches 1997, 47 (*Dragmaticon* 2.5.4). On Thierry of Chartres formulation, see Brunner 1966.

23 E.g., Cassiodorus makes the connection explicit at *Institutiones* 2.4.1 (cf. *Variae* 1.10.3), as does Claudianus Mamertus, *De statu animae* 2.3. Cf. Eriugena 1996–2003 (*Periphyseon* 3.652A). On Eriugena's "metaphysics of number" generally, see Jeauneau 2007.

authority of Numenius, that Pythagoras had called god unity (*singularitas*) and matter duality (*duitas*).[24] This duality, however, in its original indeterminate nature is itself ungenerated (*duitatem indeterminatam quidem minime genitam*), and thus this Numenian/Pythagorean claim is fundamentally dualist (though some Pythagoreans, Numenius continues, had misunderstood their master and thought that duality was produced by the solitary monad).[25] This Numenian dualism undergirds crucial moments elsewhere in Calcidius' commentary, most notably the final remarks on the creation of the world soul (the *psychogonia*), where, in a somewhat confused and complicated chain of inferences, Calcidius hints that unity and duality are in some way analogous to the subdivisions of primary components of the world soul: double substance (undivided and divided) and two-fold nature (the same and the different).[26]

In the early sixth century, near the end of the first book of his *De institutione arithmetica* (a loose translation of the *Introductio arithmetica* by the second-century Pythagorean, Nicomachus of Gerasa), Boethius explains a "most profound teaching": that every (mathematical) form of inequality can be reduced to equality.[27] According to Boethius, inequality is an "ugly" (*dedecus*) privation of the knowable and beautiful (*decus*) limit of "the Good," and it must thereby be tempered, we might say "harmonized" or "tuned" (*temperata*), by the "limited nature of the principle associated with the Good." Boethius returns to this idea again and again in his quadrivial works on arithmetic and music. Equality is the "mother" and "root" (*mater* and *radix*) from which the multitudinous forms of inequalities grow and degenerate, and by which it has set a limit (*margo*) for their otherwise unbounded contrariety.[28] The primacy of *aequalitas* to *inaequalitas*, moreover, pertains not just to numerical calculations but also, at a more fundamental level, to the nature of the universe

24 Calcidius 1975, 297.7–10 (*In Tim.* 295): "Nunc iam Pythagoricum dogma recenseatur. Numenius ex Pythagorae magisterio Stoicorum hoc de initiis dogma refellens Pythagorae dogmate, cui concinere dicit dogma Platonicum, ait Pythagoram deum quidem singularitatis nomine nominasse, siluam uero duitatis." See van Winden 1965 for a thorough account of Calcidius on matter; for additional bibliography on this passage, see Bakhouche 2011, 2.851–852.

25 Ibid.: "sed non nullos Pythagoreos uim sententiae non recte assecutos putasse dici etiam illam indeterminatam et immensam duitatem, ab unica singularitate institutam recedente a natura sua singularitate et in duitatis habitum migrante."

26 Calcidius 1975, 101.14–102.8 (*In Tim.* 53); cf. Procl. *In Tim.* 2.153.17–25 (citing Aristrandus and Numenius).

27 The method of reducing inequality to equality described by Nicomachus and Boethius can be traced, through Theon of Smyrna 1878 (*Expositio rerum mathematicarum ad legendum Platonem utilium*, 107.15–25ff.), to Adrastus and Eratosthenes.

28 See Albertson 2014, 84.

(*omnem naturae uim rerumque integritatem* = τῶν ὅλων φυσιολογίαν), the soul in particular:[29]

> Goodness, [being] limited, tractable to knowledge (*scientia*), and forever imitable and perceptible to the soul (*animus*), is by nature first and perpetual in the beauty of a substance all its own, whereas the baseness of evil, being unlimited and resting on no principles of its own but by nature forever wandering, acquires composure from the limited nature of the principle associated with the Good by having impressed upon it, as it were, a kind of seal of the noblest form and finds respite from its fluctuating wandering. For, like a kind of ruler, the soul (*animus*), strengthened by pure intelligence, curbs excessive cupidity and immoderate, unbridled irascibility and, in a way, reduces these forms of inequality to a temperate goodness.

As Jean-Yves Guillaumin has commented, this passage betrays "a Neoplatonic philosophical substrate: in the world as in the domain of number, equality, which pertains to the Same, is a principle prior to inequality, which pertains to the Different. Hence, all can be reduced to the equality, even inequality."[30] But without signaling the transition, the final sentence of the passage has slid into a discussion of late Platonic psychology, specifically the Platonic tripartition of the soul into its "limited" rational aspect (*ratio*, λογιστικόν) and its "unlimited" irrational appetite (*cupiditas*, ἐπιθυμία) and spiritedness (*iracundia*, θυμός). When reason, the soul's immortal and rational aspect, is the leader (*rector*) and constrains the mortal and quasi-irrational psychological faculties (*cupiditas* and *ira*) then harmonized goodness (*temperata bonitas*) obtains in the soul.[31]

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29 Boethius 1999, 80.6–15 (*Inst. ar*. 1.32): "bonitas definita et sub scientiam cadens animoque semper imitabilis et perceptibilis prima natura est et suae substantiae decore perpetua, infinitum uero malitiae dedecus est, nullis propriis principiis nixum, sed natura semper errans a boni definitione principii tamquam aliquo signo optimae figurae impressa componitur et ex illo erroris fluctu retinetur. Nam nimiam cupiditatem iraeque immodicam effrenationem quasi quidam rector animus pura intelligentia roboratus adstringit, et has quodammodo inaequalitatis formas temperata bonitate constituit." Cf. Nicomachus 1866, 65.1–13 (*Intr. ar*. 1.23.4–5).

30 Boethius 1995, 67, n. 201.

31 Cf. Calcidius 1975, 272.21–24 (*In Tim*. 267): "optima porro symphonia est in moribus nostris iustitia, uirtutum omnium principalis, per quam ceterae quoque uirtutes suum munus atque opus exequuntur, ut ratio quidem dux sit, uigor uero intimus, qui est iracundiae similis, auxiliatorem se rationi uolens praebeat." See Hicks 2017, 56–61 for further commentary on this passage.

In the twelfth-century, Thierry of Chartres' commentary on the *De institutione arithmetica* identified the precise utility of this "most profound teaching": it pertained to the "penetration and understanding of physics, i.e., natural philosophy."[32] In Thierry's theological commentaries on Boethius, the "Pythagorean" distinctions between, on the one hand, *singularitas* and *duitas* (*unitas* and *alteritas* in Thierry's language)[33] and *aequalitas* and *inaequalitas* on the other,[34] are carefully and inextricably intertwined: *unitas*, free of *alteritas*, engenders *aequalitas*, from which *inaequalitas* descends.[35] But Pythagorean inequality was not only a matter of metaphysics (or theology); as Thierry knew well, it is also the well-spring of music.

2 Measuring Sounds: Pythagorean Harmonics

The basics of "Pythagorean" music theory and its relationship to the competing approach associated with Aristotle's student Aristoxenus can be dispensed with summarily. Pythagorean harmonics championed the thesis that musical relations are fundamentally and essentially quantitative; musically meaningful pitch relations (e.g., the octave, the fifth, and the fourth) are reducible to whole numerical ratios (2:1, 3:2, and 4:3, respectively), specifically multiple and superparticular ratios. Any pitch relations that are irreducible to whole-number ratios are not (and cannot be) musically meaningful relations. What Pythagorean harmonics cannot readily quantify, however, is the distance (διάστημα) or difference (ὑπεροχή; lit., "excess") between pitch relations on a numerical (or geometrical) continuum. Hence, when describing pitch relations as intervals (διαστήματα), the primary reference is not the distance between the terms in the ratio, but a spread across (διὰ) strings: διὰ τεσσάρων ("across four [strings]," or a fourth), διὰ πέντε ("across five [strings]," or a fifth), and διὰ πασῶν ("across all [strings]," or an octave).

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32 Thierry of Chartres 2015, 145.

33 Thierry of Chartres 2015, 180.679–681 (ad *Inst. ar.* 2.27): "Itaque Pythagoras omnia constare dixit ex unitate et alteritate, id est ex materia et forma, ideoque ex mutabili et immutabili, et sic habent omnia constare ex diverso, materia, et ⟨ex eodem⟩, forma."

34 Thierry of Chartres 2015, 145.1195–1198: "Omnis natura quamdiu se in suo statu conservat, integra est et incorrupta, sicut omnis aequalitas. At quae exorbitaverit et declinaverit a suo principio ad vitia deflectens, quasi ab aequalitate ad inaequalitatem labitur, quae fine non clauditur."

35 See Albertson 2014, 110–115; 151; and 349, n. 60 for references; see also Thierry of Chartres 2015, 50–57.

Intervals as measurable (if not, strictly speaking, quantifiable) distances find their most natural expression in the Aristoxenian tradition, for Aristoxenus took as primary not numerical quantity but the sense-perceptible continuity (or discontinuity) of vocal sound, subjected to rigorous Aristotelian empiricism.[36] Central to the Aristoxenian enterprise is the notion of musical motion (κίνησις) across an abstract pitch space (τόπος), analogous to the traversal of distances from point to point. The vocal motion of speech is continuous (συνεχής), for it does not articulate discrete points within a well-defined pitch space, whereas the vocal motion of a singing voice is intervallic (διαστηματική), moving discretely from note (φθόγγος) to note, with each note articulating a single "incidence of the voice on one pitch" (φωνῆς πτῶσις ἐπὶ μίαν τάσιν).[37] Hence, an Aristoxenian interval (διάστημα) is "what is bounded by two notes that do not have the same pitch, since an interval appears, so to speak, to be a certain difference [διαφορά] between pitches and a space [τόπος] capable of receiving notes higher than the lower of the pitches bounding the interval, and lower than the higher of them."[38] Here we not only can but must speak of intervals as pitch distances, for Aristoxenus, in line with Theophrastus and other Peripatetics,[39] claimed that numerical quantity had nothing to do with the sense-perceptible presentation of sound to the ears (or to the mind). To say that the tone is the ratio 9:8 has little to no bearing on the sense-perceptible reality of music, and thus Aristoxenus and his adherents had no qualms about dividing the tone into two, three, or four equal parts – a position impossible in Pythagorean harmonics, as the whole tone (9:8) cannot be generated by squaring (or cubing, etc.) any rational proportion.[40] As any good Pythagorean knew, the whole tone was properly divided into a minor semitone, the *leimma* (λεῖμμα), in the ratio 256:243 (the "remainder" when two tones are subtracted from a fourth), and a major semitone, the *apotome* (ἀποτομή), in the ratio 2,187:2,048 (the "remnant" of subtracting the *leimma* from the tone). Finally, the Pythagorean *comma*

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36 See Gibson 2005, 23–30.

37 All of these terms are initially defined and discussed in Aristoxenus 1954, 1.8.14–10.10; the formal definition of φθόγγος occurs at 1.15.14–24.

38 Ibid., 1.15.25–33: "διάστημα δ' ἐστὶ τὸ ὑπὸ δύο φθόγγων ὡρισμένον μὴ τὴν αὐτὴν τάσιν ἐχόντων. φαίνεται γάρ, ὡς τύπῳ εἰπεῖν, διαφορά τις εἶναι τάσεων τὸ διάστημα καὶ τόπος δεκτικὸς φθόγγων ὀξυτέρων μὲν τῆς βαρυτέρας τῶν ὁριζουσῶν τὸ διάστημα τάσεων, βαρυτέρων δὲ τῆς ὀξυτέρας." I follow here Barker's translation with minimal modifications (Barker 1989, 136).

39 E.g., Theophrastus, *De musica* (preserved in Porphyry's *Commentary on Ptolemy's Harmonics*): "καὶ γὰρ εἰ πᾶν διάστημα πλῆθός τι, τὸ δὲ μέλος ἐκ διαφορῶν φθόγγων, τὸ μέλος ὅτι ἀριθμὸς τοιόνδε ἂν εἴη· ἀλλ' εἰ μηδὲν ἄλλο <ἢ> ἀριθμός, πᾶν ἀριθμητὸν μετέχοι ἂν καὶ μέλους, ὅσον καὶ ἀριθμοῦ" (Porphyry 1932, 62.7–10).

40 A true semitone would be $3{:}2\sqrt{2}$.

(κόμμα), in the ratio 531,441:524,288, can be obtained by subtracting the *leimma* from the *apotome*.

The most famous instantiation of the basic Pythagorean ratios occurs in the division of the world soul in Plato's *Timaeus* (35b4–36b3). The enigmatic stuff of the world soul is divided first into parts according the powers of two and three (1, 2, 3, 4, 9, 8, 27). These parts are next divided by the imposition of harmonic and arithmetic means between each number; in the third and final stage, all the epitritic intervals (4:3) are further subdivided by epogdoic intervals (9:8), which leaves in each epitritic interval the remainder (λεῖμμα) 256:243. Plato's mathematical recipe for the construction of a well-tuned world soul leaves too much underdetermined for it to be construed straightforwardly as a recipe for a well-tuned musical system.[41] Hence, commentators ancient and modern have sought repeatedly to construct its scale, and one primary conduit for Pythagorean harmonics – outside of the late-ancient encyclopedias and music-theoretical textbooks by Martianus, Boethius, Cassiodorus, and Isidore of Seville – were Calcidius' commentary on the *Timaeus* and Macrobius' commentary on Cicero's *Dream of Scipio*, both of which grappled with the musical implications of Plato's oblique division.

Calcidius furnished his account (*In Tim.* 40–50) with a series of three "lambda" diagrams that circulated widely throughout the Middle Ages, in both philosophical and music theoretical contexts.[42] Macrobius' discussion, however, proved more troubling. At 2.1.25, he analyzes the five fundamental consonances (*symphoniae*) not only according to the Pythagorean tradition (by offering numerical ratios), but also according to Aristoxenian tradition (by breaking the primary consonances into their component intervals).[43] Macrobius signals the incongruity of his division of the fourth into two tones and a (real) semitone with the standard Pythagorean account, since he notes that it is not really true, as he must, for the moment, ignore the discrepancy

41 E.g., if we multiply the base sequence (1, 2, 3, 4, 8, 9, 27) by 6 to accommodate the harmonic and arithmetic means in whole numbers (as was standard in the exegetical tradition, e.g., Calcidius 1975, 89.22–91.19 [*In Tim.* 41–42]), the resultant sequence and its intercalated means (6, 8, 9, 12, 16, 18, 24, 27, 32, 36, 48, 54, 81, 108, 162) contains several non-epitritic ratios (32:27, 81:54, 162:108), whose proper subdivision is not prescribed (see Barker 2007, 320–322).

42 See Huglo 2005 and 2008.

43 Macrobius 1963, 99.11–18 (*In Som. Scip.* 2.1.25): "symphonia διὰ τεσσάρων constat de duobus tonis et hemitonio, ut minutias quae in additamento sunt relinquamus, ne difficultatem creemus, et fit ex epitrito; διὰ πέντε constat ex tribus tonis et hemitonio et fit de hemiolio; διὰ πασῶν constat de sex tonis et fit de duplari; verum διὰ πασῶν και διὰ πέντε constat ex novem tonis et hemitonio et fit de triplari numero; δίς autem διὰ πασῶν continet tonos duodecim et fit ex quadruplo."

between a Pythagorean minor semitone (256:243) and the (Aristoxenian) semitone employed in this context (*ut minutias quae in additamento sunt relinquamus ne difficultatem creemus*). Hesitations duly noted, he continues headlong with the striking claim that the diapason consists of six tones. This is a fundamental Aristoxenian position, which is both mathematically and conceptually at odds with Pythagorean harmonics. Ptolemy famously critiqued the Aristoxenian position by proving in *Harmonics* 1.11 that if six successive tones are constructed "by means of ratio," the extreme interval of this construction will differ from the octave (2:1) by a ratio approximating 75:74 (i.e., the comma, 531,441:524,288, also approximated by Boethius as falling between 75:74 and 74 :73).[44]

The twelfth-century natural philosopher William of Conches commented on both Plato and Macrobius. In his remarks on Macrobius' account of the *symphoniae*, he correctly adduces the admixture of Aristoxenian elements and rightly compares Macrobius' Aristoxenian line with Boethius' Ptolemaic sympathies.[45] In his commentary on the *Timaeus*, William explains in meticulous detail the ratios that obtain between the initial limits and the interposed means of Plato's world soul, appeals at length to Boethius' *De institutione arithmetica* to explain the three different kinds of proportionality, and finally gives a full mathematical proof that the minor semitone consists in (and only in) the ratio 256:243, a proof that amounts to a short, quasi-independent treatise on the division of the tone.[46] The base text for the discussion was Boethius' *De institutione musica* 2.28–30, but William's mathematical method reveals a

44 Boethius 1867, 286–291 (*Inst. mus.* 3.12).

45 William of Conches, *Glosae super Macrobium* (MS Bamberg, Staatsbibliothek, Class. 40 [H.J.IV.21], fol. 21r*a*, comment. ad 2.1.25): "Hic aperte innuit Macrobius se uelle minutias contineri in diatessaron cum duobus tonis et semitonio minori, et inde habet Aristoxenum auctorem. Vnde uidetur contrarius Boethio qui in Musica nullas uult esse consonantiis minutias, et inde habet Ptolemaeum auctorem" (cf. *Inst. mus.* 3.2–4, 5.14; see also Peden 1998, 153–154). The *Glosae Colonienses super Macrobium* interprets the *minutiae* as "fractions" and suggests that these fractions could be computed in whole numbers provided the calculation uses large enough numbers, even if those numbers remain a mystery. *Glosae Colonienses super Macrobium*, comment. ad 2.1.25 (Caiazzo 2002, 257.8–10): "UT MINUTIAS RELINQUAMUS. Habitudo enim semitonii per minutias computatur, nisi aliquis ad numerorum misterium multum ascendat, ut integris inveniatur. Minutias autem propter difficultatem vitamus." Cf. Ibid., comment. ad 2.1.20 (Caiazzo 2002, 256.1–6): "quomodo in binario possit dyatessaron et dyapente computari, quas supra diximus ex quaternario et ternario constare? Solutio: quod per minutias omnia illa intervalla computabuntur, et licet propter penuriam numerorum non sint actualiter, tamen proportiones illorum ibi sunt naturaliter, et per minutias computare idem est quod et ad maiorem numerum recurrere."

46 William of Conches 2006, 140–153 (*Glosae super Platonem* 80–86).

pedagogical distillation of Boethius' calculations. Boethius' demonstration that the minor semitone consists in the proportion 256:243 is not light reading, and though he gives his readers a few basic *regulae* to grasp along the way, these rules are neither numerous nor frequent. William makes liberal use of such "shortcut" *regulae* for his calculation of the ratio. Moreover, several of these rules are either not found, or at least not formulated as such, in Boethius' mathematical texts. William, for instance, offers a *regula in musica* that determines whether two numbers are in a given superparticular proportion,[47] and likewise a rule for generating numbers that contain a specified part.[48] A mid-twelfth century commentary on the *Timaeus* by an otherwise unknown Hisdosus closely follows William's approach: he, too, offers considerable detail on the harmonic construction of the world soul, which amounts to a full-blown treatise on microtonal mathematics that proves how to calculate the ratio of the minor semitone and (as William did) employs a series of *regulae* as guideposts along the way.[49]

In music-theoretical and philosophical contexts alike, the Pythagorean tradition of mathematical harmonics remained ubiquitous throughout Late Antiquity and the Middle Ages. While the late-antique sources did convey some admixture of Aristoxenian thought (notably his definitions of pitch and interval, cited above), Aristoxenus' surviving treatises were not translated until the middle of the sixteenth century, and his alternative to Pythagoreanism received little attention before the Renaissance.[50] But Aristoxenus' own critique of his Pythagorean predecessors reveals that ratios were not the only hallmark of Pythagorean musical thought. He criticizes his predecessors for relying on two principles "extraneous," in his view, to a properly grounded music theory: "ratios of numbers and relative speeds."[51] We have dealt

47 Ibid., 152 (86.7–12): "Est enim regula in musica quod si duo numeri dicantur esse in aliqua superparticulari proportione et uelimus probare utrum ita sit an non, multiplicemus nomen a quo proportio denominatur per differentiam; si inde fit minor numerus sunt in illa proportione, sin aliter non sunt in illa."

48 Ibid., 150 (85.1–5): "Est alia regula artis arismeticae. Si in aliquibus numeris in aliqua proportione constitutis quaeramus partem quam non habeant, multiplicemus illos nomine partis quam quaerimus. Qui inde fient, in eadem erunt proportione et partem quam quaerimus optinebunt."

49 Hicks 2016, 33.846–848: "Illos uero numeros in quibus duos tonos continuos et lim⟨m⟩a ⟨contineri dicit Plato⟩ qui inuenire desiderat, hanc regulam artis arismeticae cordetenus retineat." Ibid., 34:874–875: "Oportet ergo hos numeros dimittere et alios sumere, quos per hanc facile quiuis reperiet regulam."

50 See Palisca 1993.

51 Aristoxenus 1954, 2.32.26; transl. Barker 1989, 149.

with the first; the second – "relative speeds" – takes us into the terrain of Pythagorean acoustics.

3 Pythagorean Acoustics

What exactly do the numbers in harmonic ratios quantify? Put another way, what are the natural philosophical grounds for the mathematicization of harmony? The answer to this question belongs to the science of acoustics, and though it remains preliminary to the science of harmonics proper, the topic was explored in depth by early Pythagoreans, as Aristoxenus suggests. Accordingly, the acoustical knowledge deemed necessary for an account of harmonics is summarized by Boethius in the first book of the *De institutione musica* at 1.3, i.e., the beginning of the treatise proper after the prooemium of chapters 1 and 2 (which concludes "This is enough by way of introduction; now we must begin the discussion of the fundamentals of music").[52] Thus we can assume that Nicomachus began his *Eisagoge* (Boethius' source) with a discussion of acoustics – a traditional Pythagorean starting point attested as early as Archytas.[53] In this opening chapter, Boethius defines sound as movement, produced by a blow, that is transferred to and transmitted through the air to the ears, and he quantifies this movement to establish a correspondence between number and pitch. In chapter 14, he describes this airy diffusion of sound with the Stoic example of waves in water.[54]

Boethius ultimately subscribes to a Peripatetic view of sound production and transmission. For Aristotle, the striking of a (potentially) resonant object

52 Boethius 1867, 189.12–13 (*Inst. mus.* 1.2): "Sed proemii satis est. Nunc de ipsis musicae elementis est disserendum."

53 Porphyry 1932, 56.11–12: "πρᾶτον μὲν οὖν [*sc.* τοὶ περὶ τὰ μαθήματα] ἐσκέψαντο, ὅτι οὐ δυνατόν ἐστιν εἶμεν [*scripsi cum Huffman*, ἦμεν Düring] ψόφον μὴ γενηθείσας πληγᾶς τινων ποτ' ἄλλαλα," etc., on which see Huffman 2005, 129–130. For a concise account of Pythagorean acoustics, see Barker 2014, 186–188.

54 Boethius 1867, 200.7–21 (*Inst. mus.* 1.14): "Tale enim quiddam fieri consuevit in vocibus, quale cum [in] paludibus vel quietis aquis iactum eminus mergitur saxum. Prius enim in parvissimum orbem undam colligit, deinde maioribus orbibus undarum globos spargit, atque eo usque dum defatigatus motus ab eliciendis fluctibus conquiescat. Semperque posterior et maior undula pulsu debiliore diffunditur. Quod si quid sit, quod crescentes undas possit offendere, statim motus ille revertitur et quasi ad centrum, unde profectus fuerat, eisdem undulis rotundatur. Ita igitur cum aer pulsus fecerit sonum, pellit alium proximum et quodammodo rotundum fluctum aeris ciet, itaque diffunditur et omnium circum stantium simul ferit auditum. Atque illi est obscurior vox, qui longius steterit, quoniam ad eum debilior pulsi aeris unda pervenit." See the parallels cited in Gottschalk 1968, 445, n. 3.

produces a movement within the medium (generally air, but water is allowed as well), and this movement is transmitted to the ear.[55] But it is only movement that is transmitted, not anything within the medium nor the medium itself. Thus, Aristotle would likely object[56] to the "missile" theory of sound found in Archytas (frag. 1)[57] and perhaps alluded to by Plato (*Tim.* 80a) – sound is not a body, neither air nor water (*De anima* 419b18), but a movement within a bodily medium. High and low sounds are produced by (but not identified as) fast and slow movements respectively. *De anima* 420a31–33 makes this clear: "It is not the case that the sharp [i.e., high-pitched] is swift and the heavy [i.e., low-pitched] slow: rather, the movement of the former acquires its quality because of its speed, that of the latter because of its slowness."[58] Aristotle thus continues to think of pitch in terms of variable speed, but the speed in question is the speed of the originating blow, not the speed of transmission, a crucial modification of the theory maintained by Archytas and Plato.[59]

Boethius begins his acoustical preliminaries: "Consonance, which rules every musical melody (*omnem musicae modulationem*), requires sound; sound cannot be caused without a blow or strike; a blow or strike cannot occur unless it is preceded by motion. If everything were immobile, nothing could strike anything else so that one thing impelled another, but if everything remained stationary and still, no sound could arise."[60] This adheres closely to the logic of the Euclidean *Sectio canonis*,[61] but Boethius' continuation – "hence, sound

55 *De anima* 420a4ff.

56 Cf. the Peripatetic *Problemata* 899b1–7.

57 βέλη: Porphyry 1932, 57.7–9. See Huffmann 2005, 140–141.

58 *De anima* 420a31–33: "οὐ δὴ ταχὺ τὸ ὀξύ, τὸ δὲ βαρὺ βραδύ, ἀλλὰ γίνεται τοῦ μὲν διὰ τὸ τάχος ἡ κίνησις τοιαύτη, τοῦ δὲ διὰ βραδυτῆτα"; transl. Barker 1989, 79.

59 See, for instance, Aristotle, *Sens.* 448a20–b2, as against Plato *Tim.* 80a–b; cf. Boethius 1867, 221.12–16 (*Inst. mus.* 1.30): "Plato autem hoc modo fieri in aure consonantiam dicit. Necesse est, inquit, velociorem quidem esse acutiorem sonum. Hic igitur cum gravem praecesserit, in aurem celer ingreditur, offensaque extrema eiusdem corporis parte quasi pulsus iterato motu revertitur."

60 Ibid., 189.15–22 (1.3): "Consonantia, quae omnem musicae modulationem regit, praeter sonum fieri non potest, sonus vero praeter quendam pulsum percussionemque non redditur, pulsus vero atque percussio nullo modo esse potest, nisi praecesserit motus. Si enim cuncta sint inmobilia, non poterit alterum alteri concurrere, ut alterum inpellatur ab altero, sed cunctis stantibus motuque carentibus nullum fieri necesse est sonum."

61 Cf. Barbera 1991, 114.1–6 (*Sectio canonis* Intro.): "εἰ ἡσυχία εἴη καὶ ἀκινησία, σιωπὴ ἂν εἴη· σιωπῆς δὲ οὔσης καὶ μηδενὸς κινουμένου οὐδὲν ἂν ἀκούοιτο· εἰ ἄρα μέλλει τι ἀκουσθήσεσθαι πληγὴν καὶ κίνησιν πρότερον δεῖ γενέσθαι. ὥστε ἐπειδὴ πάντες οἱ φθόγγοι γίνονται πληγῆς τινος γινομένης πληγὴν δὲ ἀμήχανον γενέσθαι μὴ οὐχὶ κινήσεως πρότερον γενομένης"; compare with the translation from Boethius 1867, 301.12–16 (*Inst. mus.* 4.1): "Si foret rerum omnium quies, nullus auditum sonus feriret. Id autem fieret, quoniam cessantibus motibus cunctis nullae inter se res pulsum cierent. Ut igitur sit vox, pulsu est opus. Sed ut sit pulsus, motus

is defined as a percussion of air that remains undissolved all the way to the hearing" (*idcirco definitur sonus percussio aeris indissoluta usque ad auditum*) – parts ways with the *Sectio canonis* and reveals the (probable) Nicomachean source text.[62] In line with post-Aristotelian developments in (Peripatetic) acoustical thought, Boethius quantifies pitch not in terms of the velocity of transmission, nor even (solely) in terms of the velocity of the blow,[63] but in terms of the variable rate of pulsation, which encompasses both velocity and frequency.[64] The continuation of this claim, namely that the sound produced by a vibrating string, although perceived as continuous, is actually a discrete set of pulsations, is nowhere developed in Nicomachus' extant works. Boethius' example is a spinning top with a single strip of color; once spinning "the whole top seems to be dyed red, not because it is really completely red, but because the speed of the red stripe overwhelms the uncolored sections and does not allow them to be seen."[65] This metaphor is not original to Boethius (witness the translator's gloss: *quem turbinem uocant*), and this "particulate" perspective on sound, combined with the example of "color blurs," is found twice in Porphyry's commentary on Ptolemy's *Harmonics*: once in a quotation from Heraclides' *Eisagoge*,[66] and once in an extended quotation from the

necesse est antecedat. Ut ergo sit vox, motum esse necesse est." See Barbera 1991, 115 and 231. On the acoustic theories of the *Sectio canonis*, see Creese 2010, 164–171.

62 It is very nearly a literal translation of a similar claim by Nicomachus: "καθόλου γάρ φαμεν ψόφον μὲν εἶναι πλῆξιν ἀέρος ἄθρυπτον μέχρι ἀκοῆς" (Nicomachus 1895, 242.20–21 [*Enchiridion* 4]). The Latin adheres much more closely to Nicomachus than to the parallel claim at *De anima* 420a3–4: "ψοφητικὸν μὲν οὖν τὸ κινητικὸν ἑνὸς ἀέρος συνεχείᾳ μέχρις ἀκοῆς"; e.g., Boethius' "indissoluta" is Nicomachus' "ἄθρυπτον" (both modifying the blow) and not Aristotle's "συνεχείᾳ," which characterizes the air (a crucial distinction).

63 Cf. Boethius 1867, 189.23–190.1 (*Inst. mus.* 1.3): "Motuum vero alii sunt velociores, alii tardiores, eorundemque motuum alii rariores sunt alii spissiores. Nam si quis in continuum motum respiciat, ibi aut velocitatem aut tarditatem necesse est ut conprehendat, sin vero quis moveat manum frequenti eam motu movebit aut raro. Et si tardus quidem fuerit ac rarior motus, graves necesse est sonos effici ipsa tarditate et raritate pellendi. Sin vero sint motus celeres ac spissi, acutos necesse est reddi sonos."

64 Ibid., 190.2–11: "idcirco enim idem nervus, si intendatur amplius, acutum sonat, si remittatur, grave. Quando enim tensior est, velociorem pulsum reddit celeriusque revertitur et frequentius ac spissius aerem ferit. Qui vero laxior est, solutos ac tardos pulsus effert rarosque ipsa inbecillitate feriendi, nec diutius tremit. Neque enim quoties chorda pellitur, unus edi tantum putandus est sonus aut unam in his esse percussionem, sed totiens aer fertitur, quotiens eum chorda tremebunda percusserit."

65 Ibid., 190.15–21: "velut si conum, quem turbinem vocant, quis diligenter extornet eique unam virgulam coloris rubri vel alterius ducat, et eum qua potest celeritate convertat, tunc totus conus rubro colore videtur infectus, non quo totus ita sit, sed quod partes puras rubrae virgae velocitas conprehendat et apparere non sinat."

66 Porphyry 1932, 30.1–31.21.

Peripatetic *De audibilibus*, which Porphyry attributes to Aristotle.[67] Barker has argued that these sources still operate on the assumption of pitch determination through velocity and not through frequency of impacts or pulses (since the latter is the indirect result of the former: a swifter velocity yields more frequent impacts).[68] The *Sectio canonis*, however, argues the point directly: "of motions, there are the more dense and the more rare. The more dense produce higher notes, and the more rare, lower."[69] Boethius' presentation thus combines the pitch determination argument of the *Sectio canonis* with the top metaphor as in the *De audibilibus* and Heraclides. As Nicomachus seems to have modeled his discussion of sound production on the *Sectio canonis*, and since the theory presented in 1.3 is entirely consistent with the theory of consonance attributed to Nicomachus at 1.31, it seems most economical to assume that Nicomachus is responsible for shaping the material, not Boethius. Thus, similarities with *De anima* 2.8 or the Peripatetic tradition generally is due to the Peripatetic strand of thought that entered the Greek music-theoretical tradition of "Pythagorean" acoustics.

Boethius' "wave model" of sound propagation proved influential in medieval traditions of Latin grammatical thought, in which it offered an attractive model for the transmission of a *vox* (vocal utterance), but it was not (re)incorporated into Pythagorean contexts.[70] The "particulate" view of sound as discrete pulsations, however, was revived when the Aristotelian *Problemata* received new attention in the fourteenth century. Gabriela Currie has called attention to the fourteenth-century reintegration of "Boethian" acoustics, itself a Nicomachean (post-)Peripatetic reinterpretation of Pythagorean acoustics, into a deeply Pythagorean framework in the thought of the fourteenth-century mathematician and natural philosopher, Nicole Oresme, specifically his *Treatise on the Configurations of Qualities and Motions* (*Tractatus de configurationibus qualitatum et motuum*). Citing Boethius, Oresme concurs that in sensible sound there is "a certain discreteness brought about by the interposition of pauses which sometimes are so frequent and small that they are not perceived by the ear and the whole seems to be one continuous sound, as Boethius has it in the first book of his *Music*."[71] Grounding his theory in Boethius' *De institutione musica*, Oresme conjectures that the perception of consonance arises from the "symphonic mixing of these small sounds" (*parvulorum sonorum*

67 Ibid., 67.24–77.18.

68 Barker 1989, 107, n. 40 and 236, n. 110. For an opposing view, see Gottschalk 1968.

69 Barbera 1991, 114.6–8 (*Sectio canonis* Intro.): "τῶν δὲ κινήσεων αἱ μὲν πυκνότεραί εἰσιν· αἱ δὲ ἀραιότεραι· καὶ αἱ πυκνότεραι ὀξυτέρους ποιοῦσι τοὺς φθόγγους· αἱ δὲ ἀραιότεραι βαρυτέρους."

70 See Hicks 2017, 173–188.

71 Oresme 1968, 305.

simphonica conmixtio); furthermore, all concordant mixtures arise from "harmonic numbers" (*harmonici numeri*), that is, the theoretically infinite series obtained from the powers of two and three, as in the world soul discussed above; on the converse, ugly sonorities arise from ratios of non-harmonic numbers or, uglier still, irrational ratios.[72] Currie summarizes well: "what Oresme puts forth is a mathematical justification of a hierarchical aesthetic positioning of tone qualities, where the sound particles stand within or outside the confines of the harmonic series: from the 'most beautiful' already endorsed in the Pythagorean mathematical tradition as expressions of the *tetraktys*, to the ugliest, expressed in irrational ratios."[73]

But this brief digression into the fourteenth century confronts us with one of the most basic tensions at the heart of Pythagorean music theory: the uneasy intersection of a perceptually based acoustical theory and a rationally grounded theory of numerical relationships. What is the relationship between the perception of musical intervals as harmonious and the rational determination of the ratios that govern such harmonies? What, in short, is the relationship between perception and reason? Here we must return, one last time, to Boethius' "Pythagoras."

4 Perception and Reason in Pythagorean Harmonics

As Klaus-Jürgen Sachs has rightly highlighted, "Boethius' teaching concerning *sensus* and *ratio* is among the most frequent cited topics from the *De institutione musica*," and Boethius' presentation of the "Pythagorean" position largely defined the discipline of music theory for at least a millennium.[74] In short, Boethius' Pythagoreans steer a middle course between perception (*sensus*) and reason (*ratio*), between, that is, Plato's criticism that the Pythagoreans were too concerned with the audible realm (*Resp*. 513a1–3) and Ptolemy's complaint that the Pythagoreans were excessively theoretical and "did not follow the impressions of the hearing even in those things where it is necessary for everyone to do so" (*Harm*. 1.2).

Pythagorean hesitations about the reliability of perception are of central concern to Boethius from the very outset of his treatise. It may be effortless, Boethius claims, to recognize that we somehow employ sensation for the perception of sensibles, but the precise nature of that sensation, as both activity

72 Ibid., 311–315.

73 Currie 2011, 147.

74 Sachs 1991, 171.

and content, is a matter of dispute.[75] Boethius' first example of the problem is not hearing, but vision, which remained for Boethius, as for the entire ancient tradition, the paradigmatic sense. But Boethius generalizes the epistemological and ontological problems inherent in vision to all sense-perceptible objects. And the stakes are apparently higher with regard to the "judgement of the ears" (*arbitrium aurium*), for "the faculty of hearing (*vis aurium*) strives to comprehend sounds (*sonos captat*) in such a way that it not only forms judgments about them and recognizes differences between them, but even more often it is delighted if their measure be sweet and well joined, or it is distressed if they strike the sense as ill-arranged and unconnected."[76] Although sight may offer the paradigm of sensation, hearing is yet the most valuable (if vulnerable) sense, insofar as it offers the most direct route to instruction or knowledge (*nulla enim magis ad animum disciplinis via quam auribus patet*), a claim analogous to the Aristotelian stance on the superiority of hearing for the acquisition of knowledge (*De sensu* 437a10). Hearing, moreover, is the very origin of (at least central aspects of) the discipline of music (cf. *De anima* 432a7–8). But if hearing is a necessary first principle, it alone is not sufficient; perception serves rather as a kind of exhortation or admonition (*quasi admonitio*) to the reasoning faculty to flesh out the occasionally confused and specious perceptions of the ears.[77] The sustained argument of *Inst. mus*. 1.9, ostensibly articulating the position of the Pythagoreans, makes the point clear: the judgment of the ears is blunt (*obtusa*), and without the support of *ratio*, it has no sure judgment, no comprehension of truth. But Boethius stops short of denying aural criteria (*iudicium aurium*) any role whatsoever within Pythagorean harmonics. In fact, the very phrase *iudicium aurium* allows for the judgement of sensibles within the domain of perception (which thus cannot be entirely passive).[78]

A crucial passage that points in this direction – a passage that momentarily bridges the seemingly unbridgeable gap between perception and

75 Boethius 1867, 179.2–8 (*Inst. mus*. 1.1): "Inlaboratum est enim quod sensum percipiendis sensibilibus rebus adhibemus; quae vero sit ipsorum sensuum, secundum quos agimus, natura, quae rerum sensibilium proprietas, id non obvium neque cuilibet explicabile esse potest, nisi quem conveniens investigatio veritatis contemplatione direxerit."

76 Ibid., 179.15–20: "Idem quoque de ceteris sensibilibus dici potest, maximeque de arbitrio aurium, quarum vis ita sonos captat, ut non modo de his iudicium capiat differentiasque cognoscat, verum etiam delectetur saepius, si dulces coaptatique modi sint, angatur vero, si dissipati atque incohaerentes feriant sensum."

77 Ibid., 195.18–23 (1.9): "Nam si nullus esset auditus, nulla omnino disputatio de vocibus extitisset. Sed principium quodam modo et quasi admonitionis vicem tenet auditus, postrema vero perfectio agnitionisque vis in ratione consistit, quae certis regulis sese tenens nunquam ullo errore prolabitur."

78 Ibid., 223.24–25 (1.33); 220.2–3 (1.28).

reason – occurs at 2.18, wherein Boethius ranks the consonances "according to the Pythagoreans" on the basis of their merit and measure. The diapason (the octave, 2:1) graces the top of the list, but the argument for its excellence is grounded in perception, and strikingly so. In a passage that has puzzled modern expositors, Boethius writes:[79]

> The consonance whose property sense perception (*sensus*) apprehends more readily ought to be classified as the primary and most pleasant (*prima suavisque*) consonance. For everything is apprehended through sense perception to be such as it is in itself (*quale est enim unumquodque per semet ipsum, tale etiam deprehenditur sensu*). If, therefore, the consonance that consists in the duple ratio is better known to everyone (*cunctis*), then there can be no doubt that the octave is the first of all consonances and is surpassing in merit, because it comes first in cognition.

This crucial passage seems *prima facie* to break ranks with the Pythagoreans and side, instead, with the Aristotelian claim that proper sensibles are always perceived veridically by their corresponding sense (and the proper sensible of hearing is sound: see *De anima* 418a12). There are two hints – one lexical, the other contextual – that Boethius here intends such a claim at this point in his treatise. Lexically, the appearance of the adjective "pleasant" (*suavis*) is neither innocent nor otiose here; rather, it deliberately evokes Boethius' definition of consonance, which is consistently couched in aesthetic terms that trade on an irreducibly sense-perceptible property: "pleasantness" (*suavitas*).[80] A consonance, to cite but one instance of Boethius' repeated definitions, is "a mixture of high and low sounds pleasantly (*suaviter*) and uniformly falling upon the ears."[81] Hence, the pleasantness of a consonance is first and foremost a feature of its perception. Contextually, this passage functions explicitly as a transition from the fundamentally arithmetical concerns of 2.1–17 – the theory of ratios (*proportiones*) and means (*medietates*) – to the fundamentally musical

79 Ibid., 249.22–29 (2.18): "Haec enim ponenda est maxime esse prima suavisque consonantia, cuius proprietatem sensus apertior conprehendit. Quale est enim unumquodque per semet ipsum, tale etiam deprehenditur sensu. Si igitur cunctis notior est ea consonantia, quae in duplicitate consistit, non est dubium, primam esse omnium diapason consonantiam meritoque excellere, quoniam cognitione praecedat." Surprisingly, Sachs omits this passage from his synopsis of the "most important quotations from the *De institutione musica* concerning the criteria" (Sachs 1991, 171–175).

80 On Boethius' use of *suavis*, see Hentschel 2000, 24–25.

81 Boethius 1867, 195.6–8 (*Inst. mus.* 1.8). Cf. 302.2–4 (4.1); 357.13–14 (5.7); 361.10–12 (5.11); and Nicomachus 1895, 262.1–4 (*Enchiridion* 12).

concerns of 2.18–30 – the connection between ratios and consonances, namely which ratios correspond to which musical intervals.[82] This chapter thus inaugurates a discussion of "how the Pythagoreans proved that the musical consonances are associated with the ratios discussed above," and this transition thus seeks to bridge the gap between Boethius' definitions of consonance, which are dependent upon sense perception, and arithmetical ratios, which are understood through the application of reason.[83] The octave, the sense-perceptible manifestation of the ratio 2:1, is not just the simplest mathematical ratio; it is also, in a more basic way, readily apprehended to be such through perception. The point is simple: who would deny that the octave sounds consonant? It is as easily recognized as such by any reasonable listener as a shape is recognized by a reasonable observer to be a square or a triangle (cf. *Inst. mus*. 1.1). Notably, Augustine makes an identical point at *De trinitate* 4.2, observing that even untrained listeners (*imperiti*) recognize the "consonance of one to two". Admittedly, for Augustine this point is less about perception *per se* and more about the fundamental role that the 1:2 ratio seems to play in Christian salvation (insofar as the singular death of Christ's body harmonizes [*congruit*] with our double death of body and soul), but he too thinks this point can be demonstrated to the ears on a tuned monochord. This is not yet to claim, however, that a listener would know from perception alone the real nature of consonance any more than a casual glance would reveal the mathematical nature of a triangle or square.

Nor is this claim as radically contradictory to (Boethius' presentation of) Pythagorean thought as it might seem. It well concords with Boethius' earlier claim that the Pythagoreans "investigate certain things only by the ear." These investigations include, he continues, the measuring of consonances, although the precise calculations of how the consonances differ among themselves is entrusted only to *ratio*, the judge and ruler over subservient *sensus*.[84] Similarly, early commentators on Pythagorean harmonics emphasize the foundational role of perception in establishing the basic nature of consonance, even if reason ultimately plays a trump card in some special cases (e.g., the eleventh).[85] For instance, Ptolemais' *Eisagoge* claims that "Pythagoras and his successors

82 Note the chapter's opening words: "sed de his hactenus" (but enough about this).

83 Hentschel 2000, 29–32.

84 Boethius 1867, 196.1–7 (*Inst. mus*. 1.9): "Nam nec omne iudicium dedunt auribus et quaedam tamen ab eis non nisi auribus explorantur. Ipsas enim consonantias aure metiuntur, quibus vero inter se distantiis consonantiae differant, id iam non auribus, quarum sunt obtusa iudicia, sed regulis rationique permittunt, ut quasi oboediens quidam famulusque sit sensus, iudex vero atque imperans ratio."

85 See Barbera 1984.

[…] wish to accept perception as a guide for reason at the outset, to provide reason with a spark, as it were; but they treat reason, when it has set out from these beginnings, as working on its own in separation from perception."[86] The passage in Boethius provides precisely this sort of spark; it is an attempt to fan the spark of sensation into the fire of rational knowledge.

Admittedly, Boethius is (momentarily) more optimistic about the accuracy of perceptual judgement than "orthodox" Pythagoreanism would seem to allow; nonetheless, the tension between reason and perception in Boethius' account of the "Pythagoreans" set in motion nearly a millennium of music-theoretical speculation. To write the medieval history of this Pythagorean music-theoretical speculation, however, would entail no less than a survey of the entirety of medieval harmonic theory, an impossibility here.[87] Two examples must suffice to indicate the complexities involved: first, an example of late ninth-century resistance to the Pythagorean quantitative, rational framework, even from within a (late) Platonic vantage point; second, a thirteenth-century reinterpretation of the consonances *secundum auditum* (according to hearing), which accords greater role to perception even as it builds the ratios of the consonances in question through an extension of the Platonic division of the world soul.

A passage from the late ninth-century *Musica enchiriadis* (*Music Handbook*), a text famous for its integration of Boethian authority within the monastic tradition of plainchant and the burgeoning tradition of polyphonic singing, offers what Calvin Bower has called "a thinly veiled argument with the Pythagorean position concerning the epistemology of music as articulated by Boethius."[88] The sweetness of consonant sounds and the unpleasant disagreement of discordant sounds, the author tells us, "relates to an even deeper and divine theory, to a theory remaining concealed among certain things within the most hidden places of nature."[89] Although the author, Bower argues, undoubtedly knew Boethius and the mathematical explanation for consonance and dissonance, "the scholar writing the closing passage of the *Musica enchiriadis* seems to want no part of such a reduction."[90] Instead, the author turns to Fulgentius' version of the myth of Orpheus as an alternative to Pythagoras: whenever a

86 Porphyry 1932, 25.25–30; transl. Barker 1989, 242.

87 See Sachs 1991 for a concise survey and Hentschel 2000 for a detailed investigation of the problem in the thirteenth and fourteenth centuries.

88 Bower 2002, 35.

89 Schmid 1981, 56: "Cur namque aliqui tam dulci ad invicem commixtione consentiant, alii vero soni sibi misceri nolentes insuaviter discrepent, profundioris divinaeque est rationis et in aliquibus inter abditissima naturae latentis."

90 Bower 2002, 38; see also Boynton 1999.

good man (Aristeus) grasps for "profound understanding" (Eurydice), beloved of the cantor with "the best voice" (Orpheus), "he is hindered by divine wisdom, lest she be entirely possessed." But even when the noblest sound of song (Orpheus) summons her from the underworld, "as soon as she seems to be seen, she is lost."[91] Neither reason nor perception allow the revelation of the divine mysteries that are summoned by song.

John of Garland's *De musica mensurabili*, dating from around the middle of the thirteenth century, divides *consonantiae* (which here means something more like generic "intervals") into two main classes, concords (*concordantiae*) and discords (*discordantiae*), defined as follows: "An interval is called a concord when two pitches are joined at the same time in such a way that one pitch is compatible with the other according to the sense of hearing. It is said to be a discord when the opposite obtains."[92] The role of the *iudicium aurium*, the judgement of the ears, was, as we have seen, essential to Boethius' ranking of the consonances, but the extent of John's appeal to the senses in the subtle discriminations of three new subclasses of intervals (what John calls "perfect," "medial" and "imperfect" concords and discords) goes well beyond what Boethius and the "Pythagoreans" would allow. John's sub-classifications *secundum auditum* (according to the sense of hearing), moreover, permit him to class major and minor thirds as "imperfect concords," whose ratios (81:64 and 32:27 respectively) are neither multiple nor superparticular and thus heretofore had been excluded from the realm of concords. But even within this new, perceptually driven aesthetics of concord and discord, the ratios that John associates with his newly classified intervals remain resolutely "Pythagorean" in origin. As Christian Meyer and Michel Huglo have argued, they rely on (an extension of) the powers of two and three that had governed the structure of the world soul, even though the "metaphysical" importance and Pythagorean pedigree of these ratios have been nullified through their

91 Schmid 1981, 57: "Fictum est ab antiquis Aristeum Euridicem nympham Orphei coniugem adamasse. Quemque dum illa se sequentem fugeret, a serpente extincta sit. Orpheum, cuius nomen oreo phone, id est optima vox sonat, in cantore perito seu dulcisono cantu intellegimus. Cuius Euridicem, id est profundam diiudicationem, si quis vir bonus, quod Aristeus interpretatur, amando sequitur, ne penitus teneri possit, quasi per serpentem divina intercipitur prudentia. Sed dum rursus per Orpheum, id est per optimum cantilenae sonum, a secretis suis acsi ab inferis evocatur, imaginarie perducitur usque in auras huius vitae dumque videri videtur, amittitur, scilicet quia inter cetera, quae adhuc ex parte et in enigmate cernimus, haec etiam disciplina haud ad plenum habet rationem in hac vita penetrabilem."

92 John of Garland 1972, 1.67: "Concordantia dicitur esse, quando duae voces iunguntur in eodem tempore, ita quod una vox potest compati cum alia secundum auditum. Discordantia dicitur contrario modo."

very much non-Pythagorean classification.[93] By the later fourteenth century, the nullification of a Pythagorean "metaphysics" of number was largely complete: Johannes Boen, for instance, describes the preference for thirds and sixths (over octaves and fifths) as matter of time and place.[94] Different people simply like different sounds in different historical periods and regions, Pythagoras included. Boen historicizes as a matter of taste what had been a universal metaphysics of number.

5 Conclusions

Despite abundant testimony to the continuity of Pythagorean thought across the Middle Ages, we cannot speak of medieval "Pythagoreanism(s)" in the same way that we can of medieval Platonism(s) and medieval Aristotelianism(s). Accounts of Pythagoras and the ancient *Pythagorici* are largely consigned to doxographical contexts; they belong not to contemporaneous debates but rather to an imagined history of philosophical and mathematical thought. But if Pythagoreanism *per se* was no longer viable, it was precisely within the history of philosophy and the tradition of music-theoretical speculation that Pythagoreanism gained considerable traction. It was not a question of what Pythagoras "really" thought – though medieval authors were optimistic about the recoverability of Pythagorean thought[95] – but rather a question of how

93 See Huglo 2005, 315–316 and Meyer 2007, 69–73. John's ratios are as follows. Perfect concords: 1:1 (unison), 2:1 (octave); medial concords: 3:2 (fifth), 4:3 (fourth); imperfect concords: 81:64 (the ditone, or major third), 32:27 (semiditone or minor third); imperfect discords: 16:9 (the semiditone with diapente, or minor seventh), 27:16 (the tone with diapente, or major sixth); medial discords: 9:8 (the whole tone), 128:81 (the semitone with diapente, or minor sixth); perfect discords: 243:128 (the ditone with diapente, or major seventh), 256:243 (the semitone), and 729:512 (the tritone). See John of Garland 1972, caps. 9–10 (1.67–75).

94 Frobenius 1971, 76–77: "Diverse namque regiones diversos cantus exigunt, ut in hoc experimento – dum scolas Oxonienses in Anglia colui, quam regionem a Comitatu Hollandie, loco mee nativitatis, solum mare discriminat – audito, quod layci ibidem et clerici, senes, iuvenes et indifferenter omnes tertiis et sextis tantam atribuebant affectionem quodque, duplis et quintis postpositis, ipsas solas invocantes quasi adorare videbam; vehementer attonitus de tam vicine regionis diversa natura continue ammirabar. [...] Tertia vero aut sexta per se nec Pytagore nec Ptolomei temporibus umquam consonuit, saltem in regione ipsorum, nostre autem regioni, qua tertiis et sextis, et nullomodo sesquitertie proportioni per se, insistimus."

95 E.g., both Calcidius (1975, 177.2 [*In Tim.* 136]) and Macrobius (1963, 25.25–31 [*In Som. Scip.* 1.6.41]) accept the *Golden Verses* as authentically Pythagorean; Adelard of Bath claims that Pythagoras "wrote down his findings" (*officio stili usus est*), lest they be lost

to deploy appropriately the Pythagorean inheritance within the new cultural, philosophical, and musical contexts of (largely) Christian patterns of thought.

It must also be stressed that much of what circulated under Pythagoras' name has little or nothing to do with the "historical" Pythagoras. The Pythagoras and Pythagoreanisms I have discussed are *not* those of modern historians of philosophy, but, as already noted, of an imagined history inherited and embellished by medieval authors. (Nor is this imagined history solely the product of the medieval imagination: the available evidence had already been tampered with for generations.) In vain would we search amid the copious medieval testimony of Pythagoras and the Pythagoreans for "authentic" Pythagorean fragments, though some of the surviving *testimonia* are of considerable, if second- or third-hand, antiquity.[96] Rather, the medieval Pythagoras was deeply conditioned by the late-ancient overwriting (aptly characterized by Christoph Riedweg as a "palimpsest") of Pythagorean positions with what is essentially late Platonic doctrine.[97] In the memorable phrase of Apuleius, "Plato Pythagorized in many respects" (*Plato pythagorissat in plurimis*: *Florida* 15.25; cf. Aëtius 2.6.6), which eventually transforms – through the medium of Augustine (e.g., *c. acad.* 3.17.37) *et al.* – into a simple, factual statement by William of Conches: "Plato was a Pythagorean."[98] It was this Platonizing Pythagoreanism that deeply colored the primary late-ancient Latin accounts of Pythagoras that in turn formed the essential foundation for Pythagoras' long medieval *Nachleben*, which lived on in philosophical commentaries, speculative treatises on music theory, and even the occasional bar song.

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to posterity (Adelard of Bath 1998, 54); Hugh of St. Victor attributes to Pythagoras a work entitled *Methen tetrados*, which he describes as "a book on quadrivial teaching" (1939, 49 [*Didascalicon* 3.2]), a claim perhaps resulting from a misreading of Martianus, *De nuptiis* 2.107 (*via* Remigius of Auxerre 1962, 150 [*Comment. in Mart.* 44.20]); finally, the *Turba philosophorum* (translated into Latin in the twelfth century from Arabic) posed as a "first-hand" account of a gathering of philosophers presided over by Pythagoras (on which see Ruska 1931).

96 E.g., the citations of Eubulides and Hippasus at *Inst. mus.* 2.19; the paraphrases from Philolaus and Archytas at *Inst. mus.* 3.5–8 and 3.11, respectively; the Numenian positions articulated by Calcidius at *In Tim.* 256; etc.

97 Riedweg 2005, 128; cf. O'Meara 1989, 16.

98 William of Conches 1997, 47 (*Dragmaticon* 2.5.4); cf. Id., 2006, 23 (*Glosae super Platonem* 12.18).

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CHAPTER 3

Nicomachean Number Theory in Arabic and Persian Scholarly Literature

Sonja Brentjes

Nicomachus of Gerasa (2nd century CE) wrote a philosophically oriented introduction to elementary number theory. The philosophical underpinning of his work and the replacement of formal proofs by particular numerical examples are two main features that set it apart from Euclid's treatment of number theoretical themes in Books VII–IX of the *Elements*. Further differences between the treatment of number theory by the two ancient writers concern the more general and abstract character of the statements and their broader topical range in the *Elements*, Book VII–IX compared to those found in the *Introduction to Arithmetic*. Euclid proves, for instance, theorems on prime and compound numbers, among them the existence of infinitely many prime numbers or the representation of any natural number as the product of an ordered set of prime numbers. Nicomachus also discusses themes not found in the *Elements*, but they concern other classes of numbers such as polygonal numbers and their partial sums or arithmetical proportions. These differences do not signify, however, a complete, clear-cut separation between these two approaches to number theory, either in Antiquity or in Islamicate societies from the ninth to the nineteenth centuries.[1] The differences and overlaps between these two ancient approaches to number theory were reflected upon by various authors of mathematical or philosophical treatises and books. Ibn al-Haytham (d. after 1040), for instance, offered the following evaluation focusing on the methodological aspect:

> The properties of numbers exhibit themselves in two ways: the first is by induction, for, if one follows the properties of numbers one by one, and if one distinguishes them, in distinguishing and considering them, one discovers all of their properties. This is shown in the work *al-Arithmāṭīqī* <*sc*. Nicomachus' *Introduction to Arithmetic*>. The other way that exhibits the properties of numbers proceeds by means of demonstrations

1 For a different view, see Rashed 1994, 276–277.

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> and deductions. All the properties of number [*sic*] grasped by demonstration are contained in these three books of <Euclid> or in what is based on them.[2]

Abū Jaʿfar al-Khāzin (d. 971) seems to have looked on both authors and their methods and themes in a less opposing manner:

> It is appropriate to derive from the fundamental theorems the immediate conclusions without trying to increase the number of propositions. This applies to results and questions which Nicomachus has given us in number theory as well as for those in the *Elements*, which Euclid has taken from number theory into his three arithmetical books (components), that he proved with the help of lines and that he crowned with the study of perfect numbers, which are the highest goal.[3]

The main topics inspired by Nicomachus' *Introduction to Arithmetic* found in Arabic and Persian mathematical texts concern the definition of one as the root of all numbers, the definition and properties of classes of integers, the summation of a finite number of consecutive members of such classes, for instance natural numbers, even or odd numbers, square numbers or a special type of plane numbers – the heteromecic numbers n(n+1), plane and solid figurative numbers like triangular, pentagonal, hexagonal, cubic, pyramidal, scalene, or brick numbers, perfect, deficient, and abundant numbers, as well as specific kinds of ratios and proportions. An important reason for including such topics in teaching texts, both during the period of mainly private teaching efforts until about the early twelfth century and in classes held by madrasa teachers afterwards was the factorization of integers that the classification of numbers and the determination of proper divisors helped to teach.

A good part of Nicomachus' treatment of ratios does not appear in number theoretical texts, but in works on theoretical music, which was seen since Antiquity as the fourth mathematical discipline after number theory, geometry, and astronomy.[4] Philosophical interpretations were often excluded from the treatment of these topics in mathematical texts. When they occur, they appear primarily in the context of discussions about the definitions of one or unity and number. They can also be found in philosophical and religious debates, in particular in discussions about emanation. Encyclopaedias often

2 Quoted after Rashed 2015, 334.
3 Translated after Anbouba 1979, 154.
4 Nicomachus 1926, 184–185.

include chapters on number theory, either presenting and reflecting on topics from the *Introduction to Arithmetic* and the *Elements* or combining them with material from other mathematical fields.

Novel results are primarily connected with the search for rules generating new kinds of numbers such as balanced numbers or the calculation of new specific representatives of known rules. Such studies also raised new questions, which in several cases have not yet found an answer even today. An example is the question of whether odd perfect numbers exist. New approaches include, in particular, the application of algebra and the use of number theoretical questions in combinatorics and the generation of magic squares (or the harmonious disposition of numbers, as those figures are called in Arabic texts).[5] Outside of treatises on mathematical knowledge, results and methods from the *Introduction to Arithmetic* are known from medical works.

Further ancient Greek treatises that include Neo-Pythagorean number theoretical topics such as those by Theon of Smyrna (d. after 132), Iamblichus (fl. *ca* 300), or Johannes Philoponos (d. *ca* 575) do not seem to have been translated into Arabic, although individual themes or concepts such as the amicable numbers, not found in Nicomachus' *Introduction to Arithmetic*, seem to have been transmitted through unknown means.

In my contribution, I will trace the appropriation of Nicomachus' work through translations, paraphrases, and extracts in Arabic and Persian scholarly texts between the early ninth and, approximately, the seventeenth centuries. I will discuss the impact that the work, its perception as belonging to Pythagorean doctrine, and individual themes from it had on mathematical discussions, teaching practices, or the development of new ideas. I will also point out the use of individual themes from it outside of number theory proper and make an effort to situate number theoretical knowledge in its sociocultural contexts.

1 Translations

Nicomachus' *Introduction to Arithmetic* was twice translated into Arabic, once in the early ninth century from Syriac, the other time probably in the second half of the century from Greek. The names of the two translators are known. The first was the metropolitan of the Church of the East in Harran, Mosul, and Hazza Ḥabīb b. Bahrīz (late eighth-early ninth centuries). He undertook his work for the later founder of the Tahirid dynasty Ṭāhir b. al-Ḥusayn (d. 822),

5 Djebbar 1999 and 2000; Sesiano 1991, 2003 and 2017; Rashed 2015.

possibly during the latter's governship of al-Jazīra, the region in which Ḥabīb was active, from 813 to 819. The Syriac version is believed to have been produced in the later eighth or early ninth century. Its translator is unknown. Both the Syriac version and its Arabic translation are lost. A Hebrew translation of a revised and commented paraphrase of the Arabic text undertaken by Qalonymos ben Qalonymos (1286–after 1328) in 1317, perhaps in Arles, is the only extant trace of this product of a multilingual and multicultural knowledge sphere as it had emerged in Baghdad, the capital of the new caliphal dynasty, the Abbasids (r. 750–1258), founded in 762. Christians, Jews, Muslims, Zoroastrians, and most likely adherents of other late antique religious cults had come to the new capital to profit from the opportunities offered by the new dynasty and its cultural and economic policies. Although we still do not understand precisely the complex processes that led to the translation of a substantial body of scientific texts from Greek, Syriac, Middle Persian, and even Sanskrit into Arabic, we know that many of the mathematical texts were translated for members of the Abbasid court, for caliphs, princes, grand viziers and other members of the high and powerful civilian as well as military elite. One of the famous scholarly members of this social group was Abū Yūsuf Yaʿqūb b. Isḥāq al-Kindī (*ca* 800–after 870). He was the Arab scholar who taught Ḥabīb b. Bahrīz's Arabic translation of Nicomachus' work to a number of students, criticizing both the translation as well as Nicomachus' text. Freudenthal and Lévy argue that the paraphrase was the independent work of one of those students.[6] I find this difficult to accept because it contradicts the culture of teacher-student relationship not merely during the ninth century, about which we actually do not know very much, but also in much later times when the sources are more abundant and explicit. A student writing an independent work on oral lectures without any involvement of the teacher in the production of the text is an unusual phenomenon. Moreover, the quotes attributed to al-Kindī in the third person singular do not signify that they were written by someone else than the Arabic philosopher. This form was the standard type of authorial parlance in that period. Thus, they rather point to al-Kindī as the author of the Arabic paraphrase. In a later paper, Freudenthal and Zonta propose to situate the final version of the Arabic paraphrase after al-Kindī's death. A second stage of its production seems to have taken place according to the extant text due to a reader's request to provide him with Nicomachus' philosophical introduction from the beginning of his work.[7] In a third stage, Freudenthal and Zonta believe, this later description of the first five sections

6 Freudenthal and Lévy 2004, 481–483, 514–517; Freudenthal in the present volume.
7 Freudenthal and Zonta 2007, 68–69.

of the Greek text together with the student's introductory remarks became attached to the earlier "lecture notes."[8]

The chapter headings of Qalonymos' Hebrew translation of the paraphrase of Nicomachus' book and al-Kindī's commentaries show that the entire text of the *Introduction to Arithmetic* had been translated into Arabic. After an introduction by al-Kindī's student follows a discussion of the philosophical content of the first five chapters of Book I of the *Introduction to Arithmetic* combined with comments by al-Kindī. The next eighteen sections elaborate the content of Book I. Book II is explained in twenty sections. This division of the paraphrase is thus shorter than that found in Nicomachus' work. In a number of cases, material found in the *Introduction* in separate chapters has been integrated into a single section. Furthermore, the placement of the content of the Hebrew paraphrase does not always coincide with the corresponding content of the *Introduction*. Terminological problems complicate the evaluation of the Hebrew text, which so far has not been completely translated.[9]

A summary of Nicomachean number theory is found in a book on history by the ninth-century official of the administration of the Tahirids in Khurasan Ibn Wāḍiḥ al-Yaʿqūbī (d. *ca* 897/8), who later worked for the Tulunids (r. 868–905) in Cairo. It is thought that he finished his history called *Taʾrīkh* (*Annals*) shortly before 873, the last year of Tahirid rule in Northeastern Iran and Central Asia. He believed that Nicomachus' work consisted of three parts in addition to an introduction which presented philosophical views on the arrangement of the world according to numbers and the bi-partite division of numbers into even and odd. This suggests that he did not summarize a translation of Nicomachus' work, but some paraphrase, maybe the lost *Introduction to Number Theory* by al-Kindī. It is less likely that his source was the Arabic original of the extant Hebrew paraphrase translated by Qalonymos, since the latter does not present itself in such a tripartite structure. According to al-Yaʿqūbī, the first part of this work deals with the subdivisions of even and odd numbers, and the second part treats quantity on its own, namely deficient, perfect, and abundant numbers. This shows that Book I of Nicomachus' work split into two separate parts. The third part discusses two types of quantity in connection: connection through equality and connection through difference. The second type divides into connections of the greater and the smaller. To this third part belong the kinds of the multiple, the increase by a part, the increase by at least two parts, the multiple increased by a part, and the multiple increased by several parts. These are the five objects that al-Kindī uses in one of his medical

8 Ibid., 69.

9 Brentjes 1988, 18–23.

treatises, which links medical crises with Nicomachean number theory (see below). Finally, al-Yaʿqūbī lists single topics which cannot be ascribed to specific chapters of Nicomachus' *Introduction*. The description of these topics suggests that al-Yaʿqūbī had problems fully understanding the content of the different sections. Similarly, the Hebrew paraphrase equally indicates that its translator found it not always easy to translate the Arabic paraphrase.[10]

The second translator was the Sabian scholar and head of the Sabian community of Baghdad, Thābit b. Qurra (d. 901). He translated and revised a number of Greek mathematical and astronomical works. In a number of cases he cooperated with other scholars, in particular the member of the Church of the East, physician and translator Isḥāq b. Ḥunayn (d. 911). Nicomachus' book was, however, the most extensive and thematically variegated Greek work that he translated alone. He translated it directly from Greek into Arabic. At least one copy of his translation is extant today and was edited by Kutsch.[11]

Thābit's translation suggests that he had access to a Greek text of Nicomachus' *Introduction* that deviates in some instances significantly from the one edited by Hoche.[12] In general, Thābit follows closely enough the Greek text, translating, however, the more general or philosophical sentences either in a condensed or simplifying manner or in a somewhat more elaborate form. Additions, which recognizably had come later into the text through the integration of marginalia, can also be found. If they were indeed part of the Greek text available to Thābit, he did not correct the translation even in easily detectable cases. In other instances, Thābit's Greek text seems to have been faulty, while at times his formulations are more precise than in Hoche's edited text. The first might result from scribal errors in the transmission of the Greek text, since their grammatical properties cannot be explained through mistakes in the transmission of the Arabic translation. The second deviation reflects more or less clearly the use of Nicomachus' work in a learning or teaching environment in Late Antiquity, through which explanations of unclear passages were added to the text. Such cases can be found in Chapter 13 on relatively prime numbers or in Chapter 19 on the heteromecic numbers $n(n + 1)$.[13] Further differences concern the formal layout of the text and the tables. The Arabic text edited by Kutsch contains chapter headings summarizing the subsequent content, while Hoche's Greek text is structured through alphanumerical letters. It remains unclear though whether this numbering was provided by Nicomachus,

10 Ibid., 18–20, 26–28.
11 Kutsch 1958.
12 See Nicomachus 1866.
13 Brentjes 1988, 32–35.

added by a later copyist, or introduced by Hoche. Formal differences can also be found with regard to some of the tables or diagrams. Sometimes the Arabic text transmits a smaller or a wrongly placed table. Sometimes the edited Greek text offers tables from different manuscripts in the footnotes. In the case of the sieve of Eratosthenes, for instance, the Arabic text presents instead a multiplication table explicitly attributed by Thābit to a Greek source, while Hoche put three different tables of the sieve into the footnotes and none into the text itself.[14] This shows that users of the Greek text strengthened the didactic properties of Nicomachus' work and that Thābit followed the by then established practice of ninth-century translators and searched for and worked with more than one Greek manuscript.

The terminology of later Arabic and Persian texts on number theoretical topics suggests that Thābit's translation did not have a wide circulation. The number theoretical parts of Ibn Sīnā's (d. 1037) Arabic and Persian philosophical encyclopaedias use in some cases a terminology that either agrees with that chosen by Thābit or is recognizably similar to it. An example is the description of relatively prime numbers. Thābit calls them, following Nicomachus, on the one hand "secondary and compound, when they are considered individually" and on the other hand "prime and not compound, when they are connected with another number."[15] The only difference in Ibn Sīnā's parlance is the replacement of "when they are connected with" by "in comparison with."[16] The same expressions can be found in the encyclopaedia *Mafātīḥ al-ʿulūm* (*Keys to the Sciences*) on scholarly terminology and the meaning of important concepts for a broad range of religious, philological, philosophical, medical, mathematical, astronomical, and alchemical disciplines and topics by the administrator Muḥammad b. Aḥmad al-Khwārazmī (tenth century). This indicates that Thābit's translation was known in tenth-century Iran. A much larger group of Arabic and Persian texts uses, however, the word *mutabāyin* (different, opposite, diverse) for relatively prime numbers. This is a terminology found in one of the Arabic versions of Euclid's *Elements*. There are other expressions that seem to indicate a kind of standardization of number theoretical terminology based on the early translations or summaries of Nicomachus' and Euclid's works.[17] Overall, the terminology preferred by authors of number theoretical treatises or chapters since the tenth century shows little impact from Thābit's

14 Kutsch 1959, 46; Hoche 1866, 30–31.
15 Kutsch 1959, 31.
16 Erevan, Matenadaran, MS arab. 45, f. 120v, 19–20.
17 Brentjes 1988, 32–39.

translation. In contrast, his new work on amicable numbers inspired many later writers.

2 Number Theoretical Reflections by Authors of the Ninth Century and Their Disciplinary Contexts

As in other contexts, the translations of the *Introduction to Arithmetic* quickly inspired newly composed texts on Nicomachean number theory as a whole or on individual themes drawn from it. Only few of such works have been preserved until today. Among the lost ones are al-Kindī's own *Introduction to Number Theory* and a text by his former student and later prominent scholar and court official Aḥmad b. al-Ṭayyib al-Sarakhsī (executed in 899). Preserved are a text on hidden or imagined numbers and two medical treatises with number theoretical topics by al-Kindī, and Thābit b. Qurra's text on amicable numbers. Recently, an epitome of a work called *Book on Number Theory*, which perhaps refers to Nicomachus' *Introduction*, has been found in the Royal Library in Rabat, Morocco. But it has not been published yet.[18]

In his text on arithmetical games, al-Kindī wishes to provide theoretical foundations for such kinds of exercises. He differentiates the topic into two parts: the knowledge of how to determine an imagined number and the knowledge of the properties of numbers. The second branch leads him to discuss components of number theory. Following his inclination to create from a starting point a systematical division into a growing number of subdivisions – an inclination which is also observable elsewhere in his work –, al-Kindī presents the various classes of numbers and their subordinate forms: even and odd, even-times-even, even-times-even-times-odd, even-times-odd of the even numbers and prime numbers, composite numbers, and relative prime numbers as subunits of the odd numbers. While these classes fully agree with Nicomachus' book, al-Kindī claims here as his source his own, but lost *Introduction to Number Theory*.[19]

In his two medical texts, al-Kindī treats critical days in case of acute disease, and remedies. The first treatise is called *Epistle Concerning the Causes of Crises in Acute Illness*. Glen Cooper believes it was written at some time in the 830s.[20] Al-Kindī aimed at constructing a theoretical foundation for Hippocrates' mainly experiential description of the critical days during a sickness. He used

18 Djebbar 2004, 304, n. 7.
19 Brentjes 1988, 25.
20 Cooper 2011, 28.

Nicomachus' work as one of his resources introducing his short treatise with a condensed survey on odd, even, square, linear, and heteromecic numbers and their properties. His justification for this approach was the idea that critical days form a series and thus can be analyzed accordingly. He starts with two ideas found in the *Introduction of Arithmetic*, namely that the number is the cause of the numbered and that the monad is the cause of the number.[21] Then follows a longer discussion of the relationship between the monad on the one hand and odd, even, and square numbers on the other. The monad is the cause of odd, even, and square numbers, among which odd numbers are more closely related to the monad and more fundamental than even numbers. Al-Kindī presents various arguments taken from the *Introduction to Arithmetic* why that is the case. Partitioning odd numbers means to form the sum of both classes of numbers, while even numbers can only be partitioned in either one of them.[22] The one, derived from the monad in an unclear manner, because the monad cannot be a number, keeps its "sameness" when multiplied once, twice, or thrice with itself, thus forming a linear, plane, or solid number. Even numbers do not participate in this preservation of identity, while square numbers do. They result from forming the sum of consecutive odd numbers beginning from one. The odd numbers are thus their cause.[23] Moreover, they are the product of two equal numbers and thus possess a root. If one forms the sum of consecutive even numbers beginning from one the result are line numbers, which have no roots.[24] If one omits one when adding even numbers, one comes to the heteromecic numbers $n(n + 1)$, which are formed from two different components. Al-Kindī concludes from this identification of the odd numbers as superior to even numbers and closer to the monad that the crises on odd-numbered days, receiving their power from the odd numbers, will lead to improvement, while crises happening on even-numbered days lead to a worsening or even death.[25]

Felix Klein-Franke believes that, in all likelihood, al-Kindī derived the concepts and arguments of this numerical introduction to his medical letter to an unknown recipient from the *Introduction to Arithmetic*.[26] Cooper contradicts him indirectly by claiming that al-Kindī appropriated only "the most rudimentary parts" from Nicomachus' work. He speculates that he might have used Euclid's *Elements*, because there, musical proportions are allegedly discussed. Since al-Kindī does not discuss proportions in this introductory part, this

21 Klein-Franke 1975, 171.
22 Ibid., 171–172; Nicomachus 1926, 191.
23 Klein-Franke 1975, 172; Nicomachus 1926, 243.
24 Klein-Franke 1975, 172–173.
25 Ibid., 164, 173.
26 Ibid., 169.

speculation with its spurious argument does not contribute to an understanding of al-Kindī's sources beyond the *Introduction to Arithmetic*. Al-Kindī's identification of the monad as the root of the integers Cooper links to Aristotle's *Metaphysics*.[27] Since Nicomachus does not describe the monad explicitly in such a manner and treats the issue only in side remarks when discussing specific classes of numbers, it is indeed an open question which text might have inspired al-Kindī's repeated statements. But Aristotle's side remark about unity not being a number does not include the idea of it being their root. It is found in Book N, which was not available in Arabic even in the tenth century.[28] Hence, al-Kindī may either have relied on some other (written or oral) source for his Pythagorean-type ideas on the monad and the odd and even numbers or extracted short statements from different contexts in the *Introduction*, combined them and re-interpreted them in parts. Examples are the concept of "the same" and "the other" and the ascription of the odd and the square numbers to the first and the even numbers to the second.[29] It is also possible that, for his at times different views, al-Kindī depended on the modifications which, apparently, were already introduced by the Syriac translator.[30]

Al-Kindī's second medical work with recourse to Nicomachean number theory is called *On Degrees* and tries to expand the theoretical basis introduced by Galen (second century CE) for producing compound drugs. Following ancient natural philosophy and its medical applications, al-Kindī supposes that simple drugs have stable, knowable degrees of hotness, coldness, dryness, or moistness. Compound drugs, by contrast, need to be determined according to the degrees of the simple ingredients and their ratios, whether they are to be newly composed or do exist already, but need to be analyzed in their effects. Like in the epistle on critical days, al-Kindī builds on ancient knowledge while going at the same time beyond it, for instance by applying notions and methods from Nicomachus' *Introduction to Arithmetic*. He focuses above all on proportions, that is, on Book II of Nicomachus' work. The basis of composition, al-Kindī writes, is equilibrium. Nothing can be called hot, cold, dry, or moist, if not by addition of a substance with such a quality to the equilibrium or its subtraction from a substance in equilibrium.[31] The central question of his mathematical approach is then how to achieve the desired degree of quality of a specific compound drug. He believes that the effects of simple drugs in isolation are

27 Cooper 2011, 30–31.
28 Bertolacci 2005, 247.
29 Nicomachus 1926, 190–191, 255–256.
30 Freudenthal and Zonta 2007, 68, 70.
31 Gauthier 1938, 45.

different from those exercised when the simples are combined in a composite drug. Hence, he declares to enter new territory with his search for an answer. He proceeds by adding substances of one of the four qualities to a substance in equilibrium and determining theoretically their compound effect.[32] The resource he found for achieving this goal is the classification of five ratios in Nicomachus' *Introduction*.[33] Al-Kindī described his choice as follows:

> We say that the entirety of the natural additions is of five kinds. The first is the addition of the double. Afterwards is the addition, which adds a part and this is the addition of the same and a part. Then comes the addition, which adds parts [and it is the addition of the same and its parts]. Then comes the addition of the doubled with the addition of a part, which is composed of the first and the second kind. (Finally) comes the addition of the doubled with the addition of the parts, which is composed from the first and the third kind.[34]

He then proceeds to explain the steps by references to the natural numbers and their classification. He starts from one, calling it here the cause of the numbers from which they grow, a concept he hopes will settle in the soul of the reader of his book. Starting with the series of natural numbers, al-Kindī explains that two is the double of one and one its half and hence the first kind of natural additions is the addition of the double. Moreover, the beginning of ratios proceeding from the smallest is the half. Thus, one belongs to the order of what comes after it. Hence, it is the same to call this addition or ratio. This is a simplified, but at the same time cumbersome, summary of Nicomachus' explanation of the multiple and the submultiple in Book II, chap. 18 for $n = 2$.[35] Three results from two by adding to two one part of it. Hence, it represents the addition of the same and a part. This takes up Nicomachus' concept of the superpartient (Book II, chap. 20).[36] This also holds for four in relation to three, while five is the first number representing the addition of the same and its parts, which is Nicomachus' notion of the supersubpartient (Book II, chap. 20).[37] In relationship to two, however, five is the first representative of the addition of the doubled with the addition of a part. This addition is a compound ratio, composed from the ratio of the double and the ratio of

32 Ibid., 44–46.
33 Nicomachus 1926, 193–195.
34 Gauthier 1938, Arabic text, 3.
35 Nicomachus 1926, 214.
36 Ibid., 220.
37 Ibid.

an added part. This corresponds to the first kind of Nicomachus' multiple superparticular.[38] Eight then is in relationship to three the first of the addition of the doubled with two added parts. It is a compound ratio composed from the first and the third kind. Here, al-Kindī presents an example of Nicomachus' notion of multiple superpartient.[39]

The repetitive, laborious way in which al-Kindī describes the simplest representation of the ratios and explains the "naturalness" of the position of each of them in subsequent order suggests that not only the topic was unknown to his intended audience, but also that he himself was not yet at ease with it. In order to make things easier for his audience, al-Kindī applies methods already used by Nicomachus. He supports, for instance, his inductive explanations by examples, which he describes at first verbally and then presents in tables. In this manner, al-Kindī also presents his five sequences in a table.

Thābit b. Qurra's *Treatise on the Derivation of Amicable Numbers in an Easy Way* is the main self-standing work written in the ninth century that focuses exclusively on the mathematical aspect of this type of numbers. Nicomachus only talked about abundant, deficient, and perfect numbers, explaining them through examples. Abundant is a number whose proper divisors, summed up, yield a number greater than this number. Deficient is a number for which the sum of its proper divisors is smaller than the factorized number. In the case of a perfect number, the sum of its proper divisors equals the number itself. Nicomachus named as perfect numbers 6, 28, 496, and 8,128. They result from the rule proved by Euclid at the end of Book IX of the *Elements*, which has been shown much later to be the only form of even perfect numbers: $(2^{n+1} - 1)2^n$ for 1, 2, 4, 6.[40] So far, no odd perfect numbers have been found.

TABLE 3.1 Ratios according to al-Kindī

ratio of equality	1	1	1
ratio of the double	1	2	4
ratio of the added as a part	2	3	4
ratio of the added of parts	3	5	7
ratio of the double and the added as a part	2	5	11
ratio of the double and the added as parts	3	8	13

38 Ibid., 222.
39 Gauthier 1938, 3–6; Nicomachus 1926, 224.
40 Nicomachus 1866, 39–44; Id. 1926, 209–212.

Because Nicomachus did not mention amicable numbers, this class most likely came to be known to scholars of the ninth century either through a translation of Iamblichus' commentary on Nicomachus' book or through some other channel. Iamblichus' commentary is the oldest extant text that mentions the smallest pair of amicable numbers 220, 284.[41] Amicable numbers are those, where the sum of the proper divisors of one of the two yields the other number. Iamblichus stated this as follows:

> The parts [i.e., the proper divisors] of each [of these numbers] are able to produce the other [number], according to the word about friendship that Pythagoras revealed. For when he was asked by someone "what is a friend?" he answered "another I" – as is shown in these numbers.[42]

Indeed, Thābit b. Qurra attributed amicable numbers to the tradition of "Pythagoras and the ancient philosophers of his school" and emphasized that he wrote his treatise because neither Euclid nor Nicomachus had "either mentioned or showed any interest in them."[43] He also made clear that he preferred Euclid's style of proving mathematical results in some generalized form over Nicomachus' inductive exemplification:

> For perfect numbers of the two types we have mentioned, Nicomachus described a method for determining them, but he did not prove it. As to Euclid, he described the method of determining them and demonstrated it with care in the arithmetic books of his *Elements*; and he placed them at the end of what he had reached in the latter, leading some to think that this was the end point at which he aimed and the ultimate goal of these books. As to amicable numbers, I found no one who mentioned them or took the trouble to devote himself to them. Now that the subject has come to my mind and that I have determined a demonstration about them, I should not like – in light of what has been said – to lose it by failing to establish it. I will do so once I have introduced the lemmas necessary for the task.[44]

41 Iamblichus 1975, 35 (cf. now Iamblichus 2014).

42 Quoted after Brentjes and Hogendijk 1989, 374.

43 Rashed 1994, 277.

44 Quoted after Rashed 2015, 336. For a slightly different translation of the second paragraph on amicable numbers, see Hogendijk 1985, 270.

Thābit's treatise consists of ten propositions, which can be considered as forming four groups. The first group with the first four propositions delivers preliminaries about divisors of prime or composite numbers and the properties of a geometric series with a ratio 1:2 between its terms. The second group consists of Proposition 5, which generalizes Euclid's theorem for perfect numbers to include abundant and deficient numbers, and Proposition 6, which clarifies when a number with the structure $2^n pq$, p and q being different primes, is abundant or deficient. The third group delivers, together with Propositions 7–9, three lemmas that are needed on different levels for the proof of the main theorem of the treatise, namely Proposition 10.[45]

According to the quoted introductory statement, Thābit's research yielded two results: a rule for generating amicable numbers and a proof of this rule. The rule states that $2^n pq$ and $2^n r$ are amicable, if p, q, and r are prime numbers, and p, q > 2.[46] The structure of the three prime numbers is in modern notation $p = 2^{n+1} - 1 + 2^n$, $q = 2^{n+1} - 1 - 2^{n-1}$, and $r = 2^{n+1}(2^{n+1} + 2^{n-1}) - 1$. Thābit himself formulates them slightly differently, most likely to make their form as similar as possible to the structure given by Euclid for perfect numbers.[47] Hogendijk's analysis of the proof shows that Thābit applied Euclid's methodology of aiming at a general proof by establishing the veracity of the claim for a generally described special case.[48] Thus, Thābit merged a Pythagorean topic with Euclidean methodology. This approach continued to inspire later writers about amicable and other classes of numbers, although Nicomachus' inductive way was also followed by other authors of Arabic, Persian, and Ottoman Turkish chapters or entire texts on this theme.

The kind of quasi-general demonstrative method implies that Thābit had to find a new pair of amicable numbers to check his procedure. Hogendijk suggests that the case for which Thābit established his rule was n = 4. This yields the new pair 17,296 and 18,416.[49] Another important property of Thābit's work is that the pair for which he had established and proved his rule is not the next pair of amicable numbers after 220, 284, which is 1184, 1210. Hogendijk concludes that Thābit's pair most likely was not discovered by trial and error, but in connection with the search for a generating rule.[50]

The analysis of Thābit's procedure and the example of 220, 284 with its divisors and partial sums indicate that while Thābit followed closely Euclidean

45 Brentjes and Hogendijk 1989, 374–375.
46 Hogendijk 1985, 271.
47 Brentjes and Hogendijk 1989, 377.
48 Hogendijk 1985, 270–271.
49 Ibid., 271.
50 Ibid., 272.

practices and modes, he was also inspired by Nicomachaen or more generally Neo-Pythagorean number theory.[51]

3 Nicomachean Number Theory in Mathematical Treatises after 900 and before 1200

With the foundations laid by translating the two Greek works and writing the mathematical and medical treatises discussed above, the discussion of classes of numbers and types of problems and their respective treatment either with Euclidean or with Nicomachean methods spread from specialized number theoretical texts to a broader range of mathematical treatises. Specialized treatises were written repeatedly over the centuries, but number theoretical topics and methods achieved greater visibility for students, readers, or teachers of mathematical texts in works teaching number and calculation systems. A third genre of mathematical writings where Nicomachean number theory appeared to some degree concerned combinatorics. Texts on algebra and practical geometry or surveying are other genres of mathematical literature that occasionally contain number theoretical passages or even subchapters.

One of the specialized treatises on number theory written in the tenth century presents itself through its title as a commentary on Nicomachus' *Introduction*. Its author was Abū l-Qāsim ʿAlī b. Aḥmad al-Anṭākī (d. 987). Only the last part of this commentary is extant. Since it is called the third book, it might, perhaps, be related to the text summarized by al-Yaʿqūbī. Sesiano, who edited and studied it, believes that it is a combination of more than two sources by authors of different mathematical capabilities.[52] According to his analysis, this extant part of al-Anṭākī's *Commentary* consists of three unconnected, distinct parts. The first of these three parts offers definitions and propositions from Books II and VII–IX of Euclid's *Elements* as well as arithmetical identities taken from works on algebra.[53] This combination of number theoretical with algebraic topics and – in other cases – methods is a widespread characteristic of approaches among scholars of the mathematical sciences in Islamicate societies between the ninth and the thirteenth centuries, at the very least.[54] Part 2 of al-Anṭākī's *Commentary* deals with different kinds of magic squares, a topic of no relevance to Nicomachus' *Introduction*. The *Commentary*'s last book

51 Brentjes and Hogendijk 1989, 377.
52 Sesiano 2017, 10.
53 Ibid., 13–14.
54 See, for instance, Rashed 2015.

discusses hidden or imagined numbers. It contains several literal quotes from al-Kindī's treatise on the topic.[55] In one instance in Part 1 of the extant rest of the *Commentary*, there is a reference to the topic of perfect numbers discussed earlier in it.[56] Hence, it might be possible that al-Anṭākī's *Commentary* does not refer, as believed, to Nicomachus' work despite its use of the transliteration of the Greek *arithmetiké*, but perhaps to al-Kindī's *Introduction* to which al-Anṭākī added extracts from other sources determined by Sesiano.

The other extant specialized treatises on number theory also often mix different thematic and methodical strands, bringing together aspects from the *Elements* and the *Introduction to Arithmetic* with algebra, the search for abstract rules for the so-called Pythagorean theorem $x^2 + y^2 = z^2$ and its generalization for $n = 3$ or 4. Some also specialize on a single topic. During the tenth century, the best-known authors of such specialized treatises were Abū al-Ṣaqr al-Qabīṣī (fl. *ca* 950), Abū Jaʿfar al-Khāzin, Abū Maḥmūd al-Khujandī (*ca* 940–1000), and Abū al-Wafāʾ (d. 998). Al-Qabīṣī wrote a six-page treatise on classes and series of numbers and the partial sums.[57] He claims to have collected various rules from different treatises known to him adding new results established by himself.[58] This interest in the summation of finite series of integers was probably stimulated by the study of Nicomachus' book, where one finds for instance the summation of the first six consecutive odd numbers.[59] Al-Qabīṣī also presents prime, compound, relative prime, relative compound, perfect, and amicable numbers.[60] With regard to the latter, he applies Thābit b. Qurra's rule, but without naming him. A new result within the context of Nicomachean number theory is al-Qabīṣī's presentation of the partial sum of consecutive natural numbers to the power of 4 beginning from one.[61]

Abū al-Wafāʾ wrote a short treatise on definitions of classes of numbers and other categories used in different arithmetical traditions. He structured this text according to the bipartite division of Nicomachus' *Introduction to Arithmetic*. Within the two parts, the most visible trend is his unification of Euclidean and Nicomachean classes and their definitions. Beyond that, he included terms from practical texts like *root* and from other theoretical works like *compound ratio*.[62]

55 Sesiano 2019, 15.
56 Ibid.
57 Sesiano 1987.
58 Ibid., 484.
59 Nicomachus 1926, 263–264.
60 Sesiano 1987, 484–488.
61 Ibid., 487.
62 al-ʿAlī 1977, 22–29.

As components of works on arithmetic in general we find discussions of different classes of numbers and their properties in the synthesis of arithmetic of the Shafiʿī legal scholar and leading representative of Ashʿarī *kalām* Abū Manṣūr ʿAbd al-Qāhir al-Baghdādī (d. 1037) as well as in the more innovative book on arithmetic *Kitāb al-badīʿ fī al-ḥisāb* (*Book of the Wondrous on Arithmetic*) by the slightly earlier Abū Bakr Muḥammad al-Karajī (d. *ca* 1029). Al-Karajī wrote for readers well-trained in the mathematical sciences.[63] In the chapters on the foundations, he quotes primarily definitions from two Arabic versions of the *Elements*, but occasionally, he also adds a phrase taken from Nicomachus' book. An example is his addition after the definition of the odd numbers that between the odd and the even number there is always only a difference of one.[64] While this example fits the context, the next reference to a Nicomachean statement does not, since it is squeezed between the definitions of the even-times-even and even-times-odd numbers: "The one does not belong to the numbers but is its basis" (or: root).[65] Other remarks or definitions seem to be either summaries of various Euclidean or Nicomachean statements or independent formulations. An example is the description of the two subspecies of a number constructed by two factors: a square with two equal factors and a number formed by two different factors.[66] After the definition of perfect numbers, the last one in Book VII of the *Elements*, al-Karajī adds a definition of amicable numbers. Then follow definitions of various kinds of proportional numbers and quantities, which is a modification of definitions from Book V of the *Elements* for numbers in combination with descriptions found in the *Introduction to Arithmetic*. In the subsequent parts of the book, al-Karajī treats partial sums of different series of numbers, followed by perfect and amicable numbers together with a summary of theorems from Books VII–X of the *Elements*, Diophantine analysis, rules for calculating with polynomials, as well as algebraic problems.[67] In his justification for the rules of summation he takes recourse to some claims found in the *Introduction to Arithmetic* such as the statement that a (natural) number is half of the sum of any two of its equally distanced neighbors.[68] This content of al-Karaj's book shows on the one hand that he considered the summation of series, ratios, and perfect and amicable numbers as proper for a higher-level book on arithmetic and on the other that he wished to bring together the different strands of number theory

63 Rashed 1994, 281.
64 al-Karajī 1964, 8.
65 Ibid.
66 Ibid.
67 al-Karajī 1964; Rashed 1994 and 1996, 398–401, 409–410.
68 al-Karajī 1964, 17; Nicomachus 1926, 192.

known to scholars in Islamicate societies through translations and their own investigations.

Al-Baghdādī elucidated in his *al-Takmila fi al-ḥisāb* (*The Completion of Arithmetic*) the then used systems of calculation like the Indian positional decimal system, finger-reckoning and other types of oral calculation, the application of proportions to business problems, inheritance problems, tasks for finding a hidden number together with themes from number theory and Book X of the *Elements*. In the number theoretical part, he discussed, among other topics, deficient, abundant, perfect, amicable, and polygonal numbers, adding to them the equivalent numbers, a class of numbers unknown from earlier extant Greek or Arabic treatises.[69] Equivalent numbers are those whose sums of proper divisors are equal to each other. In order to study their properties, he asks to find two such numbers, whose sums of their proper divisors are equal to a given number.[70] Among other results, not known from earlier works, is the insight that the smallest odd abundant number is 945, or the rejection of Nicomachus' claim that there is one perfect number in each rank of decimal powers, because he knew that there was none between 10,000 and 100,000. When discussing figurate numbers, al-Baghdādī calculates the partial sums of the first n square, pentagonal, and pyramidal numbers.[71] Since al-Baghdādī was first and foremost a religious scholar, the fact that he also wrote on mathematical topics and did so competently and in a well-informed way shows on the one hand that at the latest in the early eleventh century leading representatives among the religious scholars appreciated various parts of what then still was called the sciences of the ancients and found nothing objectionable in such activities. On the other it foreshadows the increasing engagement of scholars primarily focusing on the study and debate of religious matters with the mathematical sciences, medicine, philosophical themes and methods, and the one or the other of the so-called occult sciences.

4 Nicomachean Number Theory in Textbooks for Teaching

The majority of the discussions of number theoretical themes in the Neo-Pythagorean tradition after 1200 took place in the context of teaching at madrasas and sometimes at mosques. The head teachers at such institutions were usually appointed for teaching law according to their respective legal school.

69 Saidan 1987, 439–440; Rashed 1994, 281.
70 Rashed 2015, 341–342.
71 Rashed 1994, 282, 298–299.

A good number of them, however, also taught texts dealing with other fields of knowledge, among them various mathematical disciplines. Although there still does not exist a good survey of how often, where and when number theoretical topics were taught in different societies and periods, it is clear that number theory was one of the minor mathematical fields present at such teaching institutions. A few examples suffice to characterize the scope and context of such teaching and the view that such teachers took on number theory. The first example comes from a century earlier and was not written by a madrasa teacher. I included it because it too is a textbook. Its author is Abū l-Rayḥān al-Bīrūnī (d. after 1048). He wrote it as a textbook of the astral sciences for a young woman called Rayḥāna in Ghazna, today Ghazni in Afghanistan. My second example is an independent teaching text on Nicomachean number theory written in the early thirteenth century by the Damascene legal scholar and madrasa teacher Ibn Fallūs (1194–1252/3).

Arithmetic is the second preparatory set of questions in al-Bīrūnī's *Kitāb al-tafhīm li-awāʾil ṣināʿat al-tanjīm* (*The Book of Instruction in the Elements of the Art of Astrology*), which follows geometry and precedes information about the structure of the universe and the mathematical tools needed for its investigation. The fourth main part deals with judicial astrology. The arithmetical part brings together all major themes of number theory with all important systems of calculation (Indian positional decimal system; Graeco-Babylonian sexagesimal system; kinds of oral calculation and more) as well as algebra. It begins in the tradition of Nicomachus and Euclid with a definition of one and a description of its properties and differences from integers. To the latter, al-Bīrūnī adds fractions, which he calls numbers, like the Brethren of Purity (see below). As a consequence, in the next section al-Bīrūnī explains how the unit as a technical concept can be divided, while for the one as a philosophical notion any division is unthinkable and needs to be rejected. In this context he introduces the unit of astronomy, the degree and its subunits: minutes, seconds, and so forth, the unit of monetary exchange, the dirham and its subunit, the *fulūs*, and the unit of measurement for corn, the *jarīb* and its sexagesimal subunit.[72] This was one of the multiple ways to weaken the ancient Greek exclusion of one from the concept of number. Then follow the definitions of all classes of numbers known from the *Elements* and the *Introduction to Arithmetic* plus the amicable numbers.[73] The remainder of the chapter deals, as said above, with other parts of arithmetic and algebra.

72 al-Bīrūnī 1934, 23, 25.
73 Ibid., 25–30.

Ibn Fallūs was a Ḥanafī legal scholar who taught arithmetic, algebra, number theory, geometry, and medicine in the thirteenth century at madrasas in Damascus and Cairo. He took his number theoretical treatise on his pilgrimage to Medina and Mecca and went with it repeatedly around the Kaʿba asking God for help. When the Ayyubid ruler of Damascus, al-Muʿaẓẓam ʿĪsā (r. 1218–1227), asked him for a fatwa allowing the consumption of date wine, he refused to comply. As a result, he had to spend some time under house arrest and may have left for Cairo afterwards.

Ibn Fallūs opens his survey of the "Secrets of Numbers" with the explicit admission that he had relied on "the book of Nicomachus, the Pythagorean, on Arithmetic."[74] As other scholars, mentioned before, Ibn Fallūs combined topics from Nicomachus' work with other mathematical themes. For that purpose, he added to the first two number theoretical chapters a "chapter on geometry and universal propositions taken from other mathematical books."[75] This chapter discusses almost exclusively arithmetical and algebraic problems such as rules for calculating the binomial triangle, algebraic formulations of theorems from Book II of the *Elements*, or traditional problems for calculating salaries and labor hours. Although Ibn Fallūs does not name any other of his sources, it is clear that he was familiar with parts of the algebraic tradition as found in the works of al-Karajī and the algebraic interpretation of Book II of the *Elements* as found already in the first chapter of the *Epistles of the Brethren of Purity* (see below).[76] As for the elements from Nicomachus' *Introduction to Arithmetic*, Ibn Fallūs deals with all themes of Book I, that is, all classes of numbers studied independently, not in a relationship, plus amicable numbers and inimical numbers. In contrast to other authors, Ibn Fallūs differentiates between three kinds of amicable and hateful numbers. It is unclear though from where he got the notion of hateful numbers and whether any later scholar picked them up.[77] The first chapter presents the definitions of the various classes and the second chapter explains their generation. Tables of perfect numbers are joined to the text as an appendix. There, one can find larger specimens than the ones documented in earlier known texts.[78] Rashed is of the opinion that the search for new representatives of perfect, amicable, deficient, and abundant numbers was primarily an interest among scholars focusing on algebra, while scholars interested in number theory as a theoretical enterprise allegedly did not

74 Berlin, Preußischer Kulturbesitz, Staatsbibliothek, MS Lbg. 199, f. 15a–32a.
75 Ibid., f. 16b.
76 Brentjes 1988, 115.
77 Berlin, Preußischer Kulturbesitz, Staatsbibliothek, MS Lbg. 199, f. 18b, 21b–22a.
78 Brentjes 1990.

invest in endless calculations.[79] This is, however, not supported by the known sources. Theoretically inclined scholars as well as teachers participated in this search. Ibn Fallūs was one of many. However, he was not a gifted mathematician. He made a number of mistakes when excerpting other mathematical works or calculating higher numbers.

5 Nicomachean Number Theory in Encyclopaedias from the Tenth to the Fourteenth Century

Nicomachean number theory, mostly under the designation of arithmetic (*ḥisāb*), appears in different types of encyclopaedias (terminological, philosophical, general, specialized) compiled by different kinds of authors (administrators, philosophers, madrasa teachers). It reflects the adoption and adaptation of ancient classifications of scholarly knowledge to views and needs of different socio-cultural groups in a number of Islamicate societies. Most of such works appeared between the tenth and the fourteenth centuries. Some were still compiled in later times but are not well studied and thus ignored here.

The writers of encyclopaedic synopses of knowledge pursued different agendas when they compiled them. Most often they apply the rhetoric of teaching and learning or at least of informing their immediate contemporaries at a court or in a town or city. The administrative and general encyclopaedic collections bring together summaries of religious, philosophical, medical, mathematical, occult, philological, and daily life knowledge fields. Philosophical encyclopaedias follow most often an Aristotelian structure and set up of knowledge. Special encyclopaedias discuss mostly between seven to twelve single disciplines from the three main domains of systematized and taught knowledge – the so-called disciplines of transmission, the rational disciplines, and the mathematical sciences. Occasionally they also contain the one or the other representative of the occult disciplines, which on average were less visible in the knowledge fields publicly recognized as teaching subjects at madrasas. In particular after the fourteenth century, their inclusion in encyclopaedic literature became more widespread.

Four of the main authors of number theoretical chapters in encyclopaedias in Arabic and Persian who wrote between the tenth and the fourteenth centuries were the so-called *Brethren of Sincerity*, a group of poorly known scholars who lived in Basra in the later tenth and perhaps the early eleventh centuries,

79 Rashed 2015, 337.

the philosopher Ibn Sīnā (d. 1037), his student and later secretary Abū ʿUbayd al-Juzjānī (d. after 1037), the administrator Abū ʿAbdallāh al-Khwārazmī (d. 997), and the madrasa teacher Muḥammad b. Maḥmūd al-Āmulī (d. after 1351). All of them worked with concepts and methods taken both from the *Elements* and the *Introduction to Arithmetic*.

In the chapters on number theory and theoretical music of their fifty-one *Epistles*, the *Brethren of Sincerity* combined the material of the *Introduction to Arithmetic* with an algebraic form of theorems from Book II of the *Elements* and arithmological reflections on the spiritual and other values of numbers, possibly informed by the *Theology of Arithmetic* attributed to Iamblichus. It is, however, not known whether and if so in which form this work has been translated into Arabic.[80]

Ibn Sīnā named the second chapter of the mathematical part of his great philosophical encyclopaedia *Book of Healing al-arithmāṭīqī*, that is, number theory. In its introduction he refers explicitly to his use of the number theoretical books of the *Elements* and the application of geometrical theorems to the multiplication, division, and ratios.[81] In the following elaborations, he also relies on Aristotle's *Categories* for the definition of the essence of numbers. Nicomachus is not explicitly mentioned but appears through statements such as that each number is half of its equi-distanced neighbours.[82] Further topics in the tradition of Nicomachus' *Introduction* and its practitioners in Islamicate societies are the calculations of partial sums of series, the explanations of the properties of the various classes of numbers including figurative numbers, and ratios. Ibn Sīnā offers a series of extensions to Nicomachus' descriptions of those series and classes, some of which can also be found in other contemporary Arabic works such as those by al-Qabīṣī and al-Baghdādī. Unknown otherwise in the literature of the period are several of the rules that he gives for abundant and deficient numbers as, for example, that every compound even-times-odd number is abundant, unless the odd part is a prime.[83] As the last part of the claim highlights, not all of the rules are (fully) correct, a feature that Ibn Sīnā's work shares with Nicomachus' *Introduction to Arithmetic* and treatises by his contemporaries. In addition, Ibn Sīnā also included a short summary of Thābit b. Qurra's rule for amicable numbers.[84] Numerous properties and results are illustrated in tables.

80 Brentjes 1984.
81 Ibn Sīnā 1975, 17.
82 Ibid., 18, 23.
83 Ibid., 33.
84 Ibid., 28.

The mathematical parts of Ibn Sīnā's shorter Arabic encyclopaedia *Book of Salvation* were written by his secretary al-Juzjānī, while the master himself is believed to have compiled those parts in the first Persian encyclopaedia of his philosophical teachings *Book of Knowledge*. However, they are often missing in the manuscripts of the *Book of Salvation*, which apparently served mainly as a primer for logic and metaphysics. In the *Book of Knowledge*, the number theoretical part is described as an abbreviation of Ibn Sīnā's expositions. The text has no tables and focuses primarily on those rules and results that are relevant in music.[85] Indeed, the general discussion of what numbers are and of the series and classes of numbers is kept very brief.[86] Ratios and proportions, i.e., material related mostly to Book II of the *Introduction to Arithmetic*, are presented in more detail.[87] Going beyond the *Introduction*, al-Juzjānī includes there an explanation of compound ratios based on Book V of the *Elements*. Based on the author's introductory words, Akhadova believes that this part was added in preparation of the chapter on theoretical music in the *Book of Knowledge*.[88]

The administrator at the Samanid court, Abū ʿAbdallāh Muḥammad b. Aḥmad al-Khwārazmī (d. 997) lived most of his life in Nishapur (Northeastern Iran). Sometime after 975, he wrote his book *Keys to the Sciences* for his colleagues who in his view did not know well enough the technical terminology and its meaning in the various fields of knowledge available in his time. The second part of his book is dedicated to the knowledge fields appropriated through translations of Greek, Syriac, and Sanskrit sources. Number theory follows as Chapter 4 medicine and is the first of the four ancient mathematical sciences. His choice of sequence thus agrees, like that of the *Brethren of Purity*, with Nicomachus' classification, while Ibn Sīnā and al-Juzjānī worked with a modified sequence that placed number theory before music. Within the chapter, however, al-Khwārazmī presents the material differently from the *Introduction to Arithmetic*. After the numbers as such, calling them the quantity on its own, he treats first the quantity in relationship, i.e., ratios and proportions, before he talks about plane and solid numbers, followed by a section on series.[89] Given al-Khwārazmī's goal to teach terminology, most of the content of the sections are classifications of terms and definitions which are illustrated by brief, simple examples. The main source of the sections is Nicomachus' book. But

85 Ibn Sīnā 1958, 190.
86 Ibid., 190–199; Akhadova 1964, 264–270.
87 Ibn Sīnā 1958, 199–216.
88 Akhadova 1964, 272–273, 280.
89 al-Khwārazmī 1409/1989, 209–217.

al-Khwārazmī also included bits and pieces from other works such as amicable numbers.[90] The last section on the chapter called *arithmātīqī* is a section on the Indian decimal positional system, the calculation with letters representing numbers, and the beginnings of algebra.[91] This is surprising since such practical matters were neither part of Nicomachus' number theoretical exposition nor part of the number theoretical Books VII–IX of Euclid's *Elements*. Although this merger is not taken up by all writers about number theory in either of the two traditions in Islamicate societies and when it is taken up, the fusions differ by either including information on calculation systems into number theory or by putting elements from number theory into treatises on number systems and rules of calculation, this changed relationship indicates a systemic shift away from a more philosophically grounded, theoretical perspective to a more inclusive and pragmatic approach.

The madrasa teacher and courtier of the Ilkhanid and Inju dynasties (1256–1335; 1325–1353) Muḥammad b. Maḥmūd al-Āmulī wrote between about 1335 and 1342 an extensive Persian encyclopaedia on the entire edifice of knowledge fields pursued in debate and teaching in various Islamicate societies. In his plan of the quadrivial disciplines he follows the structure preferred by Ibn Sīnā, placing geometry and astronomy before number theory and music after it.[92] Within the chapter, al-Āmulī agrees with the structure encountered in al-Khwārazmī's book except for the last part on calculation systems and algebra.[93] These two subject matters al-Āmulī discusses as so-called branch disciplines in another part of his book. He even describes the content of this book as being about expressions from the knowledge about the properties of numbers.[94] However, he clearly drew substantial parts of Ibn Sīnā's number theoretical chapter into his text including the use of tables in different formats.[95]

6 Nicomachean Number Theory in Conjunction with Other Mathematical Inquiries

While most of the mathematical texts dealing with Nicomachean number theory are either almost exclusively surveying the major topics of this branch of

90 Ibid., 210.
91 Ibid., 218–222.
92 al-Āmulī 1379, vol. 3, 45.
93 Ibid., vol. 3, 45–72.
94 Ibid., vol. 3, 45.
95 For the tables, see al-Āmulī 1379, vol. 3, 53–54, 58, 62–63.

mathematical knowledge or positioning it in relationship to other branches of arithmetic like calculation systems and algebra, there are a few mathematical texts whose authors used themes from the *Introduction to Arithmetic* in conjunction with methods from the *Elements* in order to further other mathematical investigations. One of such inquiries concerns combinatorics. The first inspirations to study the formation of patterns such as combinations or permutations came from lexicography and grammar in the eighth century. Other cases where such problems were dealt with are occasionally known from music, astrology, and algebra.[96] In the Maghreb, interest in such matters was apparently stimulated in the late twelfth and early thirteenth centuries through results achieved by linguists.[97] Other mathematical inquiries stimulating the production of new combinatorial results were connected to studies of series of numbers, in particular those dealing with figurative numbers. Figurative numbers are arrangements according to geometrical plane or solid figures like triangles, squares, pentagrams, or pyramids and cubes. Beside the thematic reliance of this kind of studies on works drawing on Nicomachus, one of the methods applied by the scholars in the Maghreb also took its inspiration in the last instance from his work – the arrangement of series of numbers in a table and the derivation of conclusions about their relations and sums thanks to the visible symmetries.[98] This double connection to the works of the ancient scholars in general is clearly expressed in one of the earliest extant mathematical books dealing with figurative numbers, among other themes related to the *Introduction to Arithmetic*, written by a North African scholar. Ibn Munʿim (d. 1228) wrote in the introduction to his work *The Science of Arithmetic* that he had acquired some of the knowledge he would discuss in it from the ancients, while the other parts he had found himself with the help of their methods of argumentation and the techniques of their verifications.[99] In addition to his applications of such methods in the chapters on different classes of numbers, he uses for instance the tables for solving problems concerning combinations and permutations with or without repetitions. In one task, he wished to determine the number of different tassels he could fabricate from pieces of silk in ten colours that use a given number of colours[100] (see the table at the end of the chapter). A second task referring to the production of multi-coloured tufts consists in determining the number of tassels made from a given number of

96 Djebbar 1985, 2.
97 Ibid.
98 Ibid., 3.
99 Ibid., 6.
100 Ibid., 19–26, 50–55.

silk pieces with a given number of different colours.[101] Other problems come from the linguistic tradition. Two of them ask to determine the number of permutations of letters in a word without or with repetitions.[102]

Such kinds of investigations continued among scholars from the Maghreb, as far as we know, until the fifteenth century. Traces of them can also be found in works of Mamluk scholars. Ibn al-Bannāʾ (1256–1321), for instance, takes up and continues Ibn Munʿim's work linking it explicitly to the study of figurate numbers.[103]

Combinatoric works by scholars from societies in West Asia are known from the tenth century onwards. There too, they arose first in connection with linguistic questions about the composition of roots and words derived from them. Examples of combinatorial problems can then be found in a number of different contexts such as algebra, cryptography, philosophy, or astronomy.[104] Naṣīr al-Dīn al-Ṭūsī (1201–1274), for instance, approached in this manner a metaphysical problem. Building on philosophical reflections by Ibn Sīnā on the problem of emanation, he wished to determine mathematically the emanation of multiplicity from unity by using combinatorics.[105] In number theory itself, a particularly important work was written by Kamāl al-Dīn al-Fārisī (d. 1319). In his treatise *Memoir to Friends to Demonstrate Amicability*, he showed how to find figurate numbers of any order and the relationship of this kind of inquiry to the study of proper divisors and combinations.[106]

7 Conclusions

In Islamicate societies, Nicomachean number theory played an important role in different mathematical contexts and was discussed in many specialized as well as survey texts for almost a millennium. It inspired new mathematical results, often in combination with Euclidean methods and concepts; served as a basic tool in the arithmetical education of legal scholars; was employed in philosophical and theological discussions about God and His properties; became a stable component of the knowledge necessary for the cultured gentleman at court; and delivered techniques used in several occult sciences. As

101 Ibid., 35–36, 61–63.
102 Ibid., 26–29, 55–57.
103 Rashed 2015, 154.
104 Ibid., 150–162.
105 Ibid., 164, 718–726.
106 Ibid., 163–164, 344–345.

a whole, it offered important resources across several fields of knowledge pursued by experts as well as amateurs from the top of society to its bottom.

TABLE 3.2 Table of combinations of ten colors of silk one by one, two by two…, ten by ten

Sum	**Writing the example in the table**										
1	1	Line of tassels of ten colors									
10	9	1	Line of tassels of nine colors								
45	36	8	1	Line of tassels of eight colors							
120	84	28	7	1	Line of tassels of seven c.						
210	126	56	21	6	1	Line of tassels of six c.					
252	126	70	35	15	5	1	Line of tassels of five c.				
210	84	56	35	20	10	4	1	Line of tassels of four c.			
120	36	28	21	15	10	6	3	1	Line of t. of 3 c.		
45	9	8	7	6	5	4	3	2	1	of 2 c.	
10	1	1	1	1	1	1	1	1	1	1	1 c.
All colors	**10th color**	**9th c.**	**8th c.**	**7th c.**	**6th c.**	**5th c.**	**4th c.**	**3th c.**	**2th c.**	**1st c.**	

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CHAPTER 4

The Tribulations of the *Introduction to Arithmetic* from Greek to Hebrew Via Syriac and Arabic. Nicomachus of Gerasa, Ḥabib Ibn Bahrīz, al-Kindī, and Qalonymos ben Qalonymos

Gad Freudenthal

For Mauro Zonta,
in memoriam.

Nicomachus of Gerasa wrote his Neo-Pythagorean *Introduction to Arithmetic* toward the turn of the first and second century CE.[1] More than a millennium later, in 1317, the Jewish Provençal industrious translator, scientist, and man of letters translated (a version of) that work from Arabic into Hebrew.[2] During the long process of transmission through four cultures and four languages the text underwent profound changes: (i) in the Hebrew text many interpolations can be identified, some explicitly ascribed to "Abū Youssef," i.e., al-Kindī (801–866), the so-called "first philosopher of the Arabs";[3] (ii) throughout, the text is a paraphrase rather than a literal translation; and (iii) while the manuscripts of the Hebrew text reproduce many of the tables and drawings of the Greek original, they also carry drawings and tables to which nothing corresponds in the Greek text. A century ago, the great scholar Moritz Steinschneider, to whom we owe most of what we know about medieval Hebrew translations (and much more), examined these differences and made the following resigned observation: "Einiges ist auch nach Vergleichung mehrerer mss. nicht ganz klar."[4] Three

1 Greek text: Nicomachus of Gerasa 1866 (ed. Hoche). Translations: D'Ooge 1926 and Nicomachus of Gerasa 1978.
2 Steinschneider 1893a, § 320, 517.
3 Steinschneider 1893b and 1960, 227–228.
4 Steinschneider 1893a, 517.

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recent studies, which I had the pleasure to co-author, have shed some new light on the history of the text.[5] The present publication is based on these studies.

1

Nicomachus' *Introduction to Arithmetic* reached the Arabic world twice: a first version was made from a Syriac one; this version will be discussed below. Subsequently, the work was translated into Arabic a second time, now directly from the Greek, by the noted mathematician Thābit Ibn Qurra (d. 901); this version will not concern us here.[6] The Hebrew work is entitled *Sefer ha-'aritmatiqa'* ("The book of arithmetic") and the name of the author is given, following the Arabic, as Nīqūmākhūs al-gaharshīnī. The name of the translator, Qalonymos b. Qalonymos of Arles, appears in the colophons of two manuscripts (out of eight) of the text. In these colophons, the well-known translator indicates that he completed his work on 5 Nissan [50]77, i.e., March 19, 1317, when he was 30 years old.

Sefer ha-'aritmatiqa', the Hebrew version of the *Introduction to Arithmetic*, opens with a *Prologue*, which is not part of the Greek original of the work and which sheds important light on the history of the text. Its anonymous author addresses an unnamed personality, apparently of high rank; I will call that person the *Addressee*. From the Prologue we understand that the Addressee had already studied in part the *Introduction to Arithmetic* (the "famous work"), in a version that the author of the Prologue had "corrected" or "revised" "under the authority of our master, the noble Yaʿqūb ibn 'Isḥāq aṣ-Ṣabbāḥ al-Kindī" (515.3–5).[7] I will therefore call the author of the Prologue the *Revisor*. Al-Kindī was much interested in mathematics,[8] and so it is not surprising that he was interested in Nicomachus' work, to the point of "reading" it with his students while also "revising" the text. Certain works of al-Kindī indeed contain identifiable traces of his study of the *Introduction to Arithmetic*.[9] It should be noted,

5 Freudenthal and Lévy 2004. (This publication includes a critical edition of the Hebrew text of the first part of the work, accompanied by an annotated French translation.) Freudenthal and Zonta 2007; Zonta and Freudenthal 2009. Freudenthal 2005 essentially summarizes Freudenthal and Lévy 2004.

6 Kutsch 1958.

7 The references given in brackets in the text refer to the pages and lines of the published Hebrew text and its facing French translation (Freudenthal and Lévy 2004).

8 See, e.g., Rashed 1993, 7–12.

9 Brentjes 1987, 227–229. See also Endress 1997, 55; Langermann 2003.

however, that there are only few points of convergence between Nicomachus' work and al-Kindī's metaphysics.[10]

The Prologue states that al-Kindī made the revision in order to eliminate from the Arabic text that had reached him the numerous errors introduced into it by "Ḥabib Ibn Bahrīz the Nestorian," who had translated the work from Syriac into Arabic, at the request of Ṭāhir b. al-Ḥusain, "the ambidextrous" (515.5–7). Whereas translators from Syriac into Arabic were often criticized for an excess of literalism, Ibn Bahrīz is apparently taken to task for having introduced his personal philosophical ideas into the Arabic version of Nicomachus' work; we will come back to this below.

The two persons mentioned by the Revisor are well known. Ḥabib Ibn Bahrīz was a jurist, theologian and scholar, the Nestorian metropolitan of Harrān, then Mosul and Hazza; when he was consecrated bishop, he took the name of 'Abdīshū'.[11] He translated from Syriac into Arabic medical and philosophical works, including the *Introduction to Arithmetic*. He also composed original works, of which two, one legal, the other logical, have reached us and to which we shall return. Ibn Bahrīz played a significant role in the development of Arabic logic before Ḥunain ibn Isḥāq's translations were made (873). It is striking that Ibn Bahrīz and al-Kindī wrote epitomes of the same two logical works of Aristotle (the *Categoriae* and the *De interpretatione*): in the section devoted to books of logic, Ibn al-Nadim quotes the name of Ibn Bahrīz twice, and both times his name is associated with that of al-Kindī.[12] There was thus apparently a convergence of interests between the two scholars. Moreover, al-Kindī dedicated his epistle on the causes of rain to a pupil named "Ḥabib," most likely none other than Ḥabib Ibn Bahrīz.[13] A direct connection between the two scholars is all the more likely since al-Kindī attached great importance to his contacts with different translators and, moreover, both benefited from the support of al-Ma'mūn (the other patron of Ibn Bahrīz, Ṭāhir b. al-Ḥusain, was a general of al-Ma'mūn). It thus seems that Ibn Bahrīz belonged to the circle of translators around al-Kindī and it stands to reason that the latter may

10 See al-Kindī 1974 (transl. Ivry), 20–21. Ivry concludes that "a comparison of our text and Nicomachus' yields little by way of specific comparisons" (at page 20); however, he compared al-Kindī's metaphysics with the Greek version of the *Introduction*, not with the Hebrew text, which reflects the Arabic text "corrected" by al-Kindī.

11 Troupeau 1997, and the bibliography given therein.

12 Flügel 1871, 248.27; 249.4. See further Rescher 1963, 14, and 1964, 28–29, 100.

13 This hypothesis was suggested by Steinschneider 1893a, 518, 564. The text has been edited in Bos and Burnett 2000; the dedication is on 97, 139 (transl. 161.); see also 325.

have been involved in selecting for translation this work, which responded to his theoretical interests.[14]

Ṭāhir b. al-Ḥusain is also a well-known personality, albeit of a very different profile.[15] As a general of al-Ma'mūn, he won the decisive battle that the latter fought against his brother al-Amīn, at the end of which Bagdad fell into his hands and al-Amīn was killed (813). Ṭāhir then settled for a time in the capital, where he accumulated a considerable fortune, and was later appointed governor of the territories of the caliphate to the east of Iraq (821), thus becoming the founder of the Ṭāhirid dynasty in Khorāsān. He was clearly endowed with exceptional warrior qualities – he was named "Dhul-Yamīnayn" (the ambidextrous) because in the course of a battle he cut a man into two with his left hand – but also had a penchant for the letters. Although Persian was his mother tongue, he was raised in the Arabic language, of which he had an exceptional mastery (the epistle to his son, dated 821–822, became a model of Arabic eloquence). It is thus not surprising that this close friend of al-Ma'mūn was a patron of learning too: he is known to have commissioned from scholars (including Ibn Bahrīz) at least five original works or translations.[16] Ṭāhir b. al-Ḥusain died (prematurely, possibly poisoned) in 822; this date constitutes the terminus *ante quem* for the translation of the *Introduction to Arithmetic* by Ibn Bahrīz.

2

In his Prologue, the Revisor refers to a letter he had received from the Addressee. In this letter, the latter complained that the revised text of the *Introduction to Arithmetic* was available to him only from the discussion of numbers onward. The Addressee suspected that the preceding part, to which he refers as the *Proemium* of the work, was "of great use and contains valuable information" (515.12–13) and he asked his correspondent to send it to him. The latter, the Revisor, confirms the Addressee's surmise: al-Kindī himself, he writes, stressed that the proemia of scientific and philosophical works constitute a literary genre of great significance and even commented that the proemia of the works of Nicomachus and of Ptolemy are the parts of these works

14 It is usually assumed that al-Kindī lived between 801 and 866. Given that Ibn Bahrīz completed his translation before 822 (see *infra*), the collaboration between the two scholars would have taken place when al-Kindī was only 20 years old.

15 Bosworth 1975, 90–95; Bosworth 2000.

16 See Endress 1987, 424 n. 60; Gutas 1998, 129–130.

in which the authors best explained their philosophies (517.4–8). In fact, since late Antiquity, the proemia of philosophical books constituted a codified literary genre whose purpose was to facilitate access to the works themselves.[17] The Addressee's request is therefore understandable: aware of the potential importance of the Proemium of Nicomachus' work, he wished to avail himself of it.

The Addressee's request was fulfilled. The Revisor sent the requested Proemium to the Addressee, accompanying it with a personal letter that subsequently became the Prologue as we have it in the Hebrew version (it is preserved only in that version). (Note that in what follows "Proemium" refers to the opening sections of *Introduction to Arithmetic*, construed as a proemium by the participants; "Prologue" refers to the Revisor's letter to the Addressee, inserted by the former before the Proemium.) From that Prologue we learn that at the time when the request was made to the Revisor, the revised version of the Proemium of the *Introduction to Arithmetic* did not yet exist: apparently it was only the Addressee's request that prompted the Revisor to draft it. In fact, at the end of the Proemium, just before the passage on the nature of numbers, the Revisor interpolates a remark of his own: "Here, my brother, is the whole of the Proemium of this book, up to [the passage] on the number, as you had requested. Let [the study of] this book […] be successful, and let Him, by His grace, direct you according to His will. Amen" (543.7–8). The text that the Revisor sent to the Addressee ends at this point; the bulk of the work was already in the hands of the latter. At some point in time, the Addressee combined the text that he received from the Revisor (the latter's Prologue followed by the first part of Nicomachus' *Introduction to Arithmetic*, considered as its Proemium) with the rest of the work and this reunified Arabic text (as modified by al-Kindi) became the *Vorlage* of the Hebrew version that has come down to us.

3

As already mentioned, a comparison of the Hebrew version of the *Introduction to Arithmetic* with the Greek original reveals that the two differ considerably. Thus, the Hebrew version carries numerous glosses: several of them are expressly attributed to al-Kindī, while the majority is unattributed. Some of these have a philosophical significance, indicating that they are deliberate interpolations and not the results of accidents of manuscript transmission. Moreover, throughout the text we repeatedly find the introductory formula

17 See, e.g., Westernik 1990; Mansfeld 1994; Quain 1945; Robinson 2000, 83–85; Klein-Braslavy 2002 and 2005.

"Nicomachus said" or "the author of the book said": these references to Nicomachus in the third person are para-textual elements that clearly go back to an "editor." In addition, the Hebrew text is paraphrastic throughout. The Hebrew version of *Introduction to Arithmetic* clearly resulted from heavy editing. The question arises: who is/are the editor(s) responsible for the various differences between the Greek and the Hebrew versions?

Many scholars were involved in creating, transmitting, translating and revising the text of *Introduction to Arithmetic*, and each could have played a role in its editing:

(i) Nicomachus of Gerasa, the author of the original Greek text;
(ii) The Greek commentators on the work;
(iii) The unknown translator from Greek into Syriac;
(iv) Ḥabib Ibn Bahrīz, the translator from Syriac into Arabic;
(v) Al-Kindī, who eliminated "errors" introduced by Ḥabib Ibn Bahrīz, adding at the same time his own glosses;
(vi) The anonymous Revisor, al-Kindī's student, who put in writing the corrections and glosses of his Master and wrote the Prologue;
(vii) The Addressee, who re-combined the Prologue and the Proemium he received with the bulk of *Introduction to Arithmetic*, thus creating a single continuous work;
(viii) An Andalusian scholar who (as will be seen) authored two colophons interpolated into the text;
(ix) Qalonymos ben Qalonymos, the translator from Arabic into Hebrew in 1317.

In this archaeological site, where no less than nine different layers of text are stacked on top of each other, can the different strata be identified and their paternity determined? To answer this question, let us try to appreciate the possible contribution of each of the participants, and specifically try to determine to whom the text owes its paraphrastic character and who interpolated into it the various unattributed glosses found in the Hebrew version.

4

We begin with the last link in the chain of transmission: the Hebrew translator, Qalonymos ben Qalonymos. It can be affirmed with certainty that he in no way interfered with the text: he was a prolific translator and we know his translations to have been strictly literal; nor did he have any ambition to "improve" a text he translated by interpolating additional material. This was also the general style of Hebrew translations made in Provence in the thirteenth and fourteenth

centuries.[18] The paraphrastic character and the glosses thus originated in previous stages of the transmission – they may be due to the Greek-into-Syriac translator, and/or to Ibn Bahrīz (the Syriac-into-Arabic translator), and/or to al-Kindī and his student, the Revisor.

Consider now the passage from Greek into Syriac (Stage *iii*). We do not know the identity of the translator and so the date of the translation is uncertain. Philosophical translations from Greek into Syriac evolved according to the following scheme: they began in the middle of the fifth century, continued during the sixth and seventh centuries, and came to a halt during the eighth century.[19] In the ninth century there was a new wave of translations, concomitant with the great movement of translations into Arabic. Now the translation of a specialized work such as Nicomachus', which requires an elaborate philosophical and mathematical vocabulary, cannot have been very early. Given that the work was translated into Syriac before 822 (when it was already translated into Arabic), it seems reasonable to assume that the Syriac translation dates from the end of the eighth century or the very beginning of the ninth. It is not impossible that it was Ibn Bahrīz himself who translated the text first into Syriac, then from Syriac into Arabic (Ḥunayn ibn Isḥāq did such double translations a few decades later).[20]

We next ask whether the Syriac translation was paraphrastic or literal. The Syriac version is not extant, but we have textual witnesses allowing us to determine that it was a literal translation from Greek. Remains of Ibn Bahrīz's Arabic translation from Syriac into Arabic *before* it was revised by al-Kindī are preserved in the *Ta'riḥ* by Aḥmad Ibn Abū Yaʿqūb Ibn Wadiḥ, known as al-Yaʿqūbī (d. 897), which contains a short account of Nicomachus' *Introduction to Arithmetic*.[21] The comparison of the sentences borrowed from Ibn Bahrīz with the Greek version establishes that Ibn Bahrīz's original version corresponds literally to the Greek original, implying that the Greek-into-Syriac, as also the Syriac-into-Arabic, versions (Stages *iii* and *iv*), were both *literal translations*.[22] However, we will shortly see that while al-Bahrīz translated literally, he nonetheless introduced into the work significant interpolations.

Yaʿqūbī's text has further significant information in store. A close look reveals that Yaʿqūbī was under the impression that the body of Nicomachus'

18 See, e.g., Zonta 1992, XXXI, XXXVI.
19 For what follows, see Hugonnard-Roche 1990, 132–134; Brock 1977, esp. 6–10; Brock 1983.
20 On the rationale for this translation technique, see Brock 1977, 2–3.
21 Houtsma 1883, 140–143. This section is translated in Klamroth 1888, 9–16; see also Sezgin 1974, 164–166.
22 Freudenthal and Zonta 2007.

work began only at I.6 and that the foregoing text (I.1–5) was its Proemium:[23] the entrenched notion of "proemium" was projected unto Nicomachus' work, leading its ninth-century (and possibly earlier) readers to view the work's first five chapters as its proemium, the sequel as the bulk of the work.[24] In the ninth century, we realize, the *Introduction to Arithmetic* circulated in a version in which chapters I.1–5 were somehow set apart from the rest and often circulated independently. This ties in neatly with what we saw above: at some point, the Addressee received the body of the work, with the exception of what he and the Revisor considered as the work's Proemium, which – we now realize – consisted of I.1–5. The Revisor, we saw, prepared the Proemium (I.1–5) only after the Addressee had requested it; this means that he "edited" the Proemium and the bulk of the book at two different times.

The Greek version is divided into two books, consisting of 23 and 29 chapters, respectively. The Hebrew version is divided into two books (*ma'amarim*), in conformity with the Greek division. In the Hebrew version the Proemium (corresponding to I.1–5) is not subdivided, except by short phrases of the kind "Abū Yūsuf said." The sequel, from I.6 onward, is divided into a series of unnumbered sections, each identified as a "discourse" (*dibbur*, the exact equivalent of the Arabic *qaul*) and bearing a title indicating its subject-matter: e.g., *ha-dibbur be-geder ha-mispar wa-ḥaluqato* (the discourse on the definition of number and its division), *ha-dibbur be-to'ar ha-kammah ha-ṣerufi* (the discourse on relative quantity), etc. The "discourses" are units smaller than the chapters of the Greek version, of which there is no trace here anymore. Now Yaʿqūbī knows of only three "discourses," showing that the division into the numerous short "discourses" is posterior to al-Bahrīz. We infer that it was introduced by al-Kindī's student, the Revisor, according to the Master's instructions.

The Revisor (following al-Kindī's directives) interfered in the work in two ways. First, he corrected the text, eliminating from it the "false ideas" introduced by Ibn Bahrīz. These interferences can obviously not be identified in the text that has reached us. Second, the Revisor interpolated into the text glosses, introduced by the phrase "Abū Yūsuf said."[25] Some of these are veritable quotations from al-Kindī, as we shall now see. Nicomachus opens his work by a brief discussion of the definition of philosophy that he ends by endorsing the

23 I refer to the division in the Greek version.

24 See *supra*, n. 17. Indeed, medieval Jewish authors looked for – and identified – "proemia" even in the biblical books.

25 Only once does the text of a gloss attributed to al-Kindī indicate where it ends ("end of the words of Abū Yūsuf"), so that at times it is difficult to detect where a given Kindian gloss ends.

definition given by Pythagoras.[26] This discussion is found, abbreviated and modified, in the Hebrew version (519.5–8). Then follows a long gloss introduced by the sentence "Abū Yūsuf said: The Ancients have given to philosophy several definitions" (519.11). Five definitions of philosophy are then presented, followed by a sixth, which al-Kindī presents as his own (523.4–6). Now this long gloss is found with minimal textual variants in the *Book of Definitions* attributed to al-Kindī.[27] The fact that the same passage appears in both works affords us an insight into the method of the Revisor: some of the glosses attributed to the Master he borrowed from other works of his. At the same time, it confirms the authenticity of the passages attributed to al-Kindī in the *Introduction to Arithmetic*, just as it validates the (disputed) authenticity of the *Book of Definitions*. Obviously, the Proemium may include also interpolations not identified by the phrase "Abū Yūsuf said."

The Revisor assures us that his aim has always been to summarize in the most concise way what he had heard from his Master, al-Kindī: "I will refrain from embellishing the book and adding to it," he writes (517.2–3). He further states that he avoided writing longwinded discussions, preferring "short discussion[s] that I have heard from our teacher Abū Yūsuf [al-Kindī] explaining what you wished to be explained. […] I will abandon what I [myself] wished to divulge at length, [replacing it] by his concise statement" (515.14–517.2). But the Revisor does not claim to have completely abstained from any personal comments. On the contrary, when he writes, "I have explained all that can and needs to be explained, omitting all repetition and redundancy. […] In doing so, I made it more accessible to you than it was in the text of the translator, without [however] changing the ideas" (517.12–15), he clearly asserts that he gave a personal stamp to the presentation of the ideas of both Nicomachus and al-Kindī.

In the Prologue the Revisor says nothing more about his inclination for brevity and concision. But later on, following an interpolation by al-Kindī concerning the relative distances of the earth to the celestial bodies, the Revisor intrudes with the following interjection, directed to the Addressee:

> I did not go into long [details] in this discussion, although I suppose that what the author states on this subject is not known to you,

26 Greek text in Nicomachus of Gerasa 1866, 1.1–3.

27 Text in Abū Rīda 1950/1369 H., 172–173. A revised version with an introduction and notes has been published in Al-Kindī 1976, 7–69; for our passage, see 22–23 (text), 35 (translation), 56–60 (commentary). An English translation of this passage is included in Altmann and Stern 1958, 28.

> notwithstanding your sharp mind, your perseverance in the study of the author's words, and your love for this art, and although you count among those who possess his books in their own home. Therefore, I wanted to remind you of this matter [just discussed].
>
> [However,] I have no doubt that this book of mine will eventually fall into the hands of someone who will be unfamiliar with the views of the Master, as you know them. Now, since this [foregoing] discussion [alone] cannot lead [someone] to the truth concerning the *quaesitum*, the ideas [of the unprepared reader] will be confused, the imaginings will vanish, truths will disappear, and knowledge will be lost. May God guide you right in the light of his [al-Kindī's] commentary and allow you to apprehend the splendor of His Glory.[28]

The brevity thus seems to follow the intention to make the text impenetrable to those unworthy of it.

The above reconstruction implies that the version produced under al-Kindī's authority has to be qualified as a *recension* of the *Introduction to Arithmetic*, distinct from that of Ibn Bahrīz (to be presented below). Al-Kindī, it seems, did not himself interfere with the text: he apparently left this task to his student, the Revisor, who prepared the final version of the text according to the directions he had received from al-Kindī and according to his own good judgment in the spirit of his Master. It therefore seems appropriate to refer to this recension as that of *al-Kindī/the Revisor*.

5

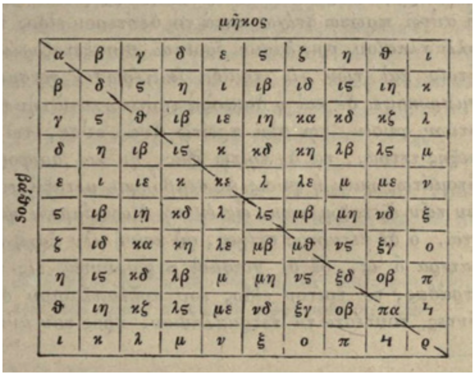
With these insights into stages *v–vi* of the transmission, we can now consider stage *iv*, the Syriac-into-Arabic translation. Let us first recall the important finding that Ibn Bahrīz's Syriac-into-Arabic translation was literal, not paraphrastic. At the same time, as we will now see, Ibn Bahrīz interfered with the text substantially. This is not surprising: we already noticed that al-Kindī observed that Ibn Bahrīz had introduced into the Arabic version his own ideas ("false ideas" in al-Kindī's judgment), precisely those that the Revisor set out to

28 MS Halle, Universitäts- und Landesbibliothek Sachsen-Anhalt Yb 4° 5, fol. 19a, collated with MSS Paris, BnF, héb. 1095, fols 195a–195b and héb. 1029, fol. 11a.

eliminate following al-Kindī's instructions. As mentioned, Ibn Bahrīz authored "epitomes" of two of Aristotle's logical works, thereby evincing that his profile was not that of a "mere translator."[29] Felicitously, we are able to identify in the Hebrew version significant interpolations that can be ascribed to Ibn Bahrīz with a near-certainty.

The Hebrew version of *Introduction to Arithmetic* contains many tables and diagrams. They can be divided into two groups according to their proximity to the Greek text:

1. *Numerical tables and diagrams*. Most of the numerical tables in the Hebrew version reflect their Greek models faithfully, where the Greek letters indicating numbers are replaced by Hebrew letters also functioning as numerals. (Compare Figures 4.1 and 4.2.)



α	β	γ	δ	ε	ϛ	ζ	η	ϑ	ι
β	δ	ϛ	η	ι	ιβ	ιδ	ιϛ	ιη	κ
γ	ϛ	ϑ	ιβ	ιε	ιη	κα	κδ	κζ	λ
δ	η	ιβ	ιϛ	κ	κδ	κη	λβ	λϛ	μ
ε	ι	ιε	κ	κε	λ	λε	μ	με	ν
ϛ	ιβ	ιη	κδ	λ	λϛ	μβ	μη	νδ	ξ
ζ	ιδ	κα	κη	λε	μβ	μϑ	νϛ	ξγ	ο
η	ιϛ	κδ	λβ	μ	μη	νϛ	ξδ	οβ	π
ϑ	ιη	κζ	λϛ	με	νδ	ξγ	οβ	πα	ϟ
ι	κ	λ	μ	ν	ξ	ο	π	ϟ	ϱ

FIGURE 4.1
Numerical Table
This table of the multiples (from × 2 to × 10) of the first ten integers is meant as an aid for generating an epimoric number or superparticular ratio, which is a number that contains a smaller number to which it is compared, plus an integral fraction (½, ⅓, ¼ …) of the latter.
REPR. FROM *INTRODUCTIONIS ARITHMETICAE LIBRI*, NICOMACHUS OF GERASA 1866 (ED. HOCHE), 51

29 See Gutas 1993, 35–36.

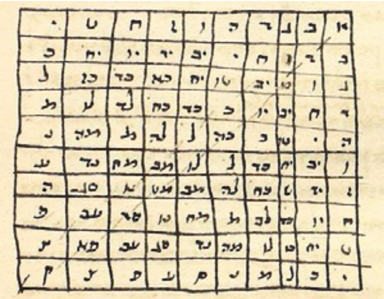


FIGURE 4.2
Numerical Table
This table, from the Ibn Bahrīz-al-Kindī (Qalonymos) version, is the faithful reproduction and translation of the corresponding Greek table, where the Greek numerals have been replaced by Hebrew numerals.
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Other numerical tables are more complete and detailed than their Greek models but follow the same general model (see Figures 4.3 and 4.4).

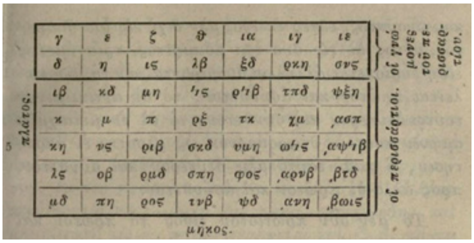


FIGURE 4.3 Numerical Table

FIGURE 4.4 Numerical Table

Figure 4.4, from the Ibn Bahrīz-al-Kindī (Qalonymos) version, is the expanded form of its Greek model in Figure 4.3. It presents the rule for generating singly even numbers, which are the product of a doubly even integer (i.e., a power of 2) multiplied by an odd integer (*Introduction to Arithmetic* I, 10). In the Greek table, the doubly even integers 4, 8, 16, …, 256 are arranged in a row below the top row of odd integers 3, 5, 7, …, 15. Each column in the third line contains the number produced by multiplying the corresponding doubly even integer in the second row by the first odd integer in the first row (3); the fourth row, the number produced by multiplying the corresponding doubly even integer in the second row by the second odd integer in the first row (5); etc. The Hebrew table is set up differently and is more complete. The two generating series are arrayed in the top row (4, 8, 16, …, 128) and the rightmost column (3, 5, 7, …, 15). The product of one term by another (a singly even number) appears where the respective row and column intersect. The Hebrew table adds something else as well: the rightmost column displays the first "doubly odd" numbers (integers divisible into two equal odd integers: 6, 10, 14, …, 30). In the Ibn Bahrīz-al-Kindī version, Nicomachus' characteristic property of a singly even number is treated at greater length than in the Greek work, both in the category of doubly even numbers and in that of doubly odd numbers.
REPR. FROM *INTRODUCTIONIS ARITHMETICAE LIBRI*, NICOMACHUS OF GERASA 1866 (ED. HOCHE), 25; HALLE, UNIVERSITÄTS- UND LANDESBIBLIOTHEK SACHSEN-ANHALT, MS YB 4° 5, FOL. 12R. BY KIND PERMISSION OF THE UNIVERSITÄTS- UND LANDESBIBLIOTHEK SACHSEN-ANHALT

Much the same holds true of the diagrams (graphical representations of mathematical statements; see Figure 4.5): the published Greek text of *Introduction to Arithmetic* carries many diagrams, but the Hebrew version has a many more, some of which correspond to a Greek model and translate it faithfully (Figure 4.6), while others have no Greek model and while they follow the same tradition, their form is more elaborate (Figure 4.7).

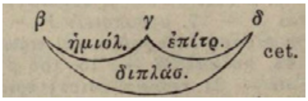


FIGURE 4.5
This diagram, found in some Greek manuscripts of *Introductionis arithmeticae Libri*, illustrates the relationship between three ratios: double, hemiolic or sesquialteral (× 1½), and epitritic (× 1⅓). Five other diagrams of the same type, found in the manuscripts but not reproduced in Hoche's edition, are reproduced in D'Ooge 1926, 235, n. 2; this diagram is also there, 234, n. 2.
FROM: *INTRODUCTIONIS ARITHMETICAE LIBRI*, NICOMACHUS OF GERASA 1866 (ED. HOCHE), 82 (APPARATUS)

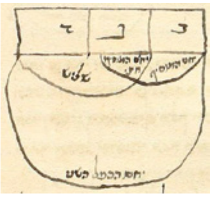


FIGURE 4.6
This diagram is the accurate reproduction and translation of the corresponding Greek diagram (Fig. 4.5), where the Greek numerals have been replaced by Hebrew numerals.
MS PARIS, BIBLIOTHÈQUE NATIONALE DE FRANCE, HÉB. 1095, FOL. 208R. REPRODUCED BY KIND PERMISSION OF THE BIBLIOTHÈQUE NATIONALE DE FRANCE

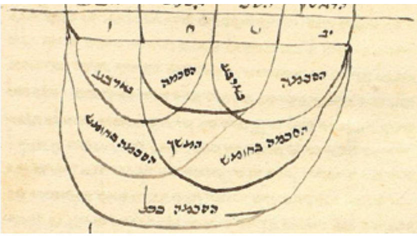


FIGURE 4.7
This diagram, absent from the Greek text, follows the Greek model (Figs. 4.5 and 4.6), but develops it further. It is inserted at the end of the book and is the most elaborated diagram of this form in the Hebrew version. It represents the numerical proportions corresponding to musical consonances: the fourth (8 to 6 or 12 to 9), the fifth (9 to 6 or 12 to 8), the octave (12 to 6), the tone (9 to 8).
MS HALLE, UNIVERSITÄTS- UND LANDESBIBLIOTHEK SACHSEN-ANHALT, MS YB 4° 5, FOL. 54R. REPRODUCED BY KIND PERMISSION OF THE UNIVERSITÄTS- UND LANDESBIBLIOTHEK SACHSEN-ANHALT

At present, little can be said about the origin of these differences. Richard Hoche (1834–1906), the editor of the Greek text (as also its two translators), placed at their center of his concerns Nicomachus' *text*, paying almost no attention to variations in tables or diagrams in different manuscripts. Moreover, Hoche did not study all the extant manuscripts.[30] This disallows drawing any conclusions from a comparison of the Greek version with its Hebrew offspring: when faced with a table or diagram in the Hebrew version not found in the printed Greek text edition, we cannot know at what stage that item entered the text tradition. In any event, as far as the numerical tables and the diagrams are concerned, they are a continuation of an existing Greek tradition, not a radical innovation.

II. *Verbal tables*. But a second group of tables in the Hebrew version have no equivalent or model at all in the Greek text: these are rectangular tables representing *verbally* a schematic synthesis of analyses offered in Nicomachus' text; they contain no numbers at all. Their purpose is to make a theoretical statement with a certain level of generalization more easily accessible and easier to retain (see Figures 4.8, 4.9, and 4.10).

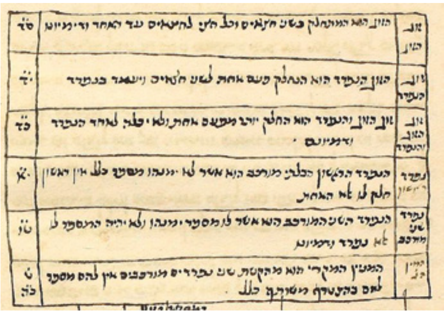


FIGURE 4.8
Textual Table
This textual table summarizes the discussions of the six species of integers: doubly even (example given: 64); doubly odd (14), singly even (24), primes (11), nonprime odd integers (15), and pairs of integers without a common divisor (9 and 25). Like the other textual tables, it does not have a Greek model.
REPRODUCED FROM: HALLE, UNIVERSITÄTS- UND LANDESBIBLIOTHEK SACHSEN-ANHALT, MS YB 4° 5, FOL. 16V. BY KIND PERMISSION OF THE UNIVERSITÄTS- UND LANDESBIBLIOTHEK SACHSEN-ANHALT

30 Hoche used nine manuscripts (Nicomachus of Gerasa 1866 [ed. Hoche], VI–VII), whereas D'Ooge lists 44 (D'Ooge 1926, 147–151).

FIGURE 4.9 Textual Table
This textual table, inserted at the end of Treatise One, summarizes the categories of numerical relationships: equality/inequality, ratios of the greater to the smaller number (five species), ratios of the smaller to the larger number (five species paralleling the previous ones).
REPRODUCED FROM: HALLE, UNIVERSITÄTS- UND LANDESBIBLIOTHEK SACHSEN-ANHALT, MS YB 4° 5, FOL. 31R. BY KIND PERMISSION OF THE UNIVERSITÄTS- UND LANDESBIBLIOTHEK SACHSEN-ANHALT.

The verbal tables constitute a true innovation, for no Greek model is known. Fortunately, they bear the unmistakable stamp of their author: Habīb Ibn Bahrīz. In fact, the latter's *Kitāb Ḥudūd al-manṭiq* (Definitions of Logic), his only philosophical work to have come down to us,[31] contains many tables whose structure and presentation are exactly identical to those of the verbal tables in the Hebrew version of the *Introduction to Arithmetic* (compare Figures 4.8, 4.9 and 4.10 with Figures 4.11 and 4.12).

Ibn Bahrīz presents these tables as one of the highlights of his *Ḥudūd al-manṭiq*. In order to expose the logical ideas of Aristotle and his commentators, he writes, he "chose the [method] of representation [*tamthīl*] and of presentation by tables [*taṣwīr*], so that [these ideas] be easier to understand

31 Published in Danišpazuh 2002, 97–126.

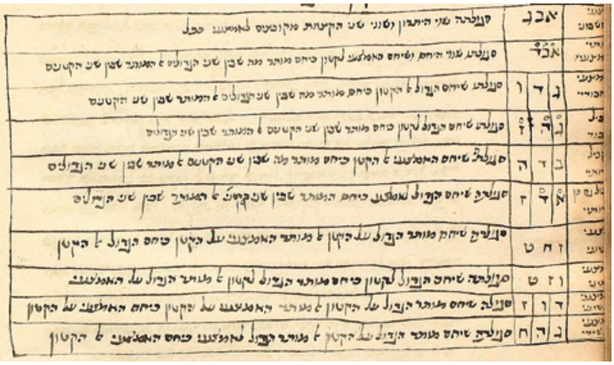
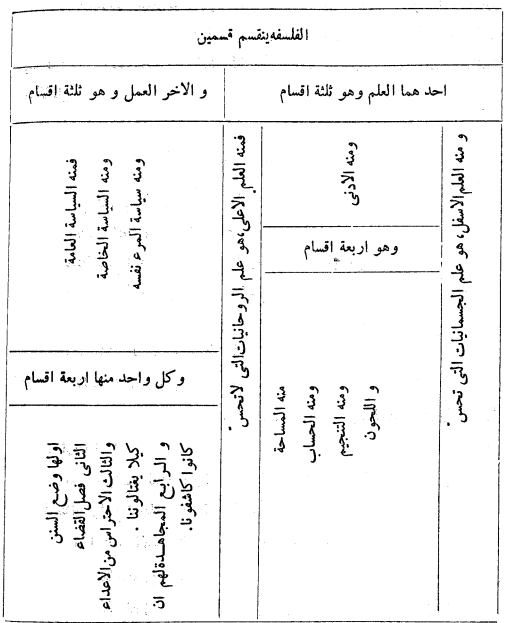


FIGURE 4.10 Numerical and Textual Table
This table is unique in that it combines the presentation of numerical and textual information. The numerical data are already in the Greek version of the *Introduction to Arithmetic* (Nicomachus of Gerasa 1866 [ed. Hoche], 144) and here they are supplemented by a summary of the ten "means".
FROM: MS NEW YORK, JEWISH THEOLOGICAL SEMINARY, 2449, F. 151V. REPRODUCED BY KIND PERMISSION OF THE JEWISH THEOLOGICAL SEMINARY, NEW YORK



الفلسفه ينقسم قسمين

احد هما العلم وهو ثلثة اقسام
فمنه العلم الاعلى وهو علم الروحانيات التى لا تحس .
ومنه الادنى
وهو اربعة اقسام
منه المساحة
ومنه الحساب
ومنه التنجيم
و اللحون
و منه العلم الاسفل وهو علم الجسمانيات التى تحس .

و الأخر العمل و هو ثلثة اقسام
فمنه السياسة العامة
ومنه السياسة الخاصة
ومنه سياسة المرء نفسه
وكل واحد منها اربعة اقسام
اولها وضع السنن
الثانى فصل القضاء
والثالث الاحتراس من الاعداء
كيلا يقتلونا .
و الرابع المجاهدة لهم ان
كانوا كاشفونا .

FIGURE 4.11 Textual Table

<table>
<tr><td colspan="3">وانما ينقسم العلم قسمان لأنّ الاشياءشيئان</td></tr>
<tr><td>والاخر غير محسوس</td><td colspan="2">احدهما محسوس وهوقسمان</td></tr>
<tr><td>وعلمه يسمّى العلم الاعلى بمنزلة الامهّات الاربع ومانشأ منها وعلمها يسمّى العلم الاسفل.</td><td>فمنه مالايفارق العدة التّى هو فيها ولا بالتوهم.</td><td>و منها ما يفارق العدّة التى هو فيها بالتوّهم فقط ، كالدائرة و الشكل المثلّث و المربع، وعلمها يسمّى علم الادب.</td></tr>
</table>

FIGURE 4.12 Textual Table
These two tables are taken from Ḥabīb ibn Bahrīz's *Kitāb Ḥudūd al-manṭiq* and are typical of this work. Their similarity to the textual tables in the Hebrew version of the *Introduction to Arithmetic* is unmistakable. This indicates that the textual tables in the *Introduction to Arithmetic* go back to ibn Bahrīz and attests to the Nestorian scholar's active contribution to this version.
REPR. FROM *AL-MANṬIQ* (*LOGIC*) *BY IBN MUQAFFAʿ* [*AND*] *ḤUDŪD AL-MANṬIQ* (*DEFINITIONS OF LOGIC*) *BY IBN BIHRIZ*, ED. DĀNESHPAZHŪH, 112, 114

and easier to retain."[32] In order to make the definitions of logic easier to understand, he further emphasizes, he endeavored to "discover, abstract, summarize, transcribe them, and represent their divisions and their definitions in tables [*taṣwīr*], so that their representation [*tamthīl*] be in front of the reader's eyes, facilitating their understanding and favoring their memorization."[33] These sentences perfectly characterize the function of the verbal-type tables appearing both in *Ḥudūd al-manṭiq* and in the Hebrew version of the *Introduction to Arithmetic*. Ibn Bahrīz, it should be noted, did not invent the tabular

32 Danišpazuh (ed.) 1978/1398 H, 97.8.
33 Danišpazuh (ed.) 1978/1398 H, 100.19–20.

representation of "divisions": as Henri Hugonnard-Roche and Gérard Troupeau have pointed out, it was inspired by Syriac scholastic models that go back to the Alexandrian tradition.[34] This incidentally explains why "primitive" forms of certain tables can be found, e.g., in Boethius' *De institutione arithmetica*.[35]

The fact that the verbal-type tables are found in *Kitāb Ḥudūd al-manṭiq* attests that Ibn Bahrīz had the necessary competence to prepare those that we find in the Hebrew version of the *Introduction to Arithmetic*. If we consider the Nestorian's educational aims as set forth in his work on logic, we can suppose that he had the motivation to "improve" also the *Introduction to Arithmetic* by adding verbal tables. Ibn Bahrīz was similarly fond of visual representations of scientific knowledge, for in his two known works he also used graphic arborescent diagrams.[36] By contrast, we do not know of any table of the same type in the works of al-Kindī: this makes it little likely that the verbal tables were added later on, by al-Kindī himself or by the Revisor, his faithful student. In general, this didactic means of presenting knowledge was infrequent.[37] Thus, assuming (as we did) that the Syriac version of the text did not contain interpolations, it seems clear that the most likely author of the verbal-type tables is Ibn Bahrīz.

It is important to note that the verbal tables are embedded in the text in a seamless way that disallows the reader to realize that they are not by

34 Troupeau 1997, 141–142; Hugonnard-Roche 1994. Verbal tables of the same kind had been used by Ibn al-Muqaffa' half a century earlier (see Danišpazuh 2002, 1–93); this work has also other similarities with that of Ibn Bahrīz (analyzed in detail in Troupeau 1997). See, as well, Endress 1992, 48 and the bibliography; Gutas 1993, 44.

35 A "primitive" version of Figure 4.10 (which combines a verbal table with a numerical table) is found in Boethius' *De institutione arithmetica* (2, 53; Boethius 1995, 174). (I am grateful to Irene Caiazzo for having drawn my attention to this fact.) It is, however, nearly identical to one found already in the *Introduction to Arithmetic* (cf. Nicomachus of Gerasa 1866 [ed. Hoche], 144; transl. D'Ooge 1926, 284). The much more developed verbal component in the table as found in the Hebrew recension seems to go back to Ibn Bahrīz. To anticipate possible misunderstandings: it is excluded that the "additional" verbal elements entered the Hebrew text directly via Boethius. Qalonymos' translations are all faithful "replicas" of the source texts, without any interpolations by the translator; moreover, in 1317, when he translated the *Introduction to Arithmetic*, Qalonymos (then aged 30) had not been in Italy and did not yet read Latin (see *supra* n. 18). The development and transmission across cultures of the extra-textual elements in the *Introduction to Arithmetic* (tables, diagrams, etc.) deserve study.

36 In *Kitāb Ḥudūd al-manṭiq*, he uses one arborescent scheme (p. 125). In his juristic work he uses a similar device to represent the law of heritages; see Selb 1970, Schema I, II and the explanations on pp. 142–143. This way of graphic representation also has its origins in the Syriac tradition; see Hugonnard-Roche 2001, esp. the folios reproduced on p. 17.

37 Endress 1987, 471.

Nicomachus himself. The first verbal-type table, for example (Figure 4.8), is introduced by the formula: "Here is the table [*ṣurah*] summarizing the divisions [of the species of number] according to what we have discussed from the beginning of *our book* up to this place";[38] the same is true of the other tables.

Ibn Bahrīz thus "intervenes" freely in the text that he translates, although his translation itself was literal, not paraphrastic. Now if Ibn Bahrīz adhered to an ideal of translation that encourages the translator to interpolate glosses in the interest of the reader, then it can reasonably be supposed that he did not limit his interpolations to tables alone. If so, he may be the author of at least a part of the passages in the Hebrew version that have no source in the Greek original and are not explicitly attributed to al-Kindī. By the same token, it seems likely that he authored the formulas "the author of the book says" or "Nicomachus says": probably these formulas were originally placed after an interpolation, to indicate that from that point on the translation of Nicomachus' text resumes. (Presumably, many of these interpolations are no longer there, having been eliminated by al-Kindī/the Revisor.) A tiny philological detail in the Hebrew text seems to confirm that these formulae have their origin in Ibn Bahrīz: the Hebrew text refers to the author of the *Introduction to Arithmetic* with the relatively rare term *maniaḥ ha-sefer* (lit. the one who "established" or "instituted" or "put down" the book), a noun derived from the root, *n.w.ḥ* (= to place, to put, to rest), whose Arabic equivalent is *w.ḍ.*ʿ. This verb is found in the same precise meaning also in Ibn Bahrīz's *Kitāb Ḥudūd al-manṭiq*.[39]

Let us now consider one specific interpolation identified in our text: we will see that it, too, was most likely introduced by Ibn Bahrīz. This interpolation is a gloss that discusses a method "called *diallelos*," a term that does not occur in the Greek text of the *Introduction to Arithmetic*. (The Greek term appears in our Hebrew text in transliteration.) As it happens, the interpolated passage derives from the Commentary on the *Introduction to Arithmetic* by Iamblichus' (*ca* 250–330).[40] Here is the passage as it appears in the Hebrew text; the phrases in italics go back to Nicomachus' Greek text (1.7), the text in roman being the interpolation:

38 MS Halle, Universitäts- und Landesbibliothek Sachsen-Anhalt Yb 4° 5, fol. 16b.

39 E.g., Danišpazuh 2002, 97.3.

40 Iamblichus 1894, 12.22–25; Iamblichus 2014, 78.17–18. The passage has been identified with the help of M. Bernard Vitrac. This is one of several Neoplatonic commentaries on Nicomachus' work; see Robbins and Karpinski 1926, 124–137; Tarán 1969.

> *The odd number is one whose parts are not equal, however you divide it; and it is impossible that its parts not be* [*one*] *odd and* [*the other*] *even.* I.e., when one of its parts is odd, the other is even. It is therefore manifest that the parts of the odd number come closest to being equal when its two parts differ by a unit, by which one exceeds the other. Indeed, in the method [lit. definition; "*geder*"] called "*diallelos*", which consists in determining one of two unknowns through the other – *for the odd number is that which differs from the even number by one unit at both its ends, be it by excess or by deficiency, and the even number is that which differs from the odd number by one unit at both its ends, either by excess or by deficiency* – well, this method does not allow one to determine the unknown odd number unless the unknown even number is known, and the even unknown number cannot be known unless the unknown odd number is known.[41]

How did this interpolation find its way from Iamblichus' Commentary into our text? It seems that no Greek manuscript of the *Introduction to Arithmetic* carries it. Nor was it the Greek-into-Syriac translator who interpolated it: for, as already mentioned, the translators into Syriac as a rule did not make interpolations. The absence of the term *diallelos* in the Syriac literature seems to confirm this assumption.[42] Since the Greek commentaries on the *Introduction to Arithmetic* were apparently unknown to Arabic writers, it seems improbable that the passage was interpolated by al-Kindī/the Revisor. These considerations leave us with Ibn Bahrīz as the most likely possibility. His intellectual "profile" as outlined above indeed makes him into an ideal suspect. But did Ibn Bahrīz have access to Greek sources? This seems possible. First, it is not excluded that he knew Greek. Moreover, we have evidence that in Ibn Bahrīz's time, Syriac translators who were confronted with difficult texts occasionally consulted Greek colleagues: for example, in a letter to Sergius, Metropolitan of Elam, dating from 799, Patriarch Timothy reports that when he translated the *Topics* from Syriac into Arabic he sought advice from Greek scholars.[43] Ibn Bahrīz may have done the same and so learned of the passage discussing the method called *diallelos*. This finding confirms the surmise reached above

41 MS Halle, Universitäts- und Landesbibliothek Sachsen-Anhalt Yb 4° 5, fols. 8b.16–9a.3, checked against MSS Paris, Bibliothèque nationale de France, héb. 1095, fols. 185b.11–20, and héb. 1029, fols. 5a.15–22. The Greek text is in Treatise I, chap. 7 (Nicomachus of Gerasa 1866 [ed. Hoche], 14.4–12; transl. D'Ooge 1926, 190–191).

42 The term does not appear in Smith 1879–1901. Thanks go to Henri Hugonnard-Roche (CNRS, Paris) for his help on this point.

43 Brock 1999, 239 (§ 8).

that many silent (unidentified) extensive interpolations may go back to Ibn Bahrīz.

Ibn Bahrīz clearly invested much thought and labor to improve the Arabic text of the *Introduction to Arithmetic* and bring it closer to the reader. Given the substantial nature of his intervention, we should henceforth think of his version of the text (= the non-extant Arabic version of the text that reached al-Kindî) as a *recension*, rather than a *translation* of the Syriac text. Ibn Bahrīz's recension of *Introduction to Arithmetic* shares the characteristics of the translations carried out in the "al-Kindī circle": first the translators tinkered and glossed their translations according to their own philosophical preferences, and, in a second move, their *spiritus rector* revised them.[44] Given that al-Kindī and Ibn Bahrīz were probably in personal contact and that their respective patrons were connected, one is tempted to think that Ibn Bahrīz prepared the Arabic version of the *Introduction to Arithmetic* at the request of al-Kindī or with his encouragement. This hypothesis allows us to understand why Ibn Bahrīz at all engaged in this enterprise: while as a lawyer he had good reasons to take an interest in logic (its study was situated at the beginning of the curriculum) and to write his epitomes, there is no apparent reason why he should have undertaken to translate such a difficult and long philosophic-mathematical work as Nicomachus'. Al-Kindī, for his part, was certainly interested in Nicomachus' work (*supra*, near n. 8), so it seems plausible to think that he engaged Ibn Bahrīz to translate it for him. (It should yet be borne in mind that al-Kindī was still young when Ibn Bahrīz translated Nicomachus' work; see *supra*, n. 14.)

6

At some point in time, already in the tenth century, the Arabic recension of Nicomachus' *Introduction to Arithmetic* reached al-Andalus. Historians have found that it was already known to mathematicians gathered around Abū 'l-Qāsim Maslama b. Aḥmad al-Majrīṭī, as well as to Ibn Sayyid in the second half of the tenth century.[45] It was also known to two Arabophone Jewish scholars – R. Abraham bar Ḥiyya of Barcelona (d. 1136) and R. Abraham Ibn Ezra

44 Gerhard Endress was the first to have identified the "circle" of translators whose *spiritus rector* was al-Kindi (Endress 1973, 192) and in this context he referred to the *Introduction to Arithmetic* (Endress 1997, 55). On other translations made in this circle and their characteristics see also: Zimmermann 1986; Fazzo and Wiesner 1993, 126ff.; Gutas 1998, 145–146.

45 Samsó 1992, 953–954.

of Tudela (1089–1164)[46] and to others.[47] But which version of the work reached them – that of Ibn Bahrīz, that of Ibn Bahrīz as revised by al-Kindī/the Revisor, or that of Thābit Ibn Qurra? The question has been answered with respect to Abraham bar Ḥiyya, who (as the late Mauro Zonta and I have shown) used the work in Ibn Bahrīz's original, unrevised Arabic version.[48] But eventually also Ibn Bahrīz's text as revised by al-Kindī/the Revisor came to al-Andalus, from where it reached Qalonymos ben Qalonymos, its Hebrew translator. This brings us to the last stage (*viii*) in the evolution of the text prior to Qalonymos ben Qalonymos, one that allegedly took place in al-Andalus.

The Hebrew text (in all eight manuscripts) carries two colophons, one at the end of Treatise I, the other at the end of the entire work. The text of the first colophon is as follows:

> This, may God direct you on the right path, will suffice up to the end of the First Treatise of the *Introduction to Arithmetic*, composed by Nicomachus of Gerasa, the Pythagorean. And it was revised [*t.q.n.*, corresponding to the Arabic *s.l.ḥ.*] in Andalus by Abu Suleiman Rabiʿ ben Yaḥyā, bishop [*usquf*] of Elvira. Help will come by studying and meditating it. God, in His mercy, will direct you, so that you may understand [it] and find in it what you seek and which is useful to your salvation. Amen. (544)

The second colophon is at the end of the book and it is shorter:

> The *Introduction to Arithmetic* is completed, [namely,] the composition of Nicomachus of Gerasa, the Pythagorean, in the revision of Rabiʿ ben Yaḥyā, bishop [*usquf*] of Elvira, the Andalusian. Praise be to God. (544).

The two colophons thus agree that *Introduction to Arithmetic* was revised by a scholar named Rabiʿ ben Yaḥyā; the first colophon adds the *kunya*, Abu Suleiman, and states that the revision was made "in Andalus," a statement which the second colophon confirms by appending the epithet "Andalusian" to name of Rabiʿ. The colophons describe Rabiʿ ben Yaḥyā as *usquf alvirah* (the Hebrew translator merely transcribed the two Arabic words). It is noteworthy that both colophons use the verb (*t.q.n.*, translating the Arabic *s.l.h.*), which is also used by the Revisor to describe his work. This confirms the Revisor's

46 Langermann 2001, 223–224.

47 Steinschneider 1893a, 519.

48 Zonta and Freudenthal 2009.

statement that he revised both the Proemium and the bulk of the book, albeit in two installments.

The explicit statements of the two colophons lead us to try to identify our Revisor as Rabiʿ ben Yaḥyā, bishop of Elvira in al-Andalus. Unfortunately, no person by this name can be found. Steinschneider already perceived the difficulty and suggested that Rabiʿ ben Yaḥyā may be a well-known scholar, Recemund, whose Arabic name could be, according to some scholars, Rabiʿ ben Zayd.[49] This person – to whom some scholars have attributed a role in drafting the Cordoba calendar – was in fact appointed bishop of Elvira by Caliph Abd al-Rahman III.[50] However, this hypothesis is invalidated by two considerations: first, the bishop in question was called "Rabiʿ ben Zayd," whereas the colophons in our text twice name Rabiʿ ben Yaḥyā; second and more significantly, the *floruit* of Rabiʿ ben Zayd is around 950–960, that is, a century after the revision of the Arabic version of *Introduction to Arithmetic* by the direct pupil of al-Kindī. Since Steinschneider, the solution of the problem has not progressed by one iota.[51]

Indeed, it seems that no Abu Sulaimān Rabiʿ ben Yaḥyā can be identified – neither in the east, around al-Kindī (the reference to al-Andalus could be erroneous), nor in the west, where a list of the bishops of Elvira since the creation of this Mozarabic episcopal See does not include this name throughout the ninth century.[52] Further, no Rabiʿ having the *kunya* "Abū Sulaimān" appears in the large computerized databases that were checked.[53] Moreover, in the present state of our knowledge on the transfer of philosophy and science from the Arabic East to the West, the very presence of a student of al-Kindī in al-Andalus in the middle of the ninth century seems unlikely.[54]

How then can we reconcile the precision of the indications of the two colophons with the impossibility to identify the person named therein? The answer suggested here is that the information according to which the Revisor's

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49 Pellat 1995.

50 Steinschneider 1874, 4–6.

51 The hypothesis that Rabiʿ ben Yaḥyā could be Rabiʿ ben Zayd, alias Recemund, has become even more unlikely than it was in Steinschneider's time. Ann Christys has shown that the identification of Recemund with Rabiʿ ben Zayd, and their link with the Cordoba calendar, is an artificial construct of historians which is not sufficiently substantiated by the facts. Thus, nothing allows us to identify our Revisor either with Rabiʿ ben Zayd, or with Recemund, or with any other Rabiʿ known to historians. See Christys 2002, 108–134.

52 See Flórez 1754, 167–171.

53 The *Onomasticon arabicum* (Paris: Institut de recherche et d'histoire des textes [IRHT] du CNRS); and the website *al-warraq.com*, through which numerous Arabic works can be searched.

54 See Samsó 1992, 953–956; Van Koningsveld 1994.

name was "Rabiʿ ben Yaḥyā" is not reliable. A close look at the colophons themselves clearly shows this. For whereas colophons are usually written in the first person, here, on the contrary, the two colophons are in the *third person*: obviously, they were not written by the Revisor himself. The two colophons thus seem to be interpolations, or reworkings of the original colophons. The following reconstruction seems plausible: if (as seems likely) the original text had colophons, they were written in the first person, without mentioning the name of the author; a later scholar who was involved in the transmission of the text, perhaps a scribe, believed that he would do well if he identified him, and he replaced the first person by the name he believed to be that of the Revisor, perhaps also adding at the same time the indications relating to al-Andalus, where the colophon shuffling was presumably done. Alternatively (although less likely), the work had no colophons at all, and both colophons were interpolated by a scribe. In any event, the identification of the Revisor as Rabiʿ ben Yaḥyā the bishop of Elvira seems to be a late fabrication and we must conclude that the information transmitted by the two colophons is probably incorrect. We should remember that the text is known to us only through the Hebrew translation made by Qalonymos b. Qalonymos, from a single Arabic *Vorlage*, so that it needed only one interpolator in a single manuscript to produce the false identification in all the manuscript witnesses of the text. In any event, the replacement of the original colophons or the interpolation must have occurred after the transfer of the sciences to the Iberian Peninsula, and thus well after the Prologue was written by the Revisor, who, as a student of al-Kindī, must have lived in the East in the second half of the ninth century.

7

The developments that culminated in the Hebrew *Introduction to Arithmetic* can now be reconstructed as follows. At an unknown date, probably at the end of the eighth century, an unidentified scholar translated Nicomachus' work from Greek into Syriac. Shortly before 822, Ibn Bahrīz prepared an Arabic translation of this translation under the patronage of Ṭahir b. al-Ḥusain. The translation was literal, but Ibn Bahrīz introduced many interpolations, some borrowed from Greek sources (like Iamblichus' Commentary). Ibn Bahrīz was connected to al-Kindī, and his work resembles other translations prepared in the latter's "circle." Ibn Bahrīz probably introduced the formulas "Nicomachus [or: the author] said" when, following an interpolation, he switched back to translating. When the work left his desk, Nicomachus' translated text and

the interpolations were still clearly distinguished (this contention will be corroborated shortly). Ibn Bahrīz's most characteristic interpolations are the verbal-type tables, a didactic device which Ibn Bahrīz used in other works. This is why his version should be described as a distinct recension of the *Introduction to Arithmetic*. Remnants of it are found in al-Yaʿqūbī's *Taʾriḫ* as well as in Abraham bar Ḥiyya's encyclopedia *Yesodey ha-tevunah u-migdal ha-emunah*. The active role played by Ibn Bahrīz in the history of the transmission of the *Introduction to Arithmetic* is one of the most important findings of the research presented here.

A few years later, the text reached al-Kindī. He made up his mind that Ibn Bahrīz had introduced into the text many errors. He manifestly was confident that he could distinguish between the authentic Nicomachean text and the "false ideas" introduced by Ibn Bahrīz: at this stage, we again conclude, the latter's interpolations were still clearly identifiable as such. (Al-Kindī did not know Syriac and thus could not compare Ibn Bahrīz's version with his model; as noted, the interpolations seem to have been marked by certain para-textual elements, such as the phrase "the author said.") Al-Kindī apparently did not himself "edit" the text that had reached him, but left the task to one of his students, the Revisor, who, however, assures us that he interfered in the text only following ideas of al-Kindī. By this time, the book was already separated into two parts: I.1–5 was considered as the Proemium of the *Introduction to Arithmetic*, the sequel was considered the book itself. At first, the Revisor corrected only the body of the work, leaving its Proemium for later. His interpolations were of two kinds. First, seven passages (of which only one in the second treatise) were interpolated and clearly identified by the introductory formula "Abū Yūsuf said," which probably goes back to the Revisor. At least some of these interpolations attributed to al-Kindī consist of passages taken from other works of his. Second, the Revisor informs us that he meddled with the text also in other ways – shortening and summarizing, embellishing and simplifying – although (he assures us) always in the spirit of al-Kindī.[55] Like Ibn Bahrīz, al-Kindī/the Revisor created a new recension of Nicomachus' *Introduction to*

55 What, then, distinguishes the interpolations that the Revisor expressly ascribed to al-Kindī from the others? If we recall that one of the interpolations that is expressly attributed to al-Kindī is borrowed *verbatim* from a known treatise by him then the following hypothesis suggests itself: when the Revisor had at his disposal a text from the very pen of al-Kindī, he inserted it in the body of the *Introduction to Arithmetic* preceded by the formula "Abū Yūsuf said"; it is a genuine citation. When he did not have such a text at his disposal and relied on his memory or on notes, then the Revisor interfered with the text without signaling it, introducing into the text an interpolation in which he summarized al-Kindī's thought in his own words. This hypothesis needs to be checked systematically.

Arithmetic. This new recension did not do away systematically with *all* of Ibn Bahrīz's interpolations, for the verbal tables were left in place and reached the Hebrew version; apparently Al-Kindī/the Revisor approved of these tables and did not consider them as "false ideas."

At this stage, the revised book (*Treatises* I.6 *sqq*. and II, without the Proemium) was made available to users. One of them – we called him the "Addressee" – wrote to the Revisor requesting the "Proemium" to the work. He already had the bulk of the book and wished to have the opening part too. The Revisor complied: he edited the Proemium as he had previously done with the rest of the book, affixed a personal letter to the Addressee (now: the Prologue), and sent both to the addressee. The latter combined all three texts and this is what became the *Vorlage* that Qalonymos was to translate. It is important to retain that the entire work was revised under the authority of al-Kindī by one and the same person –the Revisor.

With this reconstruction in mind, we realize that the numerous unattributed interpolated passages in the Hebrew version of the *Introduction to Arithmetic* must go back to either Ibn Bahrīz or to al-Kindī/the Revisor. Future research (based on a scientific edition of the entire text) will have to try to assign each interpolation to its author. This task is not necessarily beyond reach. For example, interpolations that reflect a high mathematical level are in all probability *not* by Ibn Bahrīz, for it seems unlikely that this jurist had the mathematical skill to write such passages.[56] They presumably are by al-Kindī/the Revisor.

In 1317 the recension produced by al-Kindī/the Revisor finally reached the prolific Jewish translator, Qalonymos ben Qalonymos of Arles. He prepared a faithful rendition of the Arabic text, a sort of a photograph in Hebrew of the Arabic model that had reached him. Since Qalonymos added nothing and omitted nothing, he retained all the traces left by the previous scholars involved in the long transmission process, allowing us to disentangle some threads in its complicated and long history. The Hebrew text enjoyed some popularity, as evidenced by the eight preserved manuscripts, as well as the commentaries written on it.[57]

56 E.g., at the end of Treatise I there is a passage with an explicit reference to Euclid and a mention of the Euclidean definition of proportion, which are not in the Greek original (MS Halle, Universitäts- und Landesbibliothek Sachsen-Anhalt, Yb 4° 5, fol. 28b.20–29b.17).

57 Langermann 2001.

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CHAPTER 5

Medieval Jewish Pythagoreanism. Remarks on Maimonides and on *Sefer Melakhim*

Tzvi Langermann

Judaism and Pythagoreanism are old friends.[1] Pythagorean arithmology is prominent in the thought of Philo of Alexandria, by far the most important Jewish philosopher in Antiquity.[2] The Talmudic sage Rabbi Abihu was a neighbor and contemporary of Iamblichus and may have been influenced by him.[3] *Sefer Yesira*, a concise compendium of scientific knowledge that probably dates to the early Islamic period, bears some Pythagorean characteristics.[4] Asceticism, arithmology, and contemplation of the heavens seem to have been the most important shared interests.

In this paper I will look at two medieval episodes in the history of Jewish-Pythagorean engagements. Moses Maimonides (d. 1204), the premier Jewish philosopher, was no Pythagorean. However, he has some interesting things to say about the role Pythagoreanism has played in the history of Jewish thought. Moreover, there are some streaks of arithmology in his generally Aristotelian world-view.

Sefer Melakhim ("The Book of Kings") is an unpublished and little-noticed book on arithmology and arithmetic, authored by Qalonymos ben Qalonymos (1286-after 1328), a translator and mathematician who worked at the court of Robert of Anjou. Along with the Hebrew version of Nicomachus (mediated by Ibn Bahrīz, and commented upon by al-Kindi), it is the major repository of Pythagorean lore in Hebrew letters.[5] *Sefer Melakhim* is divided into two sections (at least), one containing some mathematical theorems, the other arithmological remarks on the integers from one to ten. I will offer some

1 Both Josephus Flavius and Philo of Alexandria saw historical connections, especially with regard to the Essenes; modern scholars are as a rule more reticent. See Dillon 2002, 117–128, Atkinson and Magness 2010.

2 Moehring 1995.

3 Pines and Harvey 1986.

4 Langermann 2002.

5 Freudenthal and Levy 2004; Langermann 2001. See also the papers by Gad Freudenthal and Sonja Brentjes in this volume.

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general observations about the arithmological section and present an annotated translation of the chapter on the hebdomad.

Moses Maimonides has been burdened with many labels over the centuries, but he has yet to be labeled a Pythagorean; and I will not commit any such travesty in this paper. However, some features of his writings may invite a Pythagorean interpretation. I will call attention to some of them, and explore the interpretations in a Pythagorean vein that have been suggested by some interpreters of Maimonides' seminal *Guide of the Perplexed*.

Sefer Melakhim, by contrast, is by far the richest and most extensive Hebrew Pythagorean text; indeed, I do not know of a comparable medieval text in any language. Tony Lévy and I are engaged in a long-term – I should say, very long term – project which will, hopefully, lead to the publication of the text. In the meantime another small foretaste seems advisable.[6]

1 Moses Maimonides

There is a certain ambivalence to Maimonides' attitude. On the one hand, he clearly sides with Aristotle against the Pythagoreans. In a letter to his translator, Samuel Ibn Tibbon, Maimonides files the books of Pythagoras under the label of "ancient philosophy" whose study is a waste of time.[7] More than that: labelling Pythagoreanism as outdated and identifying it with the science adopted by the rabbis of the Talmud (and before them, accepted by and informing the visions of the prophet Ezekiel) are important for Maimonides' stance on the proper place of scientific inquiry in the religious life. The rabbinic teachings which Maimonides identifies as Pythagorean prove that the Talmudic sages pursued scientific inquiry, just as Maimonides insists every Jew – every thinking person – must. But they are important no less because Maimonides, relying primarily on Aristotle, can claim that Pythagorean teachings are no longer valid; and teachings that have been disproved are not binding, even if they are noted in canonical texts like the Talmud. The clearest example of this is found in his *Guide* II, 8, where Maimonides notes that "the entire sect of Pythagoras" believe that the celestial bodies emit harmonious sounds, and so also do some

6 I have already included a translation of one passage in my "Studies," cited in the preceding note; see also below, in the second section of this paper.

7 Shailat 1988, 593.

of the sages of the Talmud.[8] Aristotle, however, does not accept this, and neither does Maimonides.[9]

In that same chapter Maimonides states that the celestial harmonies depend on another theory, namely that the stars and planets move within fixed spheres, rather than the Aristotelian (and Ptolemaic) view that they are fixed within revolving spheres. I know of no source for this connection; in Maimonides' reading, one should speak of "the harmonies of the planets" rather than the "harmonies of the spheres." According to Maimonides' text of the Talmud, the sages admit that "the sages of the Gentiles have prevailed";[10] therefore, the theory that the heavenly motions produce harmonious sounds must be abandoned.

Despite this not surprising rejection of a famous Pythagorean teaching on the part of Judaism's premier Aristotelian, on not a few occasions Maimonides makes remarks about the significance of numbers which look to betray a Pythagorean inclination. The number four is the tetrad or tetraktys of the Pythagoreans, and one of the holiest elements in their world-view. In *Guide* II, 10, Maimonides states that "This number four is wondrous and a subject of reflection."[11] Maimonides further suggests that one should divide the concentric spheres that make up the cosmos into four classes – sun, moon, planets, and fixed stars – which would then correspond, not just in number but in some basic properties as well, to the four elements. This scheme, he avers, is his own original contribution.

In *Guide* III, 43, Maimonides undertakes to disclose the rationale underlying the Jewish holidays. He observes that Passover:

> lasts for seven days, for the period of seven days is a mean between the natural day and the lunar month. You know already that this period plays a great role in natural maters. It does so likewise in matters appertaining to the Law. For the Law always tends to assimilate itself to nature, perfecting the natural matters in a certain respect. For nature is not endowed

8 Maimonides must have had in mind doxographical reports that talk about different schools of Pythagoreans when he mentions "the entire sect of Pythagoras." For descriptions of these schools in Arabic doxographies, see especially al-Shahrastānī 1986, 201–212.

9 Jewish authors continued to debate this issue for centuries. For a late medieval chapter in this controversy, see Elior 2016. I briefly note the obtuse discussion of this same issue by the kabbalist Joseph ibn Gikatilla in Langermann 2000, 560.

10 The *textus receptus* of the Talmud preserves a somewhat different version of the debate's resolution, according to which the sage states that "their view looks to be preferable to ours." In Maimonides' version, the Jewish sage offers a full concession.

11 Maimonides 1963, 272.

> with thought and understanding, whereas the Law is the determining ruling and the governance of the deity …[12]

Earlier on he cites an otherwise unknown source that Jacob's ladder had not four but seven rungs.[13] Maimonides divides his great law code into fourteen sections, and in his *Book of Commandments* lists fourteen criteria for classifying legislation as biblical. This choice is likely not fortuitous. However, the many commentators on the *Guide* were not perked to defend, develop, or expand upon the statements concerning the numbers four or seven; their special significance seems to have been commonplace.

A remark in *Guide* I, 31, probably does not have much to do with Pythagoreanism, but I would still like to offer a few observations on it here. Maimonides proclaims that there are some things that all people agree lie beyond the capacity of humans to know, and for that reason people do not long for knowledge of this sort, since they know that there is no way of obtaining it. To this class belong the number of stars, the question whether their number is odd or even, and the number of species of plants, animals and minerals.[14] Note that this a sociological or anthropological observation, regarding certain things people have despaired of knowing. Maimonides doesn't say whether in fact they can be known; and towards the end of the lengthy introduction that he wrote to his *Commentary on the Mishnah* he optimistically describes the growth in human knowledge of the ultimate purposes (*teloi*) of plants and fruits, which seems to me to indicate that someday humankind will eventually know the purpose for which all plants have been created, which ought to mean that we will know how many species there are:

> In every generation, the benefit of plants and species of fruits that had not been apparent to those who came before come to light, and many benefits are derived from them. It is beyond the ability of an individual to fully understand the uses of all the earth's plants; this is revealed only over the course of generations by means of repeated observation (*tajriba*).[15]

12 Ibid., 571.

13 Ibid., 272.

14 It appears that Maimonides' source for the remark I am about to discuss has not been identified. Maimonides 2002 has very thorough notes throughout; but on page 71, n. 5, Michael Schwarz refers only to a study by Alexander Altmann, who cites Aristotle's *Metaphysics* XII, 3, 1070; but Schwarz does not find anything relevant there.

15 Maimonides 1964–1969, 40–41.

What interests me here is the second item: why should it be important to know if the number of stars is even or odd? Orna Harari kindly pointed out to me that the question, whether the number of stars is even or odd, was a cause célèbre in ancient Greek thought.[16] Maimonides' "source" then ought to be the skeptical tradition, rather than the Pythagorean. Nonetheless, it is worth noting that to a Pythagorean, it would be important to know if the number is even or odd: the even was regarded as unlimited (meaning here chaotic) and the odd as limited (regular and productive). (When an even number is divided in half, nothing remains but a void; but when the odd is divided, there is a remainder of one, from which more numbers can be generated). Presumably the harmonious heavens run on odd numbers. Nonetheless, I have not come across a Pythagorean thinker who takes up this question.

Maimonides' most inviting gesture towards Pythagoreanism is found in *Guide* I, 34, the chapter where Maimonides makes a very forceful case for education in the sciences. There he states: "How very many are the premises thus taken from the nature of numbers and the properties of geometrical figures from which we draw inferences concerning things that we should deny with respect to God, may He be exalted!"[17] Maimonides is speaking here within the framework of his so-called "negative theology."[18] Humans cannot know God, because to know something is to encompass or delimit it; but God has no limit. To know something is, for an Aristotelian, to know the causes of a thing – but God has no cause! However, by systematically negating attributes, characteristics, and the like that are mistakenly given to God, we can refine our conception. But what does Maimonides mean specifically by "the nature of numbers and the properties of geometrical figures"? Some medieval and modern interpreters have read some Pythagorean notions into that phrase.

Before presenting their ideas, I should first say something about Aristotle. His *Metaphysics* (1093b 11–21) ends with a sustained discussion of the Platonists – he is clearly referring to the Pythagorean ideas adopted by the Platonists – and the excessive importance they attach to numbers. However, he concludes his blistering attack with a concession of sorts, acknowledging some analogy between numbers and things, which though "accidental" in each case, nevertheless hold true for one number after the other. As I understand it, the analogy between odd and white (given by Aristotle), or between three, four, five, etc.

16 See Kuzminski 2008, 21; the debate carried on into early modern times, see Popkin 1979, 121.

17 Maimonides 1963, 74.

18 See Maimonides 1960b, 121, n. 1.

and their respective analogues is coincidental rather than causal; nonetheless, the fact that analogies exist for the numbers is itself meaningful.

Julia Annas has this to say on the passage in question:

> An interesting (because unparalleled) attempt by Aristotle to salvage something from what his opponents say. He admits that there are interesting mathematical structures reflected in nature, and that the Academy do point out formal analogies between generically different fields. But he firmly denies that this is anything more than a coincidence; in particular, the numbers do not determine the natural facts. Aristotle does not give any background to these supposed analogies here, and they seem very dubious. Although Aristotle sounds less unsympathetic than one might expect, he cannot afford to allow that they are significant, for this would surely undermine the autonomy of the different fields of inquiry, something which Aristotle is strongly committed to.[19]

Numbers are neither first principles, nor building blocks, of the cosmos, as the Pythagoreans and Platonists maintain; but Aristotle's main conclusion is only that numbers are "not separable from sensible things", not that they have no meaning beyond counting. Moreover, he seems to be saying that what we have here is a *string* of coincidences, which makes the whole story look to me to be less "coincidental". That is to say, if there is a coincidental relationship between two and one feature of the universe, and between three and another, and so on, then the relationship between the natural numbers and the universe as a whole is no coincidence. Perhaps this was a loophole that would have allowed Maimonides and other Aristotelians to find so much of interest in "the nature of numbers".

This passage might then be a good suggestion for a "source" for Maimonides' statement; but it probably has to be rejected, because the final section of Aristotle's *Metaphysics* – his lengthy and harsh attack on the "Platonists" as well the concession that I have just described – was hardly known in the Middle Ages. Maimonides probably did not find it in the Arabic version of the *Metaphysics* that he read. On the other hand, as Amos Bertolacci has observed, the absence of these passages from the end of the *Metaphysics* and their unambiguous rejection of Platonism may have facilitated the notion that the philosophies of Aristotle and Plato possess an "inner congruence":

19 Aristotle 1987, 219.

> We may wonder, though, whether the idea of an inner congruence between the philosophy of Plato and that of Aristotle can account, if not for the original extent, at least for the subsequent reception of the translations, i.e., for the actual loss of those books (A, M and N) in which Aristotle more openly criticizes Plato."[20]

2 The Commentaries to Maimonides' *Guide*

Now to the commentators on Maimonides. Shem Tov Ibn Falaquera (thirteenth century) and Asher Crescas (early fifteenth century) gloss in the same vein; they adduce examples that fit – perhaps a forced fit to our eyes – the context of the chapter in question, which is, as we have seen, Maimonides' "negative theology". Shem Tov Ibn Falaquera thought that Maimonides' remark refers specifically to the number one.[21] Multiply one by one as many times as you like, or divide it by one as often as you wish, the result is always the same, unchanging number one. Like the number one, the "True One", that is, the deity, neither multiplies nor divides. This same fixity can be illustrated by means of geometry. The circumference of a circle is fixed and unchanging because if it is divided or increased it is no longer a circular circumference. Falaquera's phrasing here is not very clear; he may intend to say that if it is divided, it becomes a set of arcs, but if increased then, perhaps, it becomes the circumference of a figure such as an ellipse. By contrast, he asserts, a straight line can be cut without the segments losing their identities as straight lines. Here is Crescas on number:

> [...] because the one is the origin of numbers, but it is not one of the numbers. All the numbers grow out of it. Should there be no 'one', there would be no numbers; but should all the numbers disappear, the 'one' would not disappear, because it is their origin and the cause of their existence. Moreover, the one neither changes nor increases, since when you multiply one by one, the product is always one. This is its true special property, and it is does not apply to the other numbers.[22]

I may suggest that Maimonides may have meant nothing more than this: though one may first learn about numbers by counting sensible objects, and

20 Bertolacci 2005, 274.

21 Ibn Falaquera 2001, 135.

22 Crescas' commentary is printed in Maimonides 1960a (the frequently reprinted edition of Ibn Tibbon's translation of the *Guide* with four commentaries), part one, 53b.

about geometrical figures by drawing figures in the sand, one eventually learns to abstract these properties from their sensible instantiations. The properties of a circle, cone, and so forth are eternally and unassailably true, even without there being any material circle or cone to point at. This seems to me to be very pertinent to Maimonides' agenda; he is arguing *inter alia* against people who can't really accept the existence of abstract entities, let alone our ability to make true statements about them. In this admittedly roundabout way, geometry is useful in denying any materiality to the deity.

The passage from *Guide* I, 34 may be the "source" for an odd reference to Maimonides' *Guide*, with citations in Latin and Hebrew, in an early modern Pythagorean work, Athanasius Kircher's *Arithmologia sive De abditis numerorum mysteriis*:

> The Rabbis, and the most celebrated amongst them, called RAMBAM [Maimonides], in his *More Nebuchim*, i.e., *Guide of the Perplexed*, supposes that these signs for numbers would otherwise have not existed, except as points or lines, out of which, as if taken from a unity [i.e., for a single number, point or line], they lead very great series (plural) of numbers. His words taken from the Hebrew are:
>
> הראשונים בני אדם כמו מסרו רבינו זל החשבונות וצורתם כצורת היוד הנקודה.
>
> [Translation of the Hebrew:] The first humans, and like them our rabbis – may their memory be for a blessing – tell us, discovered signs for calculations and their forms which are not unlike the point of the [Hebrew letter] *yod* …[23]

Nothing that resembles the Hebrew passage in the least is found in Maimonides' *Guide*, which does not take up numerical or computational notation in any form.

23 Kircher 1665, 3: "Rabbini, & inter caeteros celebrior Rambam dictus, in suo More nebuchim, id est, perpelexorum Doctore, putat huiusmodi notas numericas alias non fuisse, nisi puncta aut lineolas, ex quibus veluti ex unitate assumpta, amplissimas numerorum series educerent: verba eius ex hebraico deducta sunt:

הראשונים בני אדם כמו מסרו רבינו זל החשבונות וצורתם כצורת היוד הנקודה.

Primi filii Adam, quaemadmodum Rabbini nostri, quorum memoria in benediction, tradunt, inuenerunt signa computationem, eorumque figura non absimiles Iod puncto …"

I thank Charles Burnett for his help in preparing the translation; I have silently passed over some grammatical hiccups in the Latin. Kircher's name is frequently misprinted as Kirchner.

One more passage and comment remains for us before leaving Maimonides. Before launching his defense of the Jewish belief that the cosmos was brought into being by a willful act of God at one moment in time, Maimonides summarizes the basic tenets of Aristotle's physics. He does this because Aristotle held – more precisely, in Maimonides' view, his "later commentators" maintain that he held – to the view that the universe, as we know it today, has always existed and will always exist in the form and manner that it exists today. In order to build a fair and convincing response, and, especially, to make it clear that denial of eternalism does not entail a rejection of Aristotelianism *tout court*, Maimonides first presents in capsule form the twenty-four conclusions of Aristotle's investigations whose truth has been demonstrated; all rational people must accept them. The second of these propositions states: "The existence of magnitudes of which the number is infinite is impossible – that is, if they exist together."[24]

A Yemeni Jew who lived, I suppose, sometime around the fifteenth century, wished to connect this statement directly to the issue of the world's creation in time. He glossed the stipulation at the end of the proposition as follows:

> His saying, 'that is, if they exist together': he means by this that the things that are produced (*al-mawlūdāt*) [come to be] one after the other, and they have no limit. However, whatever exists simultaneously enters into the limit of number. Indeed, number is the root of the sciences, and they are constructed upon it. For number must have a limit. Abū Naṣr hinted at this idea. He said that one of the virtues of number is that number is the first thing that God created; [all of] reality grew out of it. This is a great secret that should not be said openly.[25]

In any event, the interpretation is far-fetched. When the name Abū Naṣr appears in conjunction with Maimonides, the reference should be to al-Fārābī, but I do not know where or if al-Fārābī ever made a statement of this sort. However, the Andalusian Muslim thinker Ibn al-Sīd al-Baṭalyawsī did propound the doctrine ascribed here to al-Fārābī.[26] His book was misattributed in some Hebrew translations to a number of other, more famous authors,

24 Maimonides 1963, 235.

25 MS London, BL Or 1423; on the rich glosses to this manuscript, see Langermann 1995.

26 The Arabic text was published, on the basis of one manuscript, in Asín Palacios 1940, who provides a Spanish translation as well. In the first chapter, al-Baṭalyawsī states that the philosophers could find no closer analogy for the procession of reality from deity than the procession of numbers from the one (Arabic, 64; Spanish translation, 100).

including al-Fārābī.[27] The Hebrew translation of his *Kitāb al-Ḥadāʾiq* ("Book of Circles") was very influential; his analogy between creation and the procession of numbers from the one was adopted by mystics as well, for example, Judah ben Nissim ibn Malka.[28]

This same doctrine is attributed to Pythagoras in other Arabic sources.[29] According to al-Shahrastānī, Pythagoras taught that "number was the first creature (*mubdaʿ*) which the Creator (*al-Bārī*) created (*abdaʿa*)."[30] A Hebrew commentary to Nicomachus' *Arithmetic* in MS Moscow, Ginzburg 340, f. 4b, associates this doctrine with the "scholars of India": "The scholars of India said that knowledge of the expansion (*hitapshtut*) of the cosmos, and the manner that it came to be from Blessed God, is [identical with] knowledge of the expansion of number [from] the one and its value relative to it." That manuscript is a very rich codex of Pythagorean and other mathematical texts from Byzantium, some translated directly from the Greek to Hebrew.[31] According to al-Shahrastānī, there was an Indian branch of the Pythagoreans; perhaps they are somehow related to this theory of the world's generation.[32]

3 The *Sefer Melakhim*

Sefer Melakhim, "Book of Kings" was written by Qalonymos b. Qalonymos for King Robert of Anjou. This same Qalonymos is the translator of a number of important mathematical treatises, including al-Kindi's version of Nicomachus' Arithmetic, the other major Hebrew book presenting Pythagorean arithmology and arithmetic.[33] *Sefer Melakhim* is extant in two manuscripts: Bavarian State Library Munich Germany Cod. hebr. 290, pp. 49–62, and a fuller version in a manuscript once in the possession of Rabbi Yosef Kapah (Qāfiḥ), and on film at the Institute of Microfilmed Hebrew Manuscripts in Jerusalem (Kapah 36, f. 215b–225a). The first part belongs to the arithmological tradition; the second

27 Steinschneider 1893, 287, n. 104; the misattribution is found in MS London, British Library Add. 21140, on the title page (f. 20a), in a later hand, which states that the authorship is ascribed to "al-Baṭalmiyy (?) or to Abū Naṣr." On this treatise and the possible influence of al-Fārābī, see Eliyahu 2015.

28 Kaufmann 1880, 2–3 and 28 of the Hebrew text; Vajda 1954, 71–72.

29 For a possible late antique parallel see Kalvesmaki 2013, chap. 4.

30 al-Shahrastānī 1986, 203.

31 Langermann 2001.

32 al-Shahrastānī 1986, 211

33 See note 5 above.

contains some arithmetical formulae. My remarks here will be limited to the first section alone.

Arithmology is a term used by historians of science to distinguish the "scientific" or "philosophical" interest (or obsession) with numbers from the mystical interest.[34] When speaking of Jewish sources, it is important to note that the arithmologist evinces no interest at all in gematriot, which looks for significance in the alphanumerical value of words. Instead, arithmologists study the arithmetical properties of numbers and their correlates in the natural and supernatural realms. Philo Judaeus, the well-known and thoroughly Hellenized Jewish philosopher from Alexandria, made much of numbers (especially the number seven) in his writings.[35] A short treatise by Abraham Ibn Ezra, *Sefer ha-Eḥad* ("The Book of the One"), belongs to this genre, and some relevant materials are found in *Sefer ha-Shem* ("The Book of the Name") by the same author.[36]

Sefer Melakhim begins, "Know that this chapter is the most glorious [part] of this science; it is as if it were its final purpose."[37] It seems, then, that the two-part excursion into number theory, which is all that remains of the treatise now, is part of a much larger work, presumably an encyclopedic presentation of the exact sciences. The first part of the surviving section has the most comprehensive survey of numbers, their correlates and special properties, that I know of in Hebrew: both their arithmetic properties, their manifestations in the natural world, and their role in the supernatural world as well (I mean, of course, angelic and the like) are taken up in detail.

In the opening paragraph we find a surprise: in one important respect, *Sefer Melakhim* is an avowedly anti-Pythagorean tract. Qalonymos explicitly denies the claim of "the ancients" that numbers are the principle or source of all sensible reality. Like Aristotle, Qalonymos holds that number is an "accident" of enumerated things – an incidental property of the beings that populate our universe, rather than the source or cause of their being.[38] He makes explicit reference to Aristotle, naming both the *Physics* and the *Metaphysics*, as well as his commentators, among the "Greeks, Arabs, and Christians". This issue in philosophy stands at the heart of the divide between the Aristotelians and the Pythagoreans. We had a whiff of it already in the first part of this paper, in the passage cited from Maimonides' *Guide* I, 34. Aristotle rejects Plato's ideas or

34 The term goes back at least to Delatte 1915.

35 Robbins, 1931; Moehring 1995.

36 Langermann and Simonsohn 2000, 167–171.

37 The opening paragraphs are found only in MS Jerusalem Kapah 36.

38 Sousedík and Svoboda 2012.

forms, self-standing immaterial entities inhabiting some higher universe, and for very much the same reasons he rejects the Pythagorean concept of number, which the Platonists appear to have accepted and assimilated to their notion of ideas. According to Aristotle, numbers are not abstract entities that exist independently of the things one counts. In his view, both the Platonic ideas and the Pythagorean numbers are abstractions that have been "reified" beyond the point of tolerance. But, as we have seen, once this critical point is scored, Aristotle is willing to concede some "coincidental" relationship between numbers and natural phenomena.

Qalonymos clearly sides with Aristotle on this pivotal question. However, he does not simply rehearse the views of the ancient Pythagoreans before rejecting them. Here follows the list he gives of the mathematical relations evident in the universe; these served as evidence for their view that numbers are abstract entities or ideas (*nivdalim*, literally "unattached [to matter]") that are the principles of being. Clearly, some of these items, notably those taken from astronomy, are not found in Aristotle. Hence we may conclude that Qalonymos is knowingly insinuating himself into an ongoing debate and living tradition to which much has been added since Aristotle: "They also found the immaterial intellects to have plurality and countability, at least with regard to their being cause and effect, or reality and quiddity.[39] They also found that most sensible entities maintain fixed proportions, for example, the size of the stellar bodies, the width of their orbs, and their eccentricities, and so also for the circles that are fixed in the eighth orb; and so also with regard to the width of the bodies of the elements …"[40] In other words, this evidence – which he will display later on, in the chapters on individual numbers – has accrued over a long period of time, and many different people have contributed to its collection.

Having made his pro-Aristotelian position clear on this matter, Qalonymos adds that numbers do possess amazing properties, some of which remain hidden; the soul rejoices in learning what it can about them. Therefore, even if numbers are only "accidents" of reality, one is justified in investigating their properties and analogues in external reality, both material and immaterial. Both numbers and geometrical figures contain much information about the

39 Aristotle maintains that immaterial entities cannot be distinguished one from the other, unless it be that one is the cause of the other, or possessing a higher degree of reality, ontological status, or rank in the scale of being (equivalent ways of expressing the same notion), or in their "quiddity" (see Maimonides, premise sixteen to *Guide*, Book II). If these immaterial beings can be distinguished one from the other, they can be counted, and thus bear some relation to number.

40 The "bodies of the elements" must refer to the idealized placement of the four elements within four concentric spheres.

nature of reality. He calls these correspondences a "secondary intention," which means that they are a desirable and intended side effect, but not the main purpose of their existence. Here again, he wishes – so I take it – to valorize arithmology to the extent possible for an Aristotelian.

In the chapters which follow, and which are devoted to the first ten integers, Qalonymos cites from a very wide range of sources. Books by Aristotle, Ibn Rushd, and other well-known figures are mentioned, along with Plato's *Timaeus*. *Sefer Yeṣira*, an early Hebrew text which figured prominently in medieval Jewish scientific thought, is cited in the chapter on the number ten.[41] However, we encounter also an impressive list of books that are rarely if ever mentioned in the Hebrew literature; Qalonymos must have read them in Arabic. These include the *Book on Occult Properties* (*Kitāb al-Khawāṣṣ*) by "Jābir bin Ḥayyān, the teacher of alchemy", cited in the chapter on the number two, and the writings of the *Ikhwān al-Ṣafāʾ* ("Brethren of Purity"), referred to several times.[42] Apollonius' *Conics* is cited in the chapter on the three; Qalonymos is the only Hebrew writer to have some acquaintance with that text.[43] Gad Freudenthal is certainly correct that Qalonymos found these texts in the library of Robert of Anjou, in whose service he entered in 1314, and to whom *Sefer Melakhim* was dedicated.[44] That same library would have held Latin books as well. Interestingly enough, Qalonymos never cites a Latin text by name, though he does refer occasionally to "Christian scholars," as we have already seen.

The two manuscripts of *Sefer Melakhim* often differ, and sometimes the divergences are significant. Occasionally neither copy presents a plausible text. Moreover, Qalonymos employs unusual terminology, presumably of his

41 Langermann 2002.

42 One of their epistles is in fact devoted to arithmetic; see Goldstein 1965. The Ikhwān are often found to be sources for Hebrew texts by modern scholars; see in particular Abrahamov 1995. The only known translation was made by our Qalonymos; it is actually a paraphrase of the "Debate between Humans and Animals"; see Ikhwān al-Ṣafāʾ 1978. The half dozen or so transcriptions of the Arabic text are all found in Yemeni manuscripts; see Langermann 1996, 150. Pythagoreanism plays a significant role in Jābir's thought; see Kraus 1986, index and passim. No alchemical works ascribed to Jābir were translated into Hebrew, nor have any fragments been identified in transcriptions of Arabic texts into Hebrew letters; but see below, note 44.

43 Langermann 2013.

44 Freudenthal 1993, 123, referring to an explicit report that Qalonymos, here called Maestro Caelo, found Jābir's book on poisons among "the treasures" of King Robert. Qalonymos must then be the translator of the lone Hebrew fragment of that book which I identified in MS St. Petersburg, National Library of Russia EVR II A 410/01; see Langermann, 1998–1999. For further reading, see Shatzmiller 2012, 163–169.

own invention and derived from his study of Arabic mathematical treatises. He provides many source references, but they cover only a small portion of the many instantiations of numbers in the natural world and beyond that he reports. A complete edition and translation is a daunting task. In an earlier study I offered a preliminary annotated translation of the chapter on the number four.[45] I close this paper with a similarly preliminary translation of the chapter on the number seven.

The Seven. It is a prime number of the first order. The prime numbers of the first order are 2, 3, 5, and 7. The number seven is the sum of the first even and the second odd,[46] and the first odd and the second even.[47] For this reason the ancient scholars called it a complete[48] number. It is in the middle between the four compound numbers, two coming before and two after it. Four and six come before it, and eight and nine come after it. When you double seven the product is fourteen, which is equal to the sum of the squares of one, two, and three, which comprise the nature of all number, as we explained above.[49] The sum of [the integers from one to] seven is a perfect number, and there is none other in the order of tens.[50]

Many things in the real [world] come in sevens.[51] Among them are the seven stars, which are the first governors of the world. For this reason, the Jewish scholars of old called them the [divine] attendants (*meshartim*).[52]

45 Langermann 2001, 233–236.

46 2 + 5.

47 3 + 4.

48 *Kolel*; this term is not found in Sarfati 1968. Cf. al-Shahrastānī 1986, 203: "[…] sept, un nombre complet (*kāmil*) parce qu'il est la réunion du pair et de l'impair."

49 The numbers one, two, and three comprise the nature of all numbers. See, for example, this paragraph in the chapter on the number two from *Sefer Melakhim*: "It is of the nature of the two that its sum and product are equal. Hence the plurality of the first effect is not absolute plurality. It is merely additive, but not a plurality of essences. In this way it is similar to the nature of numerical duality, whose product is not greater than its sum. This is not the case for all of the numbers that come after it; for all of them, their product is greater than their sum. It is, as it were, a mean between the nature[s] of unity and plurality. For this reason, it is placed, by its nature, between one and three; for the sum of one is greater than its product, but the opposite holds for three. Since the two is a mean between them, it has the nature of a mean between [the first cause? and] the first effect, insofar as it is not absolutely one but not absolutely plural."

50 Twenty-eight, the sum of the integers from one to seven, is a perfect number. See Heiberg 1901, 35 (Greek) and 50 (French).

51 Literally, "run the same way as the seven."

52 The planets govern the terrestrial world in the service of the Lord. Each planet "serves" at fixed times according to the rotations that make up the planetary week. The term is first applied to the heavenly bodies in chapter six of *Pirqei di-Rebbe Eliezer*, which is now

The [number of] days of each lunar quarter is seven, and they transform[53] the airs and natures that relate to health and illness.[54]

The terrestrial climates are seven. This division is not conventional, but rather is consequent upon a higher force, as the astral scholars[55] have said. The kinds of metals are seven: gold, silver, copper, tin, lead, iron, and mercury. Even though iron is not thought to be fusible, one can melt it by means of a hidden ruse, so that it melts quickly, like lead.[56]

There are seven types of irrational animal: predatory animals; non-predatory animals; predatory birds; non-predatory birds; winged insects; those that crawl in the dirt, i.e., insects and reptiles;[57] marine animals. From each one them many species branch out.

The sciences, as they are divided by the philosopher, are seven: nature; divine; number; geometry; the science of the configuration of the orbs; the science of music; and political science.[58] The moderns have disagreements about this division. They did not count logic, because it is not a science, but rather only a tool.[59]

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thought to be an early medieval text, but Qalonymos must have considered it to be tannaic, that is, dating from around the beginning of the Christian era.

53 Hebrew *ya'atiqu*, influenced by Arabic *naqala*, to carry, transform, that is, induce change.

54 Galen emphsized the role of the lunar quarters in his explanation for the crisis in fevers, rejecting the Pythagorean claim that it was the number seven alone that was responsible; see, e.g., Bos and Langermann 2014, 116–120. The quarterly phases of the moon were a major factor in medieval medical astrology; see Bos, Burnett, and Langermann 2006.

55 Qalonymos probably has in mind astrologers, but the Hebrew means "scholars with regard to the heavens".

56 Qalonymos may be referring to the use of a reducing agent such as charcoal for smelting iron. The technology to produce cast iron, which requires a much lower temperature, looks to have been available, but the "literacy factor" precluded its methods being recorded in the medieval literature; see Smith 1961.

57 Literally, "unclean and creeping [life-forms]."

58 This seven-fold classification likely follows the Jewish tradition of identifying the seven "pillars" of wisdom's house, mentioned in the verse from Proverbs 9:1, with the seven sciences. Qalonymos' list is almost identical with those displayed by Zerahia (Gracian) Hen, Moses da Rieti, and Judah Abrabanel (Leo Hebraeus), all discussed by Wolfson 1925, 284–285, except that these authors have ethics or practical philosophy where Qalonymos mentions political philosophy. Moreover, Qalonymos ignores any subdivision of the seven sciences; perhaps this is due to his focus here on the number seven.

59 Wolfson 1925 sees the inclusion or exclusion of logic from the sciences to be one of the points of disagreement between Aristotle, who viewed logic as a tool, and Plato, who included logic – understood, though, "in the more general sense of dialectics and as synonymous with metaphysics" – in his list. This split, and its nuanced development over the centuries, undergirds his classic study with the recurring question, is the classification before me Aristotelian or Platonic, and to what degree?

The truth is as R[abbi] M[oses] of blessed memory said in chapter forty-three of the third [part of his *Guide of the Perplexed*]: the hebdomad plays a great role in natural and legal matters.[60] This was stated as well by the author of *Sefer ha-Tamar*, at the end of his book.[61] It is one of the things that must be acknowledged.

The ages[62] of man are seven, as Ibn Sīnā said that the beginning of his book, the *Qanūn*, and [also] Hippocrates in his books (!) *On Hebdomads*.[63]

The types [categories] of quantity are seven: line; surface; body; place; time; number; speech.[64] The minimum number of months that the embryo must have so that the child can survive are seven. The reason for this length [of time] is explained well in the books of the astrologers,[65] because natural science does not suffice to account for this fully.[66]

4 Conclusions

I have demonstrated the interest of medieval Jewish thinkers in Pythagoreanism, as well as the historical importance of their writings, by means of two examples. Moses Maimonides, certainly one, if not the greatest, of all Jewish thinkers, was not in any significant sense a Pythagorean. However, certain Pythagorean notions play a non-trivial role in his thought. Moreover, Maimonides contributes some historical investigations into the adoption of Pythagorean ideas by the ancient rabbis.

60 Literally, "matters pertaining to the Torah"; this quote and its interpretations have been discussed in the first part of this chapter.

61 "Denn die Siebenzahl is den existierenden Dingen sehr natürlich, und dies Geheimnis wird dir in einer anderer Wissenschaft auseinandergesetz" (Abu Aflaḥ 1927, 58).

62 Literally, "the changes that come with the years"; once again, I prefer to translate with the well-known English phrase.

63 Ibn Sīnā does not mention the seven ages explicitly in his *al-Qanūn fī al-Ṭibb*, but he does describe the changes the body's temperaments undergo during the course of life in *Qanūn* 1.1.3.3, which figures near the beginning of his massive work on medicine. In fact, Ibn Sīnā identifies three phases, namely growth, stability, and decline, which he then subdivides. On the tract ascribed to Hippocrates, see Mansfeld 1971, 169.

64 These are the seven species of quantity identified by Aristotle in his *Categories*, chap. 6, 4b20–5a14.

65 Literally, "scholars of the stars."

66 Both "natural" and astrological justifications for a pregnancy of at least seven months can be found in the ancient and medieval texts studied by Barkai 1989. Saturn rules the eighth month, and for that reason the astrologers held that a baby born in the eighth month will not survive. The correlation between gestation periods and lunar cycles was investigated in great detail by medieval astrologers; see now Díaz-Fajardo 2017, especially p. 139.

Sefer Melakhim is one of the largest and richest medieval arithmological treatises in any language. Our small sampling has shown how the author Qalonymos expanded the genre, citing medieval authors and adding numerous examples to the well established literature on hebdomads.

The affinities between Judaism and Pythagoreanism have ancient roots. Yet it would not be correct, in my opinion, to claim that either Maimonides or Qalonymos were consciously carrying on an ancient tradition. Maimonides, in fact, views the "music of the spheres" as a false doctrine once accepted by the rabbis but now known to be incorrect. Pythagoreanism is not a "living" tradition among Jewish thinkers; that is to say, those who contribute to it do not see themselves as torch bearers in a relay race. Instead, it seems that there some features of Pythagorean thought, for whatever reason, are found to be attractive across widely varying historical contexts.

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PART 2

Pythagorean Way(s) of Life, East and West

CHAPTER 6

Popular Pythagoreanism in the Arabic Tradition. Between Biography and Gnomology

Anna Izdebska

In this chapter,[1] I will discuss the most important Arabic sources presenting and popularizing the figure of Pythagoras and his teaching: the biographical and gnomological writings that the Arabic tradition actually often mixed together. This group of texts is quite different from the main Pythagorean corpus preserved in Greek and provides additional material for studying the late antique reception of the Pythagorean tradition. At the same time, these texts are also worth studying in their own right, as witnesses to the Arabic and Islamic culture in which they were used or created. As the Pythagorean tradition had to be incorporated into a new cultural, linguistic and religious context, a new, Arabic/Islamic phase of its afterlife developed. In this way, the Arabic Middle Ages contributed to the fascinating process in which a philosophical tradition lived, changed, spread, disappeared, and flourished again across centuries and cultures, starting from Archaic Greece, which was the context in which the semi-legendary figure of Pythagoras himself lived and emerged as a cultural phenomenon.

For modern scholars, the late antique Greek biographies of Pythagoras are one of the main sources of knowledge about Pythagoras' life and thought, despite the fact that they contain deeply reworked and highly controversial material that purports to relate events which took place almost a thousand years before they were composed (that is the life of Pythagoras and the early stages in the development of the philosophical school that was associated with his name). Each of the three extant late antique biographies of Pythagoras, by Diogenes Laertius (2nd–3rd c. AD), Porphyry (3rd c. AD) and Iamblichus (3rd–4th c. AD) – and there existed more than just these – are based on much older sources, given that the biographies of Pythagoras had existed in written

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form since at least the 4th century BC (starting with the works of Aristoxenus of Tarentum and Lycon).[2]

Nevertheless, the late antique biographers were writing at a specific moment in the history of ancient philosophy and they all had their own agendas. The biography of Diogenes Laertius should be read in the context of all the other biographies contained in his *Lives and Opinions of Eminent Philosophers*, while Porphyry's and Iamblichus' texts can serve as very good examples of a peculiar genre that was popular in later Antiquity, that of the biography of a (pagan) holy man, *theios aner*, its pioneer and most famous example being the fictional *Life of Apollonius of Tyana* by Philostratus (d. AD 245).[3] As a consequence, these three biographies are a mixture of history and legend, philosophy and religion, and they were composed in order to provide an edifying example of a perfect practical implementation of specific philosophical ideas in someone's life.[4] As such, they aim to popularize philosophy in general.

Apart from Pythagoras' biographies, another branch of the popular philosophical literature devoted to Pythagoras which was transmitted in Arabic were the gnomologies (a term by which I understand here collections of sayings and short anecdotes). As it has been remarked by Dimitri Gutas, "of all the Greek writings that passed to the Arabs, the gnomologia were unquestionably the most popular."[5] This is also the reason why gnomological collections play such a crucial role in the transmission of the figure and the teaching of Pythagoras into Arabic in the Medieval period. On the one hand, there survive only a few self-standing gnomological collections attributed to Pythagoreans, the most important one being preserved in slightly different forms under a single title, the *Pythagorean Sentences*. This collection overlaps significantly with other ones preserved in Greek, such as the *Sentences of Sextus*, the *Sentences of Demophilus* or the *Sentences of Clitarchus*, as well as with some collections preserved in Syriac. On the other hand, even though the main general gnomological collections (such as the *Corpus Parisinum* or the *Gnomologium Vaticanum*) contain little gnomic material attributed to Pythagoras, such material can be found in other sources of various literary genres, such as florilegia (like Stobaeus), protreptics (Iamblichus, *Exhortation to philosophy*), commentaries (Hierocles of Alexandria, *Commentary to the Golden Verses*) and letters

2 Macris 2013, 60, 72. About all known biographies of Pythagoras preserved as well as unpreserved see Macris 2018, 708–760.

3 For Diogenes Laertius, Porphyry and Iamblichus, see Macris 2018, 753–760. For Philostratus' work, Bowie 1978; Dzielska 1986; Macris 2018, 1140–1142.

4 Macris 2013, 2014 and 2018, 752–760; O'Meara 2014.

5 Gutas 1975, 459.

(Porphyry, *Letter to Marcella*). There also exists gnomological material which remains in manuscripts and awaits proper critical editions.[6]

The pseudo-Pythagorean poem known as the *Golden Verses* also belongs to this part of the Pythagorean tradition. It was already used in this context by Stobaeus.[7] It is mostly devoted to the Pythagorean way of life and as such it is intended to have a real influence on the reader's life – rather than just to present some philosophical theories.[8] From a very early stage, the ancient sources depicted Pythagoras as a legendary figure of a divine man; at the same time, he probably left no writings at all,[9] despite being the founder of a powerful philosophical-religious-political tradition that mutated throughout Antiquity. As a result, Pythagoras' life, sayings and anecdotes about him became the preferred way of transferring ideas about what Pythagoreanism is and what the followers of Pythagoras should do.

It was this complex, multi-layered tradition that reached the Arabic world during the "Classical" period, transmitted through a variety of sources.[10] Many of these texts can be classified as "popular," by which I mean being written for the purpose of popularizing philosophical ideas and bringing attention to the figure of the philosopher as a moral teacher. Such popular philosophy would have been used in the early stages of a philosophical curriculum (as it happened with the *Golden Verses* in the case of the late antique Neoplatonists: Iamblichus' *Exhortation to Philosophy* and Hierocles' *Commentary to the Golden Verses*), but it could also have reached a wider audience of educated people, aristocrats, court officials, rulers, etc. Differently from doxography,[11] the genres of biography and gnomology remained the key tools in the process of popularization, and were sometimes separated, but most of the time mixed together, as gnomologies are often embedded in short biographical narratives (such as anecdotes, descriptions of events from a philosopher's life or of his

6 On the Pythagorean gnomological tradition, see Prochenko 2018a and 2018b.

7 Thom 2017, 80.

8 See Thom 2012, 285–287 and 2017, 77–85. Thom remarks (2017, 78) that "the importance of internalizing and practising the precepts [is] emphasized throughout by the repetition of verbs meaning 'knowing', 'learning' and 'thinking', 'mastering', 'practising', 'accustoming', 'doing', 'accomplishing' and 'labouring'." According to Macris 2013, 59–60, the *Golden Verses* were "composed to satisfy the spiritual needs of the members of a brotherhood."

9 On works (falsely) attributed to Pythagoras in Antiquity, see Macris 2018, 843–850.

10 See De Smet 2011 for some general remarks about the transmission of the heritage of Pythagoras among Islamic authors as a "voie diffuse." For general overviews of the transmission of the figure of Pythagoras in Arabic, see Rosenthal 1991, Strohmaier 2014, and Izdebska 2018.

11 On some specimens of it in Arabic see the chapter by Daniel De Smet in this volume.

interactions with students, etc.), while biographies are frequently enriched with maxims and sayings.

These two main genres of the popular philosophical writings, biography and gnomology, were closely related also in the Arabic tradition. One of the most extensive chapters devoted to Pythagoras in Arabic literature, in the *Book of the Choicest Maxims and Best Sayings*, by Abū al-Wafāʾ al-Mubashshir Ibn Fātik (11th c.) (which I discuss later on in this study), is divided (like all other chapters in this work) into two parts: a biographical one, entitled "information about Pythagoras the wise man" (*akhbār fithāghūras al-ḥakīm*) and containing an Arabic translation of an abbreviated version of Porphyry's *Life of Pythagoras*; and a gnomological one, entitled "his wisdom and wise sayings" (*ḥikmahu wa-l-adabuhu*), which is the richest collection of sayings attributed to Pythagoras that exists in Arabic (106 sayings and anecdotes).

When it comes to gnomology, the above-mentioned collection in al-Mubashshir's work is mainly based on one of the versions of the gnomological collection called *Pythagorean Sentences* (in a version that is not preserved in Greek, which makes the Arabic translation even more useful for studying the Greek Pythagorean gnomological collections). However, this was not the only Pythagorean gnomological collection preserved in Arabic. There also exists a collection of about forty or even more sayings and anecdotes about Pythagoras that was used in the Arabic gnomological history of Greek and Muslim philosophy, known as the *Ṣiwān al-ḥikma* (*The Cabinet of Wisdom*). Although this work itself is not preserved, it is accessible through three independent abbreviations, which I will discuss in more detail below. Finally, some maxims of Pythagoras can also be found in other Arabic sources, such as the inedited manuscript Istanbul Köprülü 1608[12] or in the collection of sayings and anecdotes of Greek philosophers composed by Abū l-Faraj ʿAlī ibn al-Ḥusayn Ibn Hindū (11th c.) (entitled *The Spiritual Contents of the Greek Maxims*). This author quoted fourteen sayings of Pythagoras, all of them probably taken from a completely different source than the other extant Greek and Arabic collections attributed to this philosopher.

The Pythagorean *Golden Verses* were translated into Arabic and became relatively well known as the presentation of the main teaching of Pythagoras. The poem was mixed with the gnomological literature even more strongly than in the Greek tradition. There are two commentaries to the *Golden Verses* preserved in Arabic, one attributed to Iamblichus and another one to Proclus, neither surviving in Greek. They should also be included into the group of writings that popularized the image of Pythagoras as a moral teacher and

12 For a description of the contents of this manuscript, see Gutas 1975, 42–50.

that presented his teaching as something that a reader should practice in his own life. In this chapter, I am presenting briefly the commentary attributed to Proclus, since it also contains a short biography of Pythagoras that was probably based on Iamblichus' *Pythagorean Way of Life*.

As for biographies of Pythagoras, in general, they were not so widely transmitted, at least in the preserved medieval Arabic works, as compared to the gnomological literature. Besides the commentary attributed to Proclus and the biographical part of the chapter of al-Mubashshir with its abbreviated version of Porphyry's *Life of Pythagoras*, no other Greek biography of Pythagoras seems to have reached the medieval authors who wrote in Arabic. However, there existed a peculiar narrative of his life that was transmitted subsequently by authors writing in Arabic, one that is first attested by al-ʿĀmirī (which I am also presenting below).

In what follows, I will discuss separately each of the Arabic biographical-gnomological traditions associated with Pythagoras and the Pythagoreans. I will begin by looking at the most widespread Arabic biographical tradition on Pythagoras, and then continue by discussing the Arabic reception of the two most important late antique biographies of Pythagoras, the ones by Porphyry and Iamblichus. The remaining four sections of this chapter will focus on the different Arabic gnomological traditions associated with Pythagoras and the Pythagoreans. For brevity's sake, I will restrict myself to comparing them only with the late antique Greek sources, while of course an exhaustive study of the Arabic gnomological-philosophical tradition – still a *desideratum* – would require comparing Syriac, Coptic and Byzantine collections with the Arabic ones.

1 The "Arabic Biography" of Pythagoras

It is probable that most of the Arabic writers in the Classical period had almost no clue who Pythagoras really was. The only "Muslim Pythagoreans," as they called themselves, a philosophical group known as the Ikhwān al-Ṣafāʾ,[13] who in their writings claimed to be followers of the Greek Pythagoreans, saw in Pythagoras "a wise man, a monotheist from the people of Ḥarrān."[14] Therefore,

13 On the alleged Pythagoreanism of this Islamic philosophical group, see Marquet 2006. The ideas of Y. Marquet have been recently criticised by De Smet 2007. See also Izdebska 2014, 33–36 and 2016b, esp. 362–365, as well as the chapter by Carmela Baffioni in the present volume.

14 *Ikhwān al-Ṣafāʾ* 1957, vol. III, 200.

they associated him with the only remains of the Greek culture and religion they knew, and with other historical and mythical figures of Greek sages such as Aristotle, Ptolemy and Hermes, possibly still venerated in Ḥarrān at that time.[15] They were sure that Pythagoras was a Greek wise man of a semi-divine, imam-like status but they did not know anything about his actual life.

Around the same period (i.e., the 10th century), a Muslim theologian and philosopher, Abū al-Ḥasan Muḥammad ibn Yūsuf al-ʿĀmirī (d. 992),[16] wrote the treatise *Al-Amad ʿalā ʾl-abad* (*On the Afterlife*),[17] which focuses on the Muslim doctrine of the afterlife, but begins with an outline of how Greek philosophers thought about life after death. Al-ʿĀmirī believed that Greek philosophy was in fact rooted in the Abrahamic prophecy and tried to reconcile it with Islam (as he believed that religion and philosophy generally agreed).[18] In the third chapter of his work, he offers a short history of Greek philosophy, that is, of the transmission of Eastern wisdom to Greece. His main idea was that the first five Greek philosophers, or sages, who acquired their knowledge from the East, developed it and transmitted it one to another, thus forming a chain of teachers and students: Empedocles, Pythagoras, Socrates, Plato and Aristotle (in this order).

This idea of the history of the beginnings of the Greek philosophy presented by al-ʿĀmirī's became very influential, read and copied by a number of authors.[19] However, although al-ʿĀmirī's history of Greek philosophy was so influential later on, "there is no evidence for a single earlier source which al-ʿĀmirī might have reproduced," as noted by E.K. Rowson, the editor and translator of *On the Afterlife*.[20] According to him, al-ʿĀmirī was under the influence of Christian historiography, which very often showed the history of Greek philosophy in a similar way, attributing all the important ideas of Greek philosophers to Jewish

15 On the myth of the "Sabeans of Harran" as potential intermediaries in the transmission of the Greek philosophy into Arabic, see De Smet 2011, 119–126. De Smet disputes the hypothesis of Marquet 1966 and 2006, that the Ikhwān al-Ṣafāʾ were in direct contact with the last pagans, the Sabeans from Harran – "Hellenic syncreticists" worshiping Greek philosophers as their prophets –, and that they took from them some elements of their knowldge about the Greek philosophical tradition, by oral transmission. According to De Smet, this theory has no confirmation in the existing Arabic sources. For a different opinion on this issue, see Tardieu 1986.

16 Wakelnig 2011; Rowson 1988, 3–7.

17 See more about this work in the introduction to its edition with translation and commentary by Rowson 1988, 7–51.

18 Wakelnig 2011; Allard 1975.

19 For more details about the reception of this part of al-ʿĀmirī's work, see Rowson 1988, 203–204.

20 Ibid., 204.

prophets. Nevertheless, the "selection and arrangement" of the entire story are al-ʿĀmirī's original contribution.[21]

In this story, it is Empedocles who plays the role of the first Greek philosopher, as a student of the Qur'anic sage Luqmān.[22] The second Greek philosopher, according to al-ʿĀmirī, was Pythagoras, who traveled to Egypt, where he learned geometry from the Egyptians and "the physical and metaphysical sciences from the companions of Solomon" – the Jews who came to Egypt from Syria. Then, Pythagoras brought these three disciplines of science to Greece and discovered another one there, that is, the science of melodies. The passage closes with a statement that "he claimed that he had acquired these sciences from the niche of prophecy."[23] This sentence refers to the prophet (king) Solomon as a source of Pythagoras' wisdom. The "niche of prophecy" (*mishkāt al-nubū'a*) is an Islamic expression. The word *mishkāt* ("niche") is derived from the famous *Qur'ānic* esoteric "verse of light" (XXIV, 35), interpreted mystically by several Sufis and Muslim philosophers, among them by al-Ghazālī who devoted to it his exegetical work entitled *The Niche of Lights*. The expression about the "niche of prophecy" was later repeated by other Arabic authors in their descriptions of Pythagoras.[24]

Hence, Pythagoras is presented by al-ʿĀmirī as someone who transmitted the Eastern wisdom to the West. The topos of travelling to Egypt and learning arcane knowledge from the Egyptians is well rooted in the Greek tradition.[25] The Jewish (but also Qur'anic[26]) motif is added here in order to link the history of Greek philosophy to the overall narrative of philosophy as an Eastern invention. The other Greek sages are Socrates, Plato and Aristotle, who are portrayed

21 Ibid. The author offers examples drawn from Eusebius (*Praeparatio Evangelica*, in Eusebius 1903, 410–411, 470ff, 523, 664ff), who in his attacks against pagans used to claim that the Greek philosophers (especially Plato and Pythagoras) learned their 'theology' from the Jewish exiles in Egypt after the fall of Jerusalem.

22 Luqmān was a legendary hero and sage of pre-Islamic Arabia, who appears in the Qur'an. See Heller and Stillmann 1986; De Smet 1998, 38–45.

23 Rowson 1988, 71.

24 Sa'id al-Andalusi, al-Qifti, Ibn Abī Uṣaybīʿa. See the comments of Rowson 1988, 209 and Cottrell 2016, 506.

25 The legend is attested for the first time by Isocrates (*Busiris* 28–29), probably without any intention of treating it seriously. It is later repeated and developed by several biographers of Pythagoras, and thus attested in the extant biographies: Diogenes Laertius, *Lives*, VIII 2–3; Porphyry, *Life of Pythagoras*, 6 (p. 38 in É. Des Places' Budé edition) and 11 (p. 41); Iamblichus, *On the Pythagorean Way of Life, chap.* III–IV. See more about the legendary journeys of Pythagoras in Zhmud 2012, 83–91 (Egypt: 86–88) and Macris 2018, 787–792 (Egypt: 788–789).

26 Solomon (Sulaymān ibn Dāwūd) is a Qur'anic prophet as well. See Qur'an 2:102; 4:163; 6:84; 21:78–82; 27; 34:12–18; 38:30–40.

as a chain of students and teachers who agreed in terms of their philosophical ideas, and followed what Pythagoras brought to them from the Abrahamic East. This is not a new idea either, because Jewish and Christian authors had already added new elements and new geographical destinations to the legend of Pythagoras' journey, in order to show the dependence of Greek philosophy on the Abrahamic tradition.[27]

Furthermore, al-ʿĀmirī's statement about Pythagoras acquiring his wisdom "from the niche of prophecy" was copied by later authors and mixed with a legend that certainly comes from Greek sources, namely the legend of Pythagoras hearing the harmonious sound produced by the movements of the celestial bodies.[28] Therefore, the statement that Pythagoras studied the prophetic tradition provided a foundation for presenting Pythagoras himself as quasi-prophetic and possessing super-human abilities.

He is presented in this way indeed by Abū l-Fatḥ Muḥammad al-Shahrastānī (11th–12th c.), in his monumental heresiography *Kitāb al-Milal wa-l-Niḥal* (*Book of Religions and Sects*), which contains a quotation from al-ʿĀmirī (or some intermediary source he used):

> He [*sc.* Pythagoras] gathered wisdom from the niche of prophecy and was a great sage, of firm doctrine, of strong spirit. They confirm that he saw [different] worlds through immediate perception of sense and spirit; he was so advanced in mathematics that he heard the movement of the celestial spheres and reached the place where the angels abide; and he said: I have never heard anything sweeter than the sound of their movement, neither have I ever seen anything more illuminating than their forms and figures.[29]

In this passage, Pythagoras is described as an illuminated sage who brings the wisdom from the celestial heights to the people, a wisdom that is not attainable for most mortals. Al-Shahrastānī combined the topos of Pythagoras taking

27 Jacobson 1976; Gorman 1983; Bar-Kochva 2010, 164–205 ("Hermippus of Smyrna on Pythagoras, the Jews, and the Thracians"); Macris 2018, 790.

28 Porphyry, *Life of Pythagoras*, 30; Iamblichus, *The Pythagorean Way of Life* XV, 65–67. See Burkert 1972, 350–357, Zhmud 2012, 337–346, and for the late antique interpretation of this story, O'Meara 2007. The theory that the movements of the celestial bodies make a sound which we cannot hear was criticised by Aristotle (*On the Heavens* 290b). For more literature on the hearing of the celestial music, the so-called "harmony of the spheres," see Macris 2018, 1166.

29 al-Shahrastānī 1846, 265. – Interestingly, this first person testimony is very similar to another one preserved in Greek, where Pythagoras says: "Being outside the body, I heard a melodious harmony" (see Thesleff 1965, 172.6–7).

his knowledge "from the niche of prophecy" with a story taken most probably from Porphyry's *Life of Pythagoras* (Pythagoras hearing the music of the spheres).[30] This story was well known among authors writing in Arabic. It is quoted by al-Bīrūnī (d. 1050),[31] who cites Porphyry's *Philosophical History* (the quoted passage resembles Porphyry's *Life of Pythagoras* 33–34).[32] Furthermore, it is one of the very few pieces of actual "biographical" information about Pythagoras that can be found in the *Epistles* of the Ikhwān al-Ṣafāʾ. They refer to this story twice (Epistle 5, *On Music*, 121 and 148) and they also make the connection between Pythagoras' miraculous ability to hear the movements of the spheres and heavenly bodies with the super-human "purity of the substance of his soul and the intelligence of his heart" (121). Thus, the Greek topos of Pythagoras the pagan holy man made its way to the circle of the Ikhwān al-Ṣafāʾ, who also considered Pythagoras to have a super-human status, but more as some kind of imam or pre-Islamic prophet.[33]

The same image of Pythagoras as a sage who observed the higher worlds with his own senses is also repeated in the Graeco-Arabic doxography *Opinions of the Philosophers* (XVI 5), wrongly attributed to Ammonius and based on Hippolytus' *Refutatio omnium haeresium*. It also shows all main Greek philosophers as almost univocal when it comes to their philosophical views, sharing the same monotheistic, Neoplatonic ideas.[34]

2 Traces of Greek Biographies of Pythagoras in Arabic

Despite the fact that the biographies of Pythagoras were so popular in Late Antiquity, when there must have existed many more than those known to us today, there is no evidence that the medieval Arabic authors knew even the ones we know. Among the four Greek texts presenting the story of the life of Pythagoras (the ones by Diogenes Laertius, Porphyry and Iamblichus, as well as an anonymous biography summarized by Photius) only one – Porphyry's

30 The Porphyrian passage contains four Empedoclean verses presenting Pythagoras as a sage endowed with superhuman powers; see Macris and Skarsouli 2012.

31 al-Bīrūnī 1910, vol. 1, 43; Segonds 1982, 171; Hugonnard-Roche 2012, 1465.

32 This was a lost work of Porphyry, in four books, containing biographies of philosophers, including the Seven Sages, as well as Pythagoras, Socrates and Plato. See Segonds 1982; Sodano 1997; Zambon 2012; Macris 2014, 382–384.

33 This view seems to have been perpetuated until much later; see Terrier 2019, 734–748.

34 See Rudolph 1989; Genequand 1991; De Smet 2019, as well as Daniel De Smet's chapter in this volume.

Life of Pythagoras – has been partially transmitted into Arabic. Besides, there is also a short biographical section in the Arabic commentary to the *Golden Verses* attributed to Proclus which is clearly based on the *Pythagorean Way of Life* of Iamblichus and as such was also a source of knowledge about the life of Pythagoras as depicted in its late antique version.

The earliest Arabic attestation of Porphyry's *Life of Pythagoras*, or rather of an abbreviated version of it, comes from Abū al-Wafāʾ al-Mubashshir ibn Fātik (11th c.), an Egyptian (Fatimid) scholar and historian, in his only preserved work, the *Mukhtār al-Ḥikam wa Maḥāsin al-Kalim* (*Book of the Choicest Maxims and Best Sayings*).[35] The book is devoted to twenty ancient sages, such as Homer, Solon, Hippocrates, Socrates, Plato, and Aristotle, some others belonging to the Hermetic tradition, like Hermes, Asclepius or Seth, as well as to some Church Fathers (Basil of Caesarea and Gregory) and, finally, to the Qur'anic Luqmān. Every chapter is devoted to a different person and divided into two sections, the first containing a biography and the second presenting the teachings of each of the wise men (in a form of gnomology). In the biographical part of the chapter devoted to Pythagoras,[36] the author offers the Arabic translation of Porphyry's *Life of Pythagoras*, or rather a later, heavily re-worked version of it.[37]

Al-Mubashshir's work later became a source for Ibn Abī Uṣaybīʿa (d. 1270), a Syrian historian and physician who used material taken from al-Mubashshir in his biographical history of Greek, Arabic, Persian and Indian physicians, scholars and philosophers, entitled the *ʿUyūn al-anbāʾ fī-ṭabaqāt al-aṭibbāʾ* (*Lives of the Physicians*).[38] The chapters are organized in a way very similar to al-Mubashshir's work, also divided into "biography" and "teaching/sayings" sections. In the chapter devoted to Pythagoras, al-Mubashshir's work was a substantial source for Ibn Abī Uṣaybīʿa in both sections, though not the only one.[39] What is interesting about Ibn Abī Uṣaybīʿa, however, is the fact that he

35 Cottrell 2011. The text itself was edited by ʿAbd al-Raḥmān Badawī: see al-Mubashshir Ibn Fātik 1980 [1958]. This edition remains the only one, although it was strongly criticised by Rosenthal 1960–61, who had also worked on the manuscripts of al-Mubashshir. However, he did not present his own edition.

36 al-Mubashshir Ibn Fātik 1980, 52–61.

37 Cottrell 2008 and 2016.

38 Ibn Abī Uṣaybiʿa 1882. This work is now available also in a new edition, and translated into English in Savage-Smith, Swain and van Gelder (ed.) 2020.

39 Al-Mubashshir's work was also the main source for al-Shahrazūrī (d. after 1287) in his historical-philosophical work (very similar in scope to that of al-Mubashshir) – *Nuzhat al-arwāḥ wa-rawḍat al-afrāḥ fī taʾrikh al-ḥukamāʾ* (*Promenade of Souls and Garden of Rejoicings in the History of the Philosophers*); see Cottrell 2004 (*non uidi*). Although I will not discuss this text here, it is important to note that it differs in some details from the surviving text of al-Mubashshir. A closer comparison of al-Mubashshir, Ibn Abī Uṣaybiʿaʾ

quotes some passages that are absent from the text of the life of Pythagoras quoted by al-Mubashshir but have parallels in Porphyry. He also directly points at Porphyry's *Philosophical History* (of which the *Life of Pythagoras* is believed to be the only surviving chapter) as his source. However, it is possible that he had access to better manuscripts of al-Mubashshir than those preserved today, rather than that he used Porphyry's work independently.[40]

The biography itself, although evidently derived from Porphyry's *Life of Pythagoras*, is not a simple translation of the Greek text we know today. It significantly differs from the original not only because it is abbreviated, but also because it contains some elements that are not present in the extant Greek version.[41] Of those elements, the most significant is the story of Pherecydes, the teacher of Pythagoras, who, affected by a serious illness, was carried first to Ephesus and then to Magnesia at the end of his life (a story that is completely absent from the Greek text of Porphyry's *Life of Pythagoras*).[42] Also, abbreviations are visibly made with the specific goal of ridding the text of almost all of its philosophical contents. In the Greek original, Porphyry mixed together philosophical ideas with the narrative of the philosopher's life. However, in the Arabic text all anecdotes that do not belong to the core biographical narrative are absent, and the order of the topics touched upon in the *Life* is also slightly changed. Therefore, someone, perhaps already a Greek author or a Syriac intermediary, transformed Porphyry's original into a simple biography, most probably in order to make this text fit into a collection dealing with many philosophical and religious authorities, as was later the case with al-Mubashshir himself. In the context of such collections, it was necessary to make a clear distinction between the life and the teaching of the subsequent "sages." As a result, what is left from Porphyry's *Life of Pythagoras* is the core narrative of the philosopher's life, presented in a single authoritative version (without

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and al-Shahrazūrī gives the impression that al-Shahrazūrī and Ibn Abī Uṣaybiʿaʾ had access to better manuscripts of al-Mubashshir than we have today. See al-Shahrazūrī 1988, 87–106. For this hypothesis, see Cottrell 2016, 472.

40 Cottrell 2016.

41 A comparison of the Greek text of Porphyry's *Life of Pythagoras* and the German translation of the Arabic biography, quoted by al-Mubashshir and Ibn Abī Uṣaybīʿa, was presented in Rosenthal 1937. On how this relates to the preserved Greek original of Porphyry, see Cottrell 2016, 476–478. Also in the new translation of Ibn Abī Uṣaybīʿa (Savage-Smith, Swain and van Gelder [ed.] 2020), in the chapter devoted to Pythagoras every paragraph has a reference to the parallel paragraph of Porphyry's *Life of Pythagoras* in É. Des Places Budé edition.

42 A very similar, but more extended, version of this story is preserved in Greek by Diogenes Laertius (1, 117–118). According to Rosenthal 1937, 47 Ibn Abī Uṣaybīʿa could have taken it from another chapter (now lost) of Porphyry's *Philosophical History*.

any divergences traced back to different earlier sources about Pythagoras, and without naming any sources). The Arabic text focuses on the places where Pythagoras lived or which he visited, on his teachers or students, as well as on the conflict with Cylon of Croton, the persecutions of the Pythagoreans and the death of Pythagoras himself. Consequently, this version of Porphyry's *Life* lost almost all of its original legendary, mythical and religious elements, together with its philosophical content. Part of this other material can be recognized in the gnomological section of the chapter in al-Mubashshir's work, in which the latter certainly used some Greek material. However, it is unknown if this or a similar gnomological collection was attached to the abbreviated version of Porphyry's *Life* (the reworking of the original text could have taken place already in Greek, or later in Arabic, or perhaps even through a Syriac intermediary).[43]

Apart from Porphyry's *Life of Pythagoras*, some traces of the second famous late antique "biography" of Pythagoras – Iamblichus' *Pythagorean Way of Life* – also reached Arabic readers through the translation of the commentary to the Pythagorean *Golden Verses* attributed to Proclus.[44] The Greek original of this text is lost, but it has a strongly Greek, Neoplatonic character and presents a typically Neoplatonic image of Pythagoreanism. The interpretation of the poem presented in this commentary is close to that of the other late antique philosophers known to us from the extant Greek texts. These Neoplatonists considered that the *Golden Verses* offer an introduction to philosophy: so did Hierocles of Alexandria in his commentary on the poem,[45] and Iamblichus in his *Exhortation to Philosophy*, a chapter of which is devoted to the *Golden Verses*.[46] In Arabic, besides the commentary attributed to Proclus, there is also another extant commentary, which is attributed to Iamblichus.[47] However, its alleged Greek origin is much more questionable than that of the commentary attributed to Proclus (for more about this question see below in this chapter in the section devoted to the *Golden Verses*).

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43 Since the surviving Greek text of Porphyry's *Life of Pythagoras* is incomplete and ends in the middle of a sentence, with the word *historousi*, Cottrell 2016, 483, hypothesised that the original text of Porphyry could had been followed by a Pythagorean gnomology, especially given that the *Pythagorean Sentences* were known to Porphyry and used by him in the *Letter to Marcella*.

44 This Arabic text has been edited and translated by Linley 1984. About its authorship see Daiber 1988; Izdebska 2018, 869–870 and 2019.

45 Schibli 2002; Thom 2017, 85; Aujoulat and Lecerf 2018.

46 Thom 2017, 84.

47 Daiber 1995.

The Proclean commentary begins with a short biographical introduction which has a lot of common elements with the *Pythagorean Way of Life* by Iamblichus – which was probably its direct source. It tells the legend of Pythagoras in its classic late antique version. It begins with the virgin birth of Pythagoras, preceded by a prophecy (it is explained that a "prophecy" is "heavenly communication from God"[48]), then follow his travels to Egypt and Babylon, as well as the first phase of Pythagoras' popularity. He is presented as a holy man to whom people from all countries travelled to seek wisdom. He is said to have been able to "repudiate wealth and perform miracles."[49] Afterwards, the first community of Pythagoras' students is introduced, as well as the rule of five years of silence that was obligatory in Pythagoras' school. It is only a short introduction, but it sets a very clear image of Pythagoras as a super-human wise man, similar to Jesus and the prophets. According to the author of the commentary, the *Golden Verses* were written by Empedocles, but nevertheless they present the core teaching of Pythagoras. It is the semi-divine character of Pythagoras that endows the poem with its moral and religious authority.

3 Al-Mubashshir's Gnomology

The gnomological part of al-Mubashshir's chapter[50] on Pythagoras consists of 106 sayings and anecdotes and as such it is the most extensive gnomological collection attributed to Pythagoras preserved in Arabic. It is mainly based on an unidentified gnomological collection associated with the *Pythagorean Sentences*. The collections based on the *Pythagorean Sentences* were popular in Late Antiquity in pagan as well as Christian circles from around the 2nd century.[51] It is a collection of moral prescriptions with an ascetic flavor, quite universal and easy to use. Pagan and Christian authors incorporated them into their works at the same time, and later Muslim authors followed their example, as can be seen in al-Mubashshir's work. Some collections from this tradition are also preserved in Syriac.[52] A.R. Sodano, who edited and translated a Greek collection from this group entitled the *Sentences of Demophilus*, compared

48 Linley 1984, 3.

49 Ibid., 5.

50 al-Mubashshir Ibn Fātik 1980 [1958], 62–72.

51 See Prochenko 2018. The *Pythagorean Sentences* themselves are edited (but without taking into account all the surviving manuscripts) by Chadwick 1959, 84–94. A new edition of this collection is being prepared by Katarzyna Prochenko.

52 Gildemeister 1870; Levi della Vida 1910; Wünsch 1968; Sodano 1991, 49–56; Izdebska 2018, 873–874; Arzhanov 2019, 84–90.

some sentences from al-Mubashshir with those from the Syriac translations related to this tradition. According to him, similar omissions and mistakes in the Syriac version and al-Mubashshir can indicate that the version of the sentences in al-Mubashshir could derive from a Syriac translation, even if not from the ones that we know.[53]

Whereas Ibn Abī Uṣaybīʿa collected his material in order to "reconstruct" history, al-Mubashshir's goals were different. He exposed them clearly in the introduction to his entire work. He considered his work to be above all an introduction to the teachings of the Greek sages (in the form of maxims) that might be useful and valid for Muslim (Ismaili) readers of his time. The biographical sections are considered to be introductions to the gnomological ones, offering the Arabic reader the necessary background, placing the Greek authors of the maxims within the historical context of Greek philosophy. However, it is the wise sayings, the *adāb*, and the wisdom (*ḥikma*) hidden in them that form the core of the al-Mubashshir's work. He explains in the introduction that he read a book with various sayings of Greek sages and explains the reasons why he decided to provide its contents to the Arabic audience:

> I saw in them instructions that pleased me and exhortations that remained close to my heart, while the sentences were adopted by my soul. They are of great benefit for the one who reflects on them, and of great profit for the one who acts in accordance with them. They attract to a good deed, and they point at the beauty of politics, and they awaken a desire to gather hoards for the afterlife.
>
> And this prompted me to collect in my book what I considered worthy of them: from among exhortations those that bring benefit, and instructions that are profound and anecdotes that astonish; those that contain exercise for a reader's soul and correction for his morals […], and exhort to asceticism in this passing world and alleviate earthly hardships – and there is in them the awakening of a desire of that what is close to God.[54]

Then he explains that he restricted himself to the sayings of the "divine men" who were monotheists, as well as "their successors in wisdom," and that he was not "deterred by the differences in their schools and dissimilarities of their systems, nor the antiquity of the times in which they lived."[55]

53 Sodano 1991, 66–67.
54 al-Mubashshir Ibn Fātik 1980 [1958], 2–3. Translation by Anna Izdebska.
55 Ibid.

The entire introduction is full of quotes from the Qur'ān and the hadiths, all concerned with the necessity of learning and gathering knowledge from as many sources as possible. It is clear that al-Mubashshir considered the maxims gathered in his book to be useful and interesting for mostly religious and practical reasons, and not because of their historical value. According to him, they could help his pious Muslim readers achieve practical goals such as living a good life and attaining eternal happiness in the afterlife. He directly calls the content of his book "exercise for the reader's soul," a statement that strongly reminds us of Pierre Hadot's conception of Greek philosophy as a spiritual exercise.[56] Al-Mubashshir clearly perceived Greek philosophy in the same way: as practical rather than theoretical, and having a direct impact on the lives of its followers. For al-Mubashshir, it has an important spiritual content, potentially useful in a religious context, relating to the questions of moral development, asceticism, and the desire to become as close to God as possible. It is also significant that he considered the Greek philosophers he chose to be monotheists and as such deserving to be treated as serious authorities. He trusted them more than polytheists, who in the Arabic Islamic culture would normally be associated with the pre-Islamic Arab polytheists, particularly because the medieval Arabic authors did not have a clear image of what ancient Greek religion actually was. Al-Mubashshir's attitude of almost unconditional trust in the key Greek philosophical authorities was already shared by some Christian authors like Clement of Alexandria.[57] It is also visible in a specific, late type of *florilegia sacro-profana* in which Greek philosophers are quoted alongside the Church Fathers. Among such florilegia, the most famous are the *Corpus Parisinum* and the *Loci communes* falsely attributed to Maximus Confessor.[58] In fact, one of the main sources used by al-Mubashshir, about which he wrote in the introduction, could have been a Christian florilegium, which would explain why he also included chapters about Basil and Gregory.

The gnomology attributed by al-Mubashshir to Pythagoras is mainly based on a collection belonging to the group of texts associated with the *Pythagorean Sentences*. Another source is the *Golden Verses*, but the poem is not quoted in its entirety. The gnomology contains only selected verses, separated from each other and incorporated as independent maxims in places where they fit into the context of the other sayings in a given section. The verses of the poem are not translated directly from Greek, but an earlier Arabic translation of the poem, known to us, is used here. This suggests that the verses were actually

56 P. Hadot 2001 and 2002; Chase, Clark and McGhee (ed.) 2013.
57 Macris 2019.
58 See Richard 1962; Maximus 2001; Searby 2007.

added to this gnomology, in prose, by al-Mubashshir himself. This would not be surprising, since the *Golden Verses* were translated already at the beginning of the Graeco-Arabic translation movement (8th c.) and quickly gained significant popularity.[59] Several Arabic authors treated this poem as the main source of knowledge about Pythagoras' teachings and quoted it *in extenso*, as one of the elements, or actually the only element, of a section devoted to his philosophy.[60] This practice certainly shaped the image of Pythagoras and his philosophy in the eyes of Arabic readers and was one of the reasons why his figure was much more visible in popular and practical contexts than in the context of philosophical theory or natural science.

The third source for al-Mubashshir's Pythagorean gnomology was probably a lost collection, the same that would have also contributed to the *Ṣiwān al-ḥikma* group of Arabic gnomological collections (see more details below). His fourth source was probably an unidentified work that belonged to the "mirrors for princes" type of literature. Such texts offered practical and moral advice for rulers. However, what shaped the character of the entire gnomological section is the first source, that is, the *Pythagorean sentences*. This source also agrees the most with the goals presented by al-Mubashshir in his introduction. These sentences provided a very strong monotheistic, religious, ascetic input, easy to incorporate into any monotheistic context, be it Christian (like probably the source through which they reached al-Mubashshir), pagan Neoplatonic, or Muslim. Their contents had universal appeal and this was the reason for their popularity in all these different religious milieus: they were used at the same time by antagonistic religious and philosophical groups.

The majority of the sentences focus on the relation between man and God. They advise on the ways in which man can come closer to God and how he should treat other people in order to transform his mundane struggles into an opportunity to achieve this goal. Almost all the sentences are in the form of instructions or admonitions pronounced by an illuminated teacher to his students. Therefore, the reader of these sentences has an image of Pythagoras as

59 On the Arabic translation of the *Golden Verses*, see first of all the unedited PhD dissertation of Ullmann 1959, then also Rosenthal 1941; Baffioni 1994; Izdebska 2018, 862–865. The Arabic *Golden Verses* have been translated into English by Rosenthal 1975, 118–120.

60 The *Golden Verses* are quoted: [1] in the *Ādāb al-falāsifa* of Muḥammad ibn ʿAlī ibn Ibrāhīm ibn Aḥmad ibn Muḥammad al-Anṣārī, which is a later recension of the lost *Nawādir al-falāsifa* (*Philosophers' Aphorisms*) of Ḥunayn ibn Isḥāq (d. AD 873/877): see Ḥunayn ibn Isḥāq 1985, 116–119; [2] in Ibn Miskawayh 1952, 225–228: see Dunlop 1979, 31–34; and [3] in the so-called *Philosophical Quartet*, edited by Gutas 1975. The text of the *Golden Verses* quoted in the chapter devoted to Pythagoras is not included in Gutas' edition but it is present in every manuscript he used: Aya Sofya 2460 (A), fol. 70v–76v; Aya Sofya 2822 (B), fol. 260r–263v; Paris, Arab. 202 (P), fol. 18r–20r.

a religious leader, an ascetic authority and a teacher giving clear religious and moral advice to all (which is similar to his image in the Greek gnomologies).

This entire teaching is well illustrated by one of the first maxims (#4), in the gnomological part of al-Mubashshir's chapter:

> God has the pure wisdom; therefore, the love of wisdom is connected with the love of God. And who loves God, acts based on what He loves. And who acts based on what He loves, comes closer to Him. And who comes closer to Him, finds salvation and wins.[61]

Philosophy, the love of wisdom, is explained here as the love of God, and from this identification derive both the duty to do good things and the perspective of achieving salvation. As a consequence, philosophy is easily identified with religion. Therefore, Pythagoras, a great Greek philosopher, is at the same time a great monotheist and a moral teacher for everyone who wants to become a good believer and achieve salvation.

4 The *Golden Verses* and the Graeco-Arabic Gnomological Tradition

The *Golden Verses* themselves are the best example of the popularization of the figure of Pythagoras and his teaching, a process that already started in Graeco-Roman Antiquity and continued throughout the Arabic Middle Ages.[62] This 71-line poem in hexameter, belonging to the group of the so-called pseudo-Pythagorean writings, was probably written in the Hellenistic period but became well known and widely quoted only in Late Antiquity.[63] It has no parallels in other Pythagorean pseudepigrapha, as most of the writings in this group are treatises and letters; rather, the poem resembles the genre of wisdom literature. It consists of a series of admonitions given by a teacher to his students. It begins with a recommendation to worship gods, daemons and heroes, then to honor parents and relatives and to obey the laws, which altogether means to follow the traditional religious and social rules and customs. The rest of the poem focuses on the commands regarding the best behavior towards other people, true friendship and, finally, some general ethical advice. The goal of fulfilling these recommendations is to achieve immortality, eternal happiness, and even become a god – a promise that is made at the very end of the

61 al-Mubashshir Ibn Fātik 1980 [1958], 52. Translation by Anna Izdebska.

62 Thom 1995, 13–26; 2012, 285–287 and 2017, 80–85.

63 Thom 1995, 13–58; Id. 2021 (in press).

poem.[64] The form and the contents of the poem not only show the affinity of this text with the entire genre of wisdom literature, which must have served as the source of ready material for its compiler, but also made it easy for later authors to include the entire poem or (more frequently) portions of it into gnomological collections – which are also akin to wisdom literature. Quoting the poem *in extenso* or selected verses from it in the context of wisdom literature was the main way in which the *Golden Verses* were used after they were translated into Arabic.

Consequently, the poem appears in several Arabic texts, under titles such as the *Golden Epistle/s* or the *Golden Testament/Admonition/s*, and Pythagoras is always indicated as its author. Only in the Arabic commentary attributed to Proclus is its authorship attributed to Empedocles. Thus, it was considered a moral and philosophical testament given by Pythagoras to his students, and the main text that had survived from Pythagoras himself. Therefore, it was quoted in several Graeco-Arabic gnomological and historical works as the main or even the only teaching of Pythagoras.

Besides being used as the source of knowledge about the Pythagorean philosophy, the poem was also quoted partially in and incorporated into broader gnomological collections, such as the above-mentioned chapter in al-Mubashshir's work. It was also quoted by the Ikhwān al-Ṣafāʾ, but it remains an open question as to whether they knew the entire poem or only some individual verses, such as the ending of the poem (which they quote four times).[65]

Furthermore, there exist two commentaries to the *Golden Verses* preserved in Arabic. This is certainly connected to the fact that the poem was known among the Arabic authors as the main work containing the teaching of Pythagoras. For neither of these commentaries does there exist a Greek original, nor do any extant Greek sources seem to mention them. Still, they are both attributed to late antique Neoplatonists, even though in fact they are completely different from one another.

The commentary attributed to Iamblichus consists of glosses on subsequent verses or groups of verses of the poem, quoted in the standard Arabic translation. The author seems to comment on the text of the Arabic translation, being unaware, apparently, of the Greek that underlies the Arabic. This fact, along with some other features of the text and, most importantly, substantial differences between this commentary and the interpretation of the *Golden Verses* given by Iamblichus in his *Exhortation to Philosophy* (extant in Greek), lead to the conclusion that the commentary attributed to Iamblichus

64 On this promise see Izdebska 2016a.

65 Baffioni 1994.

was probably composed directly in Arabic, and possibly only remotely rooted in some Greek source.[66] Moreover, it has generally very little in common with the Greek, in particular late antique, tradition of interpreting and reusing the *Golden Verses*. Compared to Hierocles of Alexandria's preserved work[67] as well as to Iamblichus', this Arabic commentary shows much more hesitation towards the metaphysical and theological content of the poem, focusing on the traditional values and ethical precepts it contains. Yet, these topics were of central interest for the two Greek authors whose interpretation of the poem survived until today, as well as for the Arabic commentary to the *Golden Verses* attributed to Proclus.

Since the commentary attributed to Iamblichus does not actually have a Neoplatonic character and Porphyry's *Life of Pythagoras* was transmitted into Arabic in abbreviated form, deprived of many elements specific to the original late antique text, the commentary attributed to Proclus remained the main representative of the late antique image of Pythagorean philosophy that was available to the medieval Arabic readers.[68] The author of the commentary does not focus solely on the moral teaching presented in the *Golden Verses*. Despite the inevitability of commenting on the moral content of the poem, the author, just like Hierocles of Alexandria, treated the verses of the poem as a pretext to touch upon many other questions and to present his own idea of what the actual teaching of Pythagoras was. Therefore, the commentary goes into details about metaphysical and theological questions, and these philosophical matters form the background for the moral admonitions given by the author of the poem. Moreover, the commentator does not comment on all verses of the poem. As a result, he is able to focus on questions that are not explicitly mentioned in the poem, yet were of central interest for someone who was well aware of the late antique Pythagorean tradition.[69] Thus, of all the intermediary transmitters between the Greek Late Antiquity and the Arabic Middle Ages, the commentary attributed to Proclus contains the "purest" description of the Pythagorean (or pseudo-Pythagorean) metaphysics in the version developed by the Neoplatonic philosophers.

The author's approach to interpreting the poem is clearly visible in the introduction to the commentary, where after presenting the figure of Pythagoras he introduces the poem itself:

66 Izdebska 2012a, 2016a and 2018, 870–872.

67 On which see most recently Aujoulat and Lecerf 2018.

68 For discussion about the authorship of this text see Izdebska 2019.

69 Izdebska 2016a.

> The object of the *Golden Sayings* is to inspire souls with longing for their perfection and purity, to make people human, and to guide them towards a proper way of life; man achieves perfection by means of absolute virtue, certain knowledge, and virtuous conduct. Some maintain that the *Golden Sayings* act as a guide towards divine life, the imitation of God and liberation from matter.[70]

The commentator refers to a broader opinion held by all interpreters of the poem, namely that it should be treated as a "guide" to purely religious and practical goals: divine life, imitation of God and liberation from matter. The term "guide" evokes associations with the Graeco-Roman tradition of "spiritual guidance" studied by Ilsetraut Hadot, who also placed the *Golden Verses* in this context.[71] The function of the *Golden Verses* is to "inspire souls" on their way towards perfection, virtue and knowledge. Our commentator also says that the true goal of the *Golden Verses* is to "guide" people "towards a proper way of life." What is translated here as a "proper way of life" is *sīra hasana*, where *hasana* means "good" and *sīra* can mean "a way of doing something," but it has also a broader meaning and can mean a political system. This is quite similar to the Greek word *politeia*, which can mean both a way of life of an individual and a political system. The interpretation of the *Golden Verses* as a guide towards a proper, Pythagorean way of life is very well rooted in the Greek tradition, in which the *pythagoreios tropos tou biou* was already considered a distinctive model by Plato (*Republic* X, 600 a–b).[72] The late antique authors, too, whenever writing on Pythagoras and his teaching, focused on "the Pythagorean way of life" (this is exactly the title of Iamblichus' biography of Pythagoras), as reflected in the life of Pythagoras himself and in the admonitions he gave to his students in the *Golden Verses*. Thus, the late antique philosophers (but probably earlier ones, too) considered Pythagoreanism to be more some kind of a "spiritual movement," with which they sympathized, than a set of theoretical ideas (yet these were included in the "package").[73]

One of the key elements of Pythagoreanism as described by Greek authors was the "Pythagorean friendship," a topos that occurs in virtually all descriptions of the "Pythagorean life" and of the community of the first Pythagoreans.[74]

70 Linley 1984, 7.
71 I. Hadot 1986 and 2014; Macris 2013.
72 Cf. Macris 2013, 60–61.
73 See Boyancé 1939; Cornelli 2015; for more literature on the "Pythagorean way of life" see Macris 2018, 840–841.
74 de Vogel 1966, 150–159; Thom 1997; Cornelli 2013, 67–69; Gemelli Marciano 2014, 146–147; Macris 2018, 826 and 1112.

The *Golden Verses*, too, contain admonitions about friendship, explaining whom one should choose for a friend, and prescribing forgiveness for friends. The commentary attributed to Proclus elaborates on this subject and presents a complete theory of perfect philosophical friendship, based on the pursuit of truth and virtues. He evokes the "Pythagorean friendship" writing: "Friendship was venerated among the Pythagoreans, who regarded it as a symbol of union with the gods."[75]

This sentence seems to be an echo of Iamblichus' *Pythagorean Way of Life* (XXXIII, 240), which described the Pythagorean ideal of friendship in a very similar way. Likewise, other elements of the Greek notion of the "Pythagorean way of life" appear in this commentary. These include, among other things, moderation in eating, avoiding meat, not sleeping too much (since it likens man to a vegetable and not to a constantly alert god), self-restraint in showing anger and overall emotional equanimity, controlling physical desires, being rational in all life choices, showing patience in misfortunes, undertaking a daily examination of one's conscience, etc. Therefore, the author of the commentary gives a clear picture of what the *pythagoreios bios* was according to him: it certainly represented the perfect philosophical way of life.

The presence of works like this in Arabic and the fact that in general the *Golden Verses* prevailed in quantity over other types of sources related to the Greek Pythagorean tradition had significant consequences for shaping the image of Pythagoras and of his teaching among those who encountered his name and "works" in Arabic texts. Pythagoras was emerging in the first instance as a moral teacher and his admonitions, universal and easy to integrate into any cultural or religious context, seemed to be something most valid and important for the readers of "Pythagorean texts" in the classical ages of Arabic culture.

5 The *Ṣiwān al-ḥikma* Circle of Gnomologies

Pythagoras depicted as a moral teacher and a wise man is also present in those collections of sentences that in Greek existed separately from the *Golden Verses*. Consequently, this way of presenting Pythagoras made its way into Arabic gnomologies, based for the most part on translations from Greek. One of the most influential and widely copied of the Arabic gnomologies was a

75 Linley 1984, 25. For comparison sake, see Schramm 2013 on the Neoplatonic approaches to friendship, from Plotinus and Iamblichus to Julian (clearly inspired by the Pythagorean model in the case of Iamblichus).

collection of sentences and anecdotes of Greek and Arabic philosophers known as the *Ṣiwān al-ḥikma* (*The Cabinet of Wisdom*). For several Arabic authors, this was their main source of knowledge about Greek philosophy. It was probably compiled around AD 1000 by someone from the circle of the philosopher Abū Sulaymān al-Siğistānī and was later wrongly attributed to al-Siğistānī himself (based on the testimony of Zahīr al-Dīn al-Baihaqī).[76] The original work is lost, but three other surviving Arabic collections seem to be abridgments or selections from the original *Ṣiwān al-ḥikma*. The first and most extensive one consists of 135 entries on Greek and Muslim philosophers and is called *Muntakhab Ṣiwān al-ḥikma* (*Selection of the Cabinet of Wisdom*); it was compiled by an anonymous author between AD 1191 and 1241.[77] The second abridgment, *Mukhtaṣar Ṣiwān al-ḥikma* (*Summary of the Cabinet of Wisdom*), composed by 'Umar ibn Sahlān al-Sāwī (d. *ca* 1145),[78] is much shorter (only 60 entries) and focused mainly on ethics.[79] The third excerpt from *the Ṣiwān al-ḥikma* is an anonymous gnomological collection called *Mukhtār min kalām al-ḥukamā' al-arba'ah al-akābir* (*Selection from the Sayings of the Four Major Philosophers*), called the *Philosophical Quartet* by its editor, Dimitri Gutas, because it is reduced to just four Greek philosophers: Pythagoras, Socrates, Plato and Aristotle.[80] There certainly also existed other recensions or summaries of the original *Ṣiwān al-ḥikma* that are not preserved or await to be identified.[81]

A chapter devoted to Pythagoras can be found in each of those three preserved abbreviations of the *Ṣiwān al-ḥikma*. In the *Muntakhab* the Pythagoras chapter contains 28 sayings, and 32 in the *Philosophical Quartet*, in addition to the *Golden Verses*, which are included into these chapters *in extenso*. In the *Mukhtaṣar*, which is anyway much shorter than the two other abridgments of the *Ṣiwān al-ḥikma*, the chapter on Pythagoras contains 15 sayings and does not include the *Golden Verses*. Each of the three collections is significantly different, and this is also the case for the chapters devoted to Pythagoras: they overlap only partially.

76 Dunlop 1979, Introduction; Id. 1957; al-Qadi 1981, Gutas 1975, 52–53; Id. 1982 and 1993; Daiber 1984.

77 There are two modern editions of this text: *Muntakhab Ṣiwān al-ḥikma* 1974 and Dunlop 1979.

78 Al-Qadi 1981.

79 Ibid. Preserved in a single manuscript (Istanbul Fatih 3222), it was edited by R.M. Kartanegara (1996) in an unpublished dissertation. The text of this edition is now accessible online; see *Mukhtaṣar Ṣiwān al-ḥikma* 2013.

80 Gutas 1975.

81 Id. 1982, 647.

The presentation of Pythagoras in the *Muntakhab* and in the *Philosophical Quartet* begins with a one-sentence introduction which in the second collection is preserved in a longer version:

> The first philosopher is Pythagoras. He was one of the ascetic learned men, one of the great philosophers, and one of the most eminent among the ancients.[82]

This sentence provides the only introduction to the chapter; it is followed by sentences and anecdotes. The author did not give any details about the person of Pythagoras, but he underlined how important he was by calling him "the first philosopher" (which reflects a sentence quoted earlier by the author of the *Muntakhab* in the introduction to the entire collection, that "Pythagoras was the first to call philosophy by this name"[83]), "one of the great philosophers" (which may be an allusion to al-ʿĀmirī's history of the first great Greek philosophers) and "one of the most eminent among the ancients." In this way the compiler says that Pythagoras was exceptionally important or even the most important of the Greek philosophers. The fact that he held Pythagoras in such great esteem is interesting, because generally Pythagoras was far less popular among Arabic authors and philosophers than Aristotle or Plato, partly because of the lack of actual writings by Pythagoras and the complicated biographical and philosophical tradition associated with his name. Furthermore, even in the Greek gnomological tradition itself, Pythagoras was not an important figure. Admittedly, there was a separate Pythagorean gnomological tradition of the *Pythagorean Sentences*, but in the larger, more inclusive and plurivocal, gnomological collections Pythagoras did not play any important role. As was pointed out by Dimitri Gutas, whose edition and translation of the *Philosophical Quartet* is the most comprehensive study of the Graeco-Arabic gnomologia to date, in the *Gnomologium Vaticanum* (which consists of 577 sayings) and in the *Corpus Parisinum* (447 sayings) – the two major Greek gnomological collections, which are the main sources for many other minor ones –, there are only a few sentences attributed to Pythagoras (respectively 6

82 Id. 1975, 63.

83 Also in: Ibn-an-Nadīm 1964, 245; Ibn Abī Uṣaybiʿa 1882, 70. The story that Pythagoras coined the term 'philosophy' (in the sense that a philosopher is not wise, but just a lover of wisdom) is attested widely in the Greek tradition; see, e.g., Diogenes Laertius (proem. 12), who quotes Heraclides of Pontus. For bibliography on this topic, see Macris 2018, 829–830, especially Burkert 1960; Riedweg 2004; Guida 2013; and most recently Moore 2020 *passim*, esp. 3–24 and 321–329.

and 2).[84] Even sayings belonging to the tradition of the *Pythagorean Sentences* are quoted there anonymously or attributed to other authors.

However, the original *Ṣiwān al-ḥikma* must have incorporated quite an extensive collection of sentences attributed to Pythagoras, since its three abbreviations contain 32, 28 and 15 sentences respectively, many of which do not overlap. Still, Dimitri Gutas demonstrated that, interestingly, in the *Philosophical Quartet* none of the sentences and anecdotes attributed to Pythagoras and also known in Greek is attributed to Pythagoras in the Greek collections known to Gutas: they are all attributed to other Greek philosophers.[85] Therefore, Gutas formulated a hypothesis of an Arabic "source Q" for the chapter devoted to Pythagoras in the *Ṣiwān al-ḥikma*, in which sentences attributed by the tradition to other philosophers were falsely presented as sentences of Pythagoras. He argues that this misattribution must have taken place after these sayings were translated into Arabic, since some of them can be found under non-Pythagorean attributions, consistent with the Greek tradition (and different from those given by the *Ṣiwān al-ḥikma*) – in the *Corpus Parisinum*. This collection is dated later than the sources of the Arabic source Q, and attributions in other Graeco-Arabic gnomological collections actually follow those of the *Corpus Parisinum*.

The hypothesis of an Arabic author creating a new (spurious) collection of sentences and anecdotes attributed to Pythagoras is worth discussing further in the context of the Arabic reception of the figure of Pythagoras. The introductory sentence from the *Muntakhab* and from the *Philosophical Quartet* quoted above reflects the exceptional importance that was given to Pythagoras by the author of the Arabic collection, the *Ṣiwān al-ḥikma* or the Arabic source Q. It is possible that under the influence of the legends about the Greek philosophers, this unknown author, similarly to al-ʿĀmirī, considered Pythagoras to be the most significant among the Greek sages and needed to find gnomological content that he could attribute to him. Since he could not find such material easily in the collections available to him, he started picking up sentences from every possible source. First of all, he had at his disposal the *Golden Verses* (they are quoted *in extenso* both in the *Philosophical Quartet* and in the *Muntakhab*), which are the most obvious choice, since they were considered to contain the actual teaching of Pythagoras in a gnomological form. Secondly, he incorporated the *Letter of Pythagoras to the Tyrant of Sicily*, which is the second of the Greek pseudo-Pythagorean writings attributed to Pythagoras ever translated

84 Gutas 1975, 268.

85 Ibid., 270–271.

into Arabic.[86] Moreover, he used some version of the *Pythagorean Sentences*, as well as some unidentified Neoplatonic and Neopythagorean texts and even biblical proverbs, as his sources. In addition to that, he included in his collection some sentences that the Greek tradition attributed to other philosophers, and thus he was able to create quite an extensive collection of sentences and anecdotes attributed to Pythagoras.

Obviously, he must have had a vague idea of who Pythagoras was while gathering his collection. In the introductory sentence, he calls Pythagoras an ascetic (*zāhid*).[87] This conception of Pythagoras as an ascetic, which can be also found in the Greek literature,[88] is clearly perceptible later on in the gnomology itself. Here, apart from the *Golden Verses*, also the *Letter to the Tyrant of Sicily*, in which Pythagoras refuses to visit the tyrant and his court, has a strong ascetic character. Pythagoras is presented there as a "moderate man who needs little" or "possesses everything that he needs, as he wants, every day" and "has no need for a Sicilian table."[89] While explaining the decision to decline the visit, the author of the letter writes: "A virtuous benefit does not spring from sexual intercourse or from food, but from an education leading to virtue," which sounds like a summary of the contents of the *Golden Verses*. Later on, he adds that "various old pleasures" which the tyrant enjoys "steal weak souls" and that "the ways of Pythagoras" are unattainable for the ruler who is submitted to the commands of his body. This ascetic tendency is visible also in other sentences and anecdotes present in the collection. A good example is provided by maxim Nº 9 in the *Philosophical Quartet* (Nº 11 in the *Muntakhab*):

> Make yourselves observe three things from the law: abandon anger and contentiousness, avoid eating too much, and do not sleep too much.[90]

Again, this looks like a summary of the teaching of the *Golden Verses*, and this sentence of unknown origin may be an echo of the poem itself. But clearly the author of the collection managed to gather sayings with similar content from

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86 It is preserved in Greek as a letter of Pythagoras to Hieron I, the ruler of Syracuse. See Thesleff 1965, 185; Gutas 1975, 222–225; Städele 1980, 152–153 and 186–203; Izdebska 2012b, 146–148; Ead. 2018, 860–862.

87 The term *zāhid* is the active participle of the word *zuhd*, designating Islamic asceticism (see Gobillot 2002). It appears for example in the biographies of Muslim saints of the first centuries of the Hijra. On this background, see Masotta 2017 (*non uidi*).

88 For more on the ascetic elements in Greek Pythagoreanism, see Finn 2009, 27–32 ("A Pythagorean Tradition?"); Macris 2018, 1071.

89 Transl. Gutas 1975, 224.

90 Ibid., 67.

different sources, such as the maxim Nº 11 in the *Philosophical Quartet*, which was probably extracted from an anonymous Greek ethical treatise:[91]

> It is impossible for any part of the noble, august, and divine sciences to become firmly rooted in the soul while it is imbued with filth, since like takes pleasure only in its like.[92]

Similarly, the maxim Nº 16 in the *Muntakhab* (= Nº 8 in the *Mukhtasar*):

> Do not long for things which depend on your desire, but rather long for those that are desired in the soul because of her health and awareness;[93]

or the last sentence in the *Philosophical Quartet* (Nº 32):

> Not the mutual love of souls but that of bodies ought to be prevented.[94]

This last saying is attributed to Plato in several other Arabic collections and a parallel saying is attributed to Diogenes the Cynic in the Greek tradition.[95] It directly touches upon the question of sexual abstinence, which together with moderation in food and sleep is obviously one of the pillars of asceticism. Another important element of this Pythagorean asceticism is restraining from anger and all excessive emotions. Being able to control emotions as well as desires is "the way of Pythagoras," which enables the student to follow the path of virtue and wisdom and to approach the truth and God himself.[96]

6 The Gnomology of Ibn Hindū

The third independent Arabic gnomological collection attributed to Pythagoras is preserved in the *Al-Kalim al-rūḥānīyah fī l-ḥikam al-yūnānīyah* (*Spiritual Contents of the Greek Maxims*), a collection of sentences of about fifty Greek

91 Ibid., 234–235.
92 Ibid., 69.
93 Dunlop 1979, 31. Translated from Arabic by Anna Izdebska.
94 Gutas 1975, 81.
95 Ibid., 263–264.
96 For anecdotes preserved in the Greek tradition illustrating the control of anger among the Pythagoreans, see Macris 2016, 548–550.

philosophers that was gathered by Ibn Hindū (d. 1019 or 1029), a physician and philosopher from Rey (in Persia).[97]

The chapter devoted to Pythagoras consists only of a short collection of 14 sentences and anecdotes which differ from the sentences and anecdotes found in other Arabic Pythagorean gnomologies, but also do not have obvious parallels in the surviving Greek tradition. It seems that it was translated from a lost Greek Pythagorean collection, which otherwise remains unknown. However, since it is only 14 sayings long, it is not a big collection compared to the collections from the tradition of the *Pythagorean Sentences* or the supposed source Q of the *Ṣiwān al-ḥikma* Pythagorean collection. Nevertheless, its character is much closer to the *Ṣiwān al-ḥikma* than the *Pythagorean Sentences*, mainly due to the fact that the anecdotes are mixed with sentences, while in the *Pythagorean Sentences* there are no anecdotes at all. Furthermore, Ibn Hindū also quotes the *Letter to the Tyrant of Sicily*. He (or rather his source) turns it into an anecdote in which the explanation for Pythagoras refusing to visit the tyrant is encapsulated in a sentence from the original *Letter*:

> Indeed, your reason opposes what brings benefits [to you] and your body deracinates your fundaments, so do not require me to stay with you, since getting sick along with his patients is not a condition of being a doctor.[98]

Also the last saying (number 14) – that philosophy is the "medicine of souls" (which is a widely known statement which can be found in the Neoplatonic definitions of philosophy as well as in the *Pythagorean Sentences* and other collections related to it) – overlaps with the *Philosophical Quartet* (that is the *Ṣiwān al-ḥikma* tradition).

However, many sayings which are misattributed to Pythagoras in the texts from the circle of the *Ṣiwān al-ḥikma* are in turn attributed by Ibn Hindū to other philosophers, in accordance with the Greek tradition.[99] Therefore, his source should have been different from this group of texts and closer to Greek tradition, but it is difficult to identify it more precisely.

Nevertheless, the character of the collection of Ibn Hindū is not far from the other Pythagorean collections. It has a strong Neoplatonic flavor. In anecdote number 2 Pythagoras accuses "a fat man" of "raising the walls of his prison" while in saying number 13 he compares the body to the lyre and the spirit to

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97 There are two editions of this text: Ibn Hindū 1900 and 2001.

98 Saying Nº 7; see Ibn Hindū 1900, 98. Here and below the translations are those of Anna Izdebska.

99 Gutas 1975, 272.

the music – an allusion to Plato's *Phaedo*. Finally, saying number 5 (which in fact unfolds in ten sentences) is a particularly interesting example of the way in which the tradition associated with Pythagoras was vulgarized in the Graeco-Arabic gnomological tradition:

> And he said to his son: I recommend you ten things and you should learn them: do not appear to be harsh; do not drink with the one who is too eager; do not live with a jealous one; do not take part in a discussion if you do not know the matter; do not argue with someone who is stronger than you; do not fraternize yourself with the duplicitous one; do not have business with a liar; do not increase your social relations with women; do not befriend a miser; and the tenth, and this is the central matter of this testament, in which there is the health of your soul, providing that you should not release the mystery to anyone.[100]

Apparently, the *Golden Verses* served as a model for this pseudepigraphical testament of Pythagoras. They are also called the "testament" in the Arabic tradition, having the form of admonitions given by Pythagoras to his students/followers and focusing on the way in which the students should conduct themselves. However, there are also some elements here that are rooted in the Pythagorean tradition, but not in the *Golden Verses* themselves, like the last admonition to keep the secret. Specific esoterism and obligation not to take any element of the teaching from inside the Pythagorean community to the outside were important elements of the Pythagorean legend present in the Greek biographies of Pythagoras.[101]

The admonitions given by Pythagoras in this spurious testament are all focused on the question of how one should or should not behave in one's relations with other people. This is also something that appears in the *Golden Verses*, but here it is the only content. The main message hidden in all this advice is to be rather careful in interactions with other people, as they pose a potential threat. Among various types of dangerous people mentioned here we also find women, which in fact contradicts the Greek tradition, according to which the Pythagoreans were very inclusive of women.[102] However, misogynist elements were present already in some Greek gnomological collections

100 Ibn Hindū 1900, 97–98.
101 Zhmud 2012, 150–158, 162–163; Gemelli Marciano 2014, 144–145.
102 Pomeroy 2013; Macris 2018, 1039; Dutsch 2020.

related to the *Pythagorean Sentences* or mixed with it, as it was the case with the *Sentences of Clitarchus* or the *Sentences of Menander*.[103]

This kind of mixture of elements typical of the Pythagorean tradition and foreign to it is characteristic for such a late pseudepigraphic text. It also shows the ways in which the Pythagorean tradition was vulgarized, simplified and transformed into universal, popular philosophy. Another example of a way in which this tradition was expanding and evolving were the so-called Pythagorean *symbola* – short, enigmatic maxims attributed to Pythagoras, well known in Greek and translated into both Syriac and Arabic. They were not included into the gnomological tradition as such, but existed as separate collections, much more extensive than the preserved Greek collections of symbola. Some of them reached the Arabic world through the Arabic version of Porphyry's *Life of Pythagoras*.[104]

In the Arabic gnomological tradition, Pythagoras is presented above all as a teacher, and a spiritual and moral leader of a community of his followers. The chapter devoted to him by Ibn Hindū begins with a statement that Pythagoras was the first philosopher, surrounded by his students. The later selection of anecdotes and sentences gives the impression of having been written down by someone who witnessed Pythagoras talking to his students and giving them (often mysterious) lessons. This is also the way in which Pythagoras is presented in the *Ṣiwān al-ḥikma*. All three texts from its circle are full of anecdotes showing Pythagoras who talks to people and teaches them. The actual content of this teaching was not so easy to find and sometimes had to be borrowed from other Greek wise men, but there were also still some Greek pseudepigraphs available in Arabic, such as the *Golden Verses*, the *Pythagorean Sentences* and the *Letter to the Tyrant of Sicily*, which could serve the Arabic (and Syriac or Byzantine) authors as the basis for a first idea of what the wisdom of Pythagoras was.

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103 I owe this remark to Katarzyna Prochenko.

104 For more about the Syriac and Arabic reception of the Pythagorean *symbola* see Izdebska 2021 (in press).

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CHAPTER 7

Pythagoras' Ethics and the Pythagorean Way of Life in the Middle Ages

Aurélien Robert

The influence of Pythagoras and Pythagoreanism in the Middle Ages seems at first glance limited to the sphere of the liberal arts, and more specifically to mathematics and music.[1] Yet, starting from Antiquity, some philosophers, such as Iamblichus and Porphyry, distinguished two branches within Pythagoreanism: those who dedicated themselves to the sciences, and mathematics in particular (*mathematikoi*), and those who took interest in practical philosophy (*akousmatikoi*). While some commentators deem such a distinction artificial,[2] others do not hesitate to claim that the *akousmatikoi* played a leading role in the history of ancient Pythagoreanism.[3] Did this aspect of Pythagorean philosophy completely vanish in the Middle Ages? What image do the sources available at the time convey of philosophical life as Pythagoras and his students saw it? How was this image received, interpreted or even transformed by medieval readers?

It is worth noticing that ethics is the poor cousin of studies on how Pythagoras was received in the Middle Ages. Very few studies address the topic,[4] and those that do so generally arrive at the conclusion that the number and quality of sources available did not allow the philosophers of the day to get a clear understanding of what Pythagorean ethics really was. While Pythagorean ethics came back into fashion among Roman thinkers, it seems to have slipped away in the Middles Ages, making a comeback only during the Renaissance.

In ancient Rome, the return of Pythagoreanism is believed to be due mainly to Nigidius Figulus – as briefly described by Cicero – and above all to the encyclopedic knowledge of authors such as Aulus Gellius and Varro.[5] Quickly, some moral sentences attributed to Pythagoras or to his students started to circulate in the Roman world. Some of these were undoubtedly spread by Alexander

1 See Hicks 2014 for an excellent overview of these topics.
2 Zhmud 1997 and 2012.
3 Burkert 1972; Huffman 1999.
4 For example, Joost-Gaugier 2006.
5 Flinterman 2014, and for bibliographical updates Macris 2018, 1118–1121.

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Polyhistor in his treatise *On Pythagorean Symbols*.[6] References to these sentences can be found in many Latin texts, such as Cicero's for example, in the *Tusculan Disputations*, where Pythagoras is presented as the philosopher par excellence, or in *De finibus, De officiis* and *De amicitia*, where the Pythagorean concept of friendship is explored.[7] This also brings to mind the portrait of Pythagoras in Ovid's *Metamorphoses*, and numerous other examples abound. Though a large share of this Latin literature was known in the Middle Ages, Pythagorean ethics seems to have made its true comeback in the 15th century.[8] At the time, Marsilio Ficino translated 39 *symbola* along with a commentary of the *Golden Verses* attributed to Pythagoras by Hierocles, to which were added the *symbola* collected by a man called Pandolfo Collenuccio, the translation of the *Golden Verses* and the *Lives and Opinions of Eminent Philosophers* by Diogenes Laertius, where we find an account of Pythagoras' life that provides a wealth of information on the man, his thought and his work.[9]

Without calling into question the importance of the translations and commentaries by Renaissance humanists, I would like to describe here several medieval traditions in which certain moral maxims attributed to Pythagoras or to his school of thought are conserved, discussed and sometimes even commented upon. While the presence of Pythagorean ethics remains unclear and piecemeal in the Middle Ages, and though Pythagorean thought is overshadowed by that of other major figures such as Aristotle or Augustine, the fact remains that it holds a significant and interesting place.

The main problem for medieval thinkers was to find out what could be taken from Pythagoras and made compatible with Christianity. As Andrew Hicks writes, "medieval authors were wont to pick and choose their way through Pythagorean teachings, accepting what they found useful but casting aside what they found reprehensible."[10] For this reason, this phenomenon can hardly be referred to as "medieval Pythagoreanism." Medieval thinkers were rather only inspired by Pythagorean thought for certain specific theories. The choices made regarding the sources, the organization of the anthologies and

6 On Alexander Polyhistor, who has also transmitted the so-called *Pythagorean Notebooks* (*hypomnēmata*), preserved in Diogenes Laertius, see the references gathered in Macris 2018, 739, 850, 1059, 1115.

7 On Cicero's mentions of Pythagoras see Macris 2018, 1120–1121; Balbo 2018.

8 Allen 2014.

9 On the reception of these Pythagorean texts during the Renaissance, see Celenza 1999 and 2001; on that of the *symbola* more particularly, Vuilleumier Laurens 2000. On Collenuccio, Josserand 1932. On the reception of Diogenes Laertius, Kaiser 2012, Grafton 2018 and below.

10 Hicks 2014, 419.

their use bring to the surface the image of a proto-Christian Pythagoras, similar to the figure of the medieval monk, the defender of a rigorous moral stance based on strict obedience to divine law. We will thus try to understand how this image was gradually shaped.

1 The Nature of Pythagorean Ethics

To understand how Pythagorean ethics was received in the Middle Ages, we must first remember that Pythagoras himself never wrote a single line and left to posterity only a scattered series of moral maxims and sentences reported by others, who turned them to their advantage, sometimes deforming them. These maxims are highly varied in content, yet are brought together by the same underlying will to regulate all aspects of daily life, from eating habits (for example, fasting and vegetarianism) and religious customs to superstitious beliefs and one's way of dressing.[11] Thus, Pythagorean ethics deals with a complete way of life, the *bios pythagoreios* that is inseparable from the deeply religious dimension of the doctrine.[12] Indeed, the precepts formulated in most of these maxims aim to remind men of what connects them to divinity and to regulate this relationship in the strictest possible manner. As Iamblichus had understood so well, the Pythagorean *akousmata* or *symbola*, short maxims that are allegorical and sometimes even esoteric in nature, answered different kinds of questions, with only one (what must one do and not do?) relating to ethics in the strict sense.[13] They promoted a law-abiding morality, based on divine authority, whose religious character was so prominent that it has been referred to as a "catechism of the *akousmatikoi*."[14] If we consider only the ethical aspect, certain witnesses of the Pythagorean "school" such as Aristoxenus of Tarentum (4th century BC), whose treatise on *The Pythagorean Precepts* (Gr. *apophaseis*) have been preserved in fragments thanks to Stobaeus,[15] seem to indicate that the Pythagoreans practiced asceticism and even rigoristic morality.[16] Likewise, in the *Golden Verses*, a poem that probably reflects the Pythagorean teachings of a later period, we can distinguish between the first part that presents rather common morals differing only slightly from those of other schools of philosophy, and the second part, more characteristic of Pythagoreanism, in which the

11 Huffman 2006; Thom 2013 and 2020.
12 Gemelli Marciano 2014.
13 Burkert 1972, 167–168.
14 Delatte 1915.
15 Huffman 2006, 2008 and 2019.
16 On Pythagorean asceticism more generally, see Finn 2009, 27–32; Macris 2018, 1071.

author insists on the need to follow the precepts concerning all sorts of rituals to the letter.[17]

As the reader can see, no general message about happiness, goodness or virtue is to be found in Pythagorean doxography and gnomology. Apart from the topic of friendship, highly developed by Pythagoras and his followers,[18] and often borrowed and discussed in a number of Greek and Latin texts,[19] we find above all practical judgements – some explicit, others cryptic – on what must be done in specific situations and even sometimes simple commandments that must be followed word for word. In summary, it can be said that under the guidance of the spiritual mentor Pythagoras, ethics was not only the codification of community rituals, but also a path towards individual salvation.[20]

These diverse aspects of Pythagorean ethics – asceticism, religiosity and soteriology – most likely enabled the doctrine to be partially incorporated by certain Christian authors, from the Church Fathers to medieval theologians. It is this phenomenon that we shall now try to better understand.

2 The Problem of Paganism in the Middle Ages: Can Pythagoras Be "Saved"?

Before getting into the heart of the subject, we must clarify an initial point that has long been an obstacle to studies on the reception of Pythagoras in the Middle Ages: the relationship with pagan philosophers. One of the factors believed to have prevented medieval authors from truly appropriating ancient philosophy is the view of philosophy as overly subordinated to religion and theology. Apart from the two specific cases of Plato and Aristotle, it was not until the Renaissance and the philological work of the humanists that a true secularization process began enabling the return of paganism, so that it could be studied in its own right and not criticized or transformed for apologetic purposes.[21] Ada Palmer has recently published two studies on the reception of Pythagoras during the Renaissance in which she recounts the steps in

17 Thom 1995.

18 Gemelli Marciano 2014, 146–147; cf. Macris 2018, 826 and 1112. For a skeptical approach, taking into account the fact that "compelling evidence is lacking," see most recently El Murr 2020, 568–571. For Neopythagorean developments, see Thom 1997.

19 Gegenschatz 1981; additional references on the exemplary Pythagorean friendship between Damon and Phintias in Macris 2012; 2018, 1052, 1112.

20 On this aspect of Pythagorean ethics, see Macris 2013, esp. 66–68.

21 Fubini 1990.

this process.[22] In her view, the ever stronger will to identify Pythagoras as a proto-Christian author, which began in the 15th century, provided a group of orthodox sources for the heterodoxy of later readers, in the 16th century and more particularly in the 17th century. In other words, such Christianization of Pythagoras is believed to have paradoxically brought about his return to the philosophical scene in a fully secularized form. I will not give my opinion here on the relevance of this analysis for the end of the Renaissance. Yet when the first humanists, such as Leonardo Bruni (*ca* 1370–1444), Francesco Filelfo (1398–1481) or Poggio Bracciolini (1380–1459) are laudatory, if not hagiographic, regarding Pythagoras, when they extol his virtues, his discipline and his rectitude,[23] they are but repeating, in a new language, backed by hitherto undetected sources, a message that was already highly structured in the Middle Ages. As I have already suggested, the reason for such continuity is to be found first of all in the sources that pass down the precepts of Pythagorean ethics.

In a recent book, John Marenbon recounts the history of what he calls "the problem of paganism" from Augustine to Leibniz.[24] In his view, this problem can be broken down into several sub-issues such as: Could pagans be saved? Is their virtue genuine? What could they have known about the Christian God? As he has shown, the answers vary from one author to another, ranging from a rather flexible stance accepting the fact that some pagans have been saved thanks to their virtuous life to the polar opposite viewpoint, according to which no pagan philosopher could be saved since their virtue was not genuine. Between these extremes, we find the entire range of possible theories. Without going back to the countless debates raised by this question, we should simply remember that starting in the 12th century, authors such as Peter Abelard and John of Salisbury took a more favorable stance towards some pagan philosophers, among them Pythagoras. Moreover, the figure of Pythagoras is often considered an exception, as if, more than any other pagan philosopher, he deserved to be saved.

It must be said that in medieval encyclopedias Pythagoras is presented as the inventor of philosophy – or in any case of the word "philosophy" – according to an anecdote repeated by Cicero in the *Tusculan Disputations* (V, 4) and by Augustine in *The City of God* (VIII, 2).[25] For this reason, he could have been saved for having invented this way of loving wisdom, without calling himself

22 Palmer 2016 and 2017.

23 See some examples in Joost-Gaugier 2009, 19–36.

24 Marenbon 2015. Cf. also Steel, Marenbon and Verbeke (ed.) 2012.

25 On this tradition see the references given in n. 80 of Anna Izdebska's chapter in this volume.

a wise man. Yet, he also represents, among virtuous pagans, one of those who most closely approached the truth of Christianity before the Revelation. He is thought to have been one of the first to conceive of the soul as immortal – even though his doctrine on the transmigration of the souls is not Christian – and had an intuition about the coming of monotheism.[26] Not only did he speak of God in the singular form, according to Augustine, but this God also seems to be similar to a personal god acting deliberately in the world in the shape of providence and law. What is more, as Boethius recalls in *The Consolation of Philosophy*, Pythagoras is known for this maxim: "Follow God."[27] In short, the only things he was missing in order to gain full knowledge of the Christian God were the doctrines of the Trinity and the Incarnation. That is why Augustine, in his sermon *Against the Pagans* (§36), on the one hand admits that Pythagoras was not idolatrous and did not draw upon pagan gods in terms of ethics, yet on the other hand condemns him for having transformed this truth into a lie.[28] Thus, in the Middle Ages we find a wide range of attitudes concerning the figure of Pythagoras, yet certain authors were without doubt willing to admit the sincerity of his virtue and the strength of his theology. This therefore involved sorting the wheat from the chaff. As John of Salisbury writes: "When the Pythagoreans teach us about innocence, frugality, and contempt for the world, we should listen to them; when they force souls that have ascended into the heavens back into the bodies of beasts, even Plato must be refuted, for on this point he followed Pythagoras too closely."[29] In any case, although Augustinian criticism remained present in medieval debates, it did not prevent moral maxims attributed to Pythagoras from being spread about. They first of all circulated in the form of collections of sentences and were later gradually incorporated in the *Lives of Pythagoras* starting in the 13th century. Let us now look back upon the major steps in this history.

As it is widely accepted that the oldest traces of Pythagoras' ethical and religious teachings are found in the *akousmata* – 'things heard' –, also called *symbola* or *aenigmata*, which include between 100 and 200 sentences depending

26 On this last point see Macris 2019 (focusing on Clement of Alexandria), as well as Daniel De Smet's chapter in this volume.

27 This sentence is quoted in *Consolation of Philosophy*, I, 4, 38–39; see Boethius 1999, 12. For the earlier sources transmitting this maxim, see Gemelli Marciano 2014, 136f. with n. 23; bibliography in Macris 2018, 837.

28 For some references, cf. Macris 2018, 1163.

29 *Policraticus*, 7.10, quoted by Hicks 2014, 419–420. On this argument, see Irene Caiazzo's chapter in this volume.

on the sources and the criteria used to count them,[30] we should begin by studying the reception of this corpus in medieval times.

3 The Reception of Pythagoras' *symbola* and the "*Aenigmata Aristotelis*"

The first collections of *akousmata* are believed to date back to the 5th century BC, as it appears from the first commentaries written by Anaximander the Younger of Miletus and Androcydes the Pythagorean (who could be a late apocryph, *ca* 1st century BC), of which only fragments remain.[31] The *akousmata* collected by Androcydes are probably at the root of the later quotations found in the writings of Diogenes Laertius, Porphyry and Iamblichus, yet it seems that these accounts relate directly to Aristotle's lost treatise *On the Pythagoreans* in which he is said to have discussed these *symbola*. Unlike the *Golden Verses*, whose tradition can be more or less traced, we can hardly speak of a corpus of *akousmata*, as these vary greatly from one version to another, from one quotation or commentary to the next – even though some symbols recur more often than others in the sources.

Despite the scattered, piecemeal nature of this tradition, echoes can be found very early in many pagan, Greek and Roman authors, as well as in Christian authors such as Clement of Alexandria, Origen and Ambrose of Milan.[32] Not only did these Christian authors not oppose the contents of certain of these apophthegms – they sometimes even incorporated them into their own system of thought. This appropriation by Christians was probably made easier thanks to similarities between the content of certain *symbola* and some passages of the New Testament.[33] As Johan C. Thom has illustrated,[34] early Christian literature showed several attempts to reconcile the *symbola* with the teaching of the Gospel. As regards the Latin Middle Ages, beyond

30 Burkert 1972, 166–192. See also Philip 1966, 134–150; van der Waerden 1979, 64–99. For a most complete bibliographical orientation on this topic, see Macris 2018, 821–824.

31 See Thom 2013 and 2020, and on Androcydes, Thom 2021 (in press).

32 See generally Thom 2017 and Id., forthcoming. – Clement of Alexandria discusses several *symbola* in his *Stromata*, V, 5; see the precious commentary of Le Boulluec 2009^2. (For an overview of the role of Pythagoras in Clement see Afonasin 2013; additional bibliography in Macris 2018, 1161.) – On Iamblichus' reception of the *symbola* in his *Protrepticus*, see Thom 2018.

33 For connections to the New Testament, see Hemmerdinger, 1972. See also Grant 1980; Balch 1992. Some scholars went even further, like Isidore Lévy, who saw Pythagoreanism in the construction of the figure of Jesus; see Lévy 1926 and 1927.

34 Thom 1994.

quotations gleaned here and there in Roman literature, it is primarily through the Church Fathers that certain of these sentences were passed down. From this perspective, the most important source is undoubtedly the collection of *aenigmata* contained in Jerome's *Apology against Rufinus*, which was widely circulated and commented upon.

The third part of the *Apology against Rufinus* is a testimony of the turbulent debate between Jerome and Rufinus about the influence of Origen. Their disagreement was above all theological in nature and, in this respect, Jerome accused Rufinus of having provided the Pelagians with philosophical arguments. More specifically, the debate centered on the use of pagan philosophers, and of Pythagorean books in particular. Rufinus is believed to first have accused Jerome of deception, due to his confession, in one of his letters (84, 6, 2), that in his youth he had attributed what he had read in Pythagoras, Plato and Empedocles to the apostles. For his defense, Jerome begins by calling to mind the fact that he is acquainted with none of Pythagoras' books and that he knows of the Pythagorean doctrine in a very indirect manner through his readings of Cicero and Seneca. It is here that Jerome reminds his interlocutor that Pythagoras is the author of many moral precepts that are extremely useful and, as it were, compatible with Christianity.

Three types of precepts can be distinguished among those quoted in Jerome's text. First, we find what he calls "the golden precepts," of which he gives only one example:

> One of these precepts is: "We must avoid in any way, and cut out by fire and sword and separate by contrivances of all kinds, disease (*noson/languor*) from the body, ignorance from the soul, luxury from the belly, sedition from the city, discord from the household, and in general excess from all things alike."[35]

These prescriptions, in all likelihood borrowed from Porphyry's *Life of Pythagoras*, are very general and could suit the majority of ancient Greek philosophical schools as well as many Christian authors. The restraint advocated in all spheres of life, public and private alike, in the body and in the soul, is in no way specifically Pythagorean. For this reason, when Jerome quotes other precepts, he is careful to specify – as Porphyry did – that these are also of Pythagorean lineage (*Pythagorica et illa praecepta sunt …*):

35 Jerome 1982 (*Contra Rufinum*, III, 39, 108–109); Id. 1886, 538a (English translation slightly modified here). For the Greek source, see Porphyry 1982, § 22, p. 46; cf. also Iamblichus, *On the Pythagorean Way of Life*, § 34.

> There are other precepts of Pythagoras, such as these: "Friends have all things in common"; "A friend is a second self"; "Two moments are specially to be worried about, morning and evening: that is, things which we are going to do, and things which we have done"; "Next to God we must worship truth, for this alone makes men akin to God."[36]

The first two precepts on friendship are frequently attributed to Pythagoras in ancient doxographies, but the next two are much rarer, and are quite compatible with Christianity, whether it is a question of examining our conscience, in the evening and in the morning,[37] or seeking the truth from God.[38]

This brief doxography closes with a series of *aenigmata*, which were also taken from Porphyry's *Life of Pythagoras*:

> There are also enigmas which Aristotle has collected with much diligence in his works: [1] "Never jump over the scale," that is, "Do not transgress the justice"; [2] "Never stir the fire with the sword," that is, "Do not provoke a man when he is angry and excited with hard words"; [3] "One must not touch the crown," that is, "One must maintain the laws of the cities"; [4] "Do not eat your heart," that is, "Cast away sorrow from your mind"; [5] "When you are gone, do not return," that is, "After death do not regret this life"; [6] "Do not walk on the public road," that is, "Do not follow the errors of the multitude"; [7] "Never admit a swallow into your house," that is, "Do not admit chatterers and talkative persons under the same roof with you"; [8] "Put fresh burdens on the burdened; put none on those who lay them down," that is, "When men are on the road to virtue, supply them with fresh precepts; when they abandon themselves to idleness, leave them alone."[39]

This text by Jerome was widely circulated in the Middle Ages and thus provided, in condensed form, a few of the major themes of Pythagorean ethics. The authority of Jerome and the encouragement to seek the truth from God were supposed to protect these precepts from Augustine's criticism. As for the

36 Jerome 1982 (*Contra Rufinum*, III, 39, 109); Id. 1886, 538a (transl. slightly modified). See Porphyry 1982, § 33, p. 51; § 40, p. 54; § 41, pp. 54–55.

37 On the Pythagorean self-examination or "soul-searching", coming in fact from the *Golden Verses* (quoted by Porphyry), see Thom 1995, 38–43, 163–167; Macris 2013, 83, n. 110 and 2018, 1072.

38 The term is in the singular both in Porphyry's original text and in its Latin translation.

39 Jerome 1982 (*Contra Rufinum*, III, 39, 109); Id. 1886, 538a–b (transl. slightly modified here). For the Greek source, see Porphyry 1982, § 42, pp. 55–56, quoting Aristotle.

aenigmata transmitted under the name of Aristotle, they provide a glimpse into the symbolic, occult dimension of these ethical precepts. Quite surprisingly, in this selection made by Jerome – following Porphyry – in order to show the moral value of Pythagoreanism, medieval readers mainly remembered the *aenigmata*, which were quickly circulated independently in a number of manuscripts misleadingly entitled *Aenigmata Aristotelis*.

The oldest account of these *Aenigmata Aristotelis* dates back to the 9th century, but the majority of the copies – over thirty in all – are spread out between the 12th and 15th centuries.[40] Alongside these accounts we find quotations, sometimes with commentary, of these enigmas in highly diverse literary texts. They are found copied and associated with the name of Aristotle in Sedulius Scottus' *Collectaneum miscellaneum* in the 9th century,[41] in a letter written in Germany and dating back to the 11th century,[42] in the *Florilegium Angelicum* and other similar collections,[43] in the *continuatio* of Thomas Aquinas' *De regno* written by Ptolomeus of Lucca in the 13th century,[44] or in Wolfenbüttel's *Rapularius* in the 15th century,[45] to give but a few examples.

Starting from the 14th century, the *Aenigmata Aristotelis* were also disseminated along with a moral and theological commentary entitled *Aenigmata moralizata*.[46] This text, preserved in 55 manuscripts, is sometimes attributed to Robert Holcot, a 14th-century English Dominican, most likely because it was circulated with Holcot's *Moralitates* in several manuscripts.[47] However, Beryl Smalley, in her major book on English Dominicans, had spotted the existence of this tradition and, deeming this attribution unlikely, referred to the complex relationships between the *Moralitates* and the *Gesta Romanorum*, which contain several of the same *exempla*.[48]

40 Schmidt and Knox 1985, 30–31.

41 Sedulius Scottus 1988.

42 Bulst 1949, 114.

43 Reeve 2018.

44 Ptolomeus of Lucca 1954, IV, §22: *De documentis pythagoricis sub figuris et aenigmatibus traditis*.

45 *Rapularius* 2002, 298.

46 In this annotated version, it is said that these *aenigmata* were first excerpted from the works of Aristotle by Valerius Maximus. Cf. MS Paris, Bibliothèque nationale de France, lat. 590, f. 115r. Although this attribution is erroneous, it shows the will to philologically understand the origin of these precepts.

47 These manuscripts also included the *Imagines Fulgentii* and the *Declamationes Senecae* without distinction, as if the four texts formed a single unit. In the absence of a clearly identified author, the copyists probably chose to assign the *Aenigmata moralizata* to Robert Holcot, the author of the first text in this collection.

48 Smalley 1960. See also Palmer 1991. I have compared the standard version of the *Gesta Romanorum* (2016) with two manuscripts of the *Aenigmata moralizata* attributed to

The *Aenigmata moralizata* were indeed inserted in the *Gesta Romanorum*, one of the most popular collections of moral *exempla* at the end of the Middle Ages, translated into several vernacular languages starting in the Middle Ages and published many times during the Renaissance. This text had a significant influence on medieval literature, particularly on Boccaccio (1313–1375) and Chaucer (*ca* 1340–1400), and survived in a great number of manuscripts. In the first chapter, entitled *De ponderatione vitae*, the anonymous author shows Aristotle teaching the Pythagorean *symbola* to Alexander the Great, who, we are told, "paid the closest attention to these seven counsels and profited from them as long as he lived."[49] After having recalled this brief story, the author comments on each *symbolum* from a moral and, above all, religious perspective, based primarily on Scriptural authorities.[50]

The first symbol ("Never jump over the scale") gives meaning to both the entire chapter and to its title: when Pythagoras evokes the scale, he is evoking human life. Let us imagine, he continues, a man on a ladder, placed between the poverty in which he came into the world and the poverty in which he will leave it, as the Ecclesiastes says (5:14). The same goes for sin: if we place our sins since childhood on one side of the scale, we must measure what repentance we have left on the other side before our final hour. Since we do not know how much time we have left to live, the image of the scale must encourage us to start being virtuous immediately rather than lose our way in pursuit of money and material wealth, for example.

The second symbol ("Do not stir the fire with the sword") is believed to mean "do not provoke a wrathful man with harsh words." This double metaphorical shift, from fire to anger, from the sword to the word, is explained on the one hand by the naturalist conception of moods, and on the other hand by a passage from the Psalms (56:5) in which a man's tongue is compared to "a sharp sword". Pythagoras' symbol is thus believed to mean that it is better to refrain from speaking to an angry man.

Robert Holcot (MS Paris, Bibliothèque nationale de France, lat. 590, f. 115r–119 and Heidelberg, Universitätsbibliothek, Cod. Sal. VII, 104, f. 65r–70r). They are very similar, except the introduction concerning the teaching of Alexander the Great (on which see the next paragraph), which is not present in the *Aenigmata moralizata*.

49 The Latin text has been edited several times since the Renaissance. Here I quote the English translation by Ch. Stace in *Gesta Romanorum* 2016, 96–99. The phrasing of the *symbola* is sometimes slightly different from the original. For this reason, the English translation is also different from the one quoted above.

50 For a similar approach to the Pythagorean *symbola*, see already Clement of Alexandria, *Stromateis*, book V, with Le Boulluec 2009[2] [1981[1]].

The third symbol ("Do not rail at the crown") is believed to mean "do not find fault with civic laws". Yet the author specifies immediately that the society he is referring to is in reality the Church and its laws, the Christian dogmas that no wise man can criticize ("So we must obey the words of God and his precepts and hold fast to them if we wish to come to the eternal kingdom.")

The fourth symbol ("Do not eat your heart") is paralleled with Proverbs (15:13), where it is said that sadness ravages the heart of Man as worms bore through wood. From there, the statement "do not heat the heart of a little bird" could be understood as an encouragement not to feel emotions such as sorrow, envy or hatred, due to the things that surround us here on earth. The solution to succeed in doing this is more or less the same as in the previous commentaries: "Let man therefore unite himself with God by leading a good life, and keep his remembrance of Him always fresh in his heart, and thus he will feel no grief."

The next symbols are analyzed through the same religious prism based on a Biblical reference. If based on Ecclesiastes (21:1), "Once you have started, do not turn back" [5] is taken to mean "when you have withdrawn from a state of error, do not return to the vomit of sin." Based on Matthew (7:13), it appears that the sentence "Do not walk along a public highway" [6] means "the way of the sinner, along which the greater part of the world passes, because it is broad". Finally, based on the life of Jonas, the author interprets the sentence "do not allow a twittering swallow to live in your house" [7] as meaning "do not let sin into your heart, for your conscience will always murmur against it."

This text thus ensured great success for some of Pythagoras' *symbola*. Jerome's original selection aimed to find points upon which Pythagoreanism and Christianity were compatible, and the commentary we have just read through, which consistently parallels *akousmata* and Biblical passages, only strengthened this Christian reception by showing that the hidden meaning of these maxims is found in the Bible.

Before closing this section, we must also mention another collection of seven *symbola* attributed to Pythagoras reported in the book of magic entitled *Picatrix*, the Latin translation of the Arabic work *Ghāyat al-Hakīm* (*The Goal of the Wise*), written in the 11th century and attributed to al-Majrītī.[51] While these are not the same *symbola* passed down by Jerome, they also seem to be derived from Porphyry's *Life of Pythagoras*. In any case, they were apparently not commented upon in the Latin Middle Ages, but were widely read and discussed during the Renaissance.[52]

51 *Picatrix* 1986, 228. Cf J.-P. Boudet, A. Caiozzo and N. Weill-Parot 2011 (reprint 2016).

52 Celenza 2001, 12–13.

4 Early Gnomological Collections

Alongside the *akousmata* or *symbola* there is a large body of gnomological literature related to Pythagorean ethics that is composed of moral maxims much more similar to those of other Greek philosophical schools. In this vast, shifting corpus, certain sentences are sometimes attributed to different authors depending on the collection, and the authors frequently mix up Pythagoras and his students, making it extremely difficult to take away any absolutely reliable information. Whatever the case, a part of these *gnōmai* or *apophthegmata* have been incorporated in Latin collections of *sententiae*.

The oldest and largest gnomological collection known in the Middle Ages is that of Publilius Syrus, a Latin poet and renowned author of mimes, born *ca* 93 BC Composed of some 730 aphorisms, alphabetically arranged, this collection proved a major success in the Roman world before being more broadly circulated in the Middle Ages.[53] Medieval collections of proverbs, maxims and sentences draw largely from this source, while occasionally adding new *sententiae* coming from other collections, such as those from *De moribus* (*On Moral Customs*) or *De remediis fortuitorum* (*On the remedies of fortunes*) falsely attributed to Seneca. As well, by contamination, several manuscripts of Publilius Syrus' *Sententiae* – out of the nearly 150 – attribute all of them to Seneca, a fact that allowed this text to enjoy an immense prestige. However, although these maxims do not indicate the names of their authors, some of them, whose content resembles the Pythagorean gnomological tradition, were attributed to Pythagoras in the Middle Ages.

To understand this process, we must look at another collection of *sententiae* brought together under the title *De nugis philosophorum* (*On the foolishness of courtly people*) and attributed to a certain Caecilius Balbus, a fictional character who never existed. This collection was circulated since the 9th century and played an important role in spreading some Pythagorean maxims in Latin.[54] Several sentences in this collection are also to be read in the works of Publilius Syrus, yet many others are to be found, and some of these are clearly attributed to Pythagoras. Among these, evidence has shown that several are part of the Greek tradition, while others are not, which makes them of particular interest for the history of the reception of Pythagorean ethics.

The sentences collected by the Pseudo-Caecilius Balbus are organized by topics in order to make things easier for the reader. While Pythagoras appears

53 Some manuscripts contain glosses and comments that need to be studied in detail. See Pellegrin 1976.

54 Diouron 2013.

in several sections, his maxims focus primarily on the status and attitude of the wise man. He appears as separated from the world, and above all from the crowd, as can be expected of elitist ethics. The people are presented here as the reckless, instinctive masses, driven by base desires (lust, hunger and war). If the populace acts virtuously, with justice, it is because it has been driven by necessity, and not by free will. Yet the sage is opposed not only to the people, but also to women. To someone who had told him that he preferred the conversation of women to that of philosophers, the sage would simply have answered that swine prefer muck to clear water. Further on, when speaking of women's tears, he distinguishes two types, those that are the sign of sorrow and those that set a trap for men. If we move beyond this misogyny, which we must admit is not specific to Pythagoreanism in Antiquity, the remaining sentences concern the emotions and virtues of the sage.

Concerning emotions, the maxims attributed to Pythagoras are hardly original. The wise man is said to be someone who treats the passions of the soul. Moreover, the repeated use of medical vocabulary (*viaticum*, *remedium*, *medicina*) recalls a form of Stoicism. This therapeutic dimension of philosophy is also found in a maxim on old age: knowledge (*doctrina*) is the best remedy against aging. Only two emotions appear in this collection of maxims: wrath and pride. We can immediately see that the vocabulary used lends itself to a Christian reading: the end of wrath marks the beginning of repentance (*poenitentia*) and the end of pride is a freeing of the mind. Generally speaking, these maxims thus encourage treating maladies of the soul by using a remedy that is also mental, or in any case intellectual. The letting go of passions, some of which have become part of the body, can be done only by freeing one's thought.

The analysis of virtues is consistent with the therapy of the passions mentioned above. The wise man, according to Pythagoras, must aim for goodness (*bonitas*), which is primarily characterized by rational work that is supposed to have an effect on both our beliefs and our desires. From this point of view, the positive counterpart of the passions of the soul is a certain type of hope (*bona spes*). For the rest, the recommendations contained in these sentences are quite common,[55] except for those that concern friendship. It is well known that friendship is at the heart of Pythagorean ethics, so much so that the term "Pythagorean communism"[56] has even been used. Indeed, Pythagoras is often considered the originator of the idea that all goods must be shared between

55 Concerning money, for instance, Pythagoras invites us to be content with little and not to be stingy.

56 Minar Jr. 1944.

friends.[57] Yet this is not the aspect highlighted by the Pseudo-Caecilius Balbus, who insists more on Man's ontological need for friendship ("a man without friends is like a body without a soul"; "friendships are eternal") and on the ethical principles for maintaining it. For example, we must avoid overly long debates, be wary of flattering friends and those who seek only utility, and remember that the principle of friendship is "speaking well" (*bene loqui*). As for other types of virtuous relationships, friendship is based on a positive emotion, namely trust (*fides*).

The selection made by the Pseudo-Caecilius Balbus quite clearly showed the compatibility of Pythagoreanism with Christianity, and more particularly with certain forms of medieval monasticism.[58] Indeed, the sage must live separated from the world, in a community of male friends, striving to rise above the passions and desires thanks to hope (*spes*), faith (*fides*) and repentance (*poenitentia*).

Several anthologies have collected sentences attributed to Pythagoras by the Pseudo-Caecilius Balbus. For example, in the 12th century, the *Florilegium morale oxoniense* quotes the sentences, organizing them by theme.[59] In this case, we find not only the sentences brought together by the Pseudo-Caecilius Balbus, but also other sentences, certain of which, also found in Publilius Syrus, have been reattributed to Pythagoras. For example, the sentence "Neminem cito accusaveris, neminem cito laudaveris" quoted by Publilius Syrus is attributed to Pythagoras. Some sentences from the Pseudo-Caecilius Balbus attributed to Pythagoras would also later be commented upon by William of Doncaster (12th century) in his *Explicatio aphorismatum philosophorum*.[60]

Starting from the 8th century, the number of sentences attributed to Pythagoras in *florilegia* further increased. When at the beginning of the 13th century Engelbert of Admont copied a number of maxims taken from Publilius Syrus and the Pseudo-Caecilius Balbus, he added new sentences attributed to Pythagoras that he had taken from Seneca's *Letters*.[61] In the following centuries, these Pythagorean sentences would be found outside of *florilegia* in a number of texts written in Latin, as well as in several vernacular languages.[62]

57 See Macris 2018, 841.

58 For affinities of late antique Christian monasticism with Pythagoreanism, see Jordan 1961.

59 *Florilegium morale oxoniense* 1955 and 1956.

60 William of Doncaster 1976.

61 Fowler 1978, for instance on pages 253, 269, 270, 273, 287, 290, 300.

62 For an overview of this tradition and a recent bibliography, see Schulze-Busacker 2005 and 2012.

5 Gnomological Collections Translated from Arabic

Knowledge of Pythagoras' maxims was not spread in the Middle Ages only by Latin texts from Roman or Christian Antiquity. Some of them have come to us through Latin translations of Arabic wisdom literature.[63] Among these, the compilation entitled *Kitāb mukhtār al-ḥikam wa-maḥāsin al-kilam* (*Book of the Choicest Maxims and Best Sayings*) composed in Cairo in the 11th century by Abū al-Wafāʾ al-Mubashshir Ibn Fātik, an erudite scholar from Damascus, is a reference of prime importance.[64]

Al-Mubashshir Ibn Fātik's text is composed of a series of portraits of wise men, each followed by a long series of maxims. We find sages in the Hermetic tradition, such as Hermes, Seth, Thoth or Asclepius, poets such as Homer and Solon, philosophers such as Pythagoras, Diogenes, Socrates, Plato or Aristotle, physicians such as Hippocrates and Galen, as well as Christian authors such as Gregory of Nazianzus.[65] Each chapter contains a brief biographical introduction, followed by a group of maxims that are generally organized by theme. The chapter dedicated to Pythagoras in the original Arabic features 106 maxims focusing on various subjects related to religion, ethics and dietetics.[66] The nature and number of sources used by al-Mubashshir Ibn Fātik in this text raises a number of problems, given the fact that he most likely directly borrowed from Greek texts, as well as from Arabic translations and compilations, such as those assembled by the Christian Hunayn ibn Ishaq in the 9th century.[67] Regarding the Greek sources, some passages from the chapter on Pythagoras have clear parallels with Porphyry's *Life of Pythagoras* and *Philosophical History*,[68] but al-Mubashshir Ibn Fātik perhaps also borrowed some items from the *Letter to Marcella* or from the *Pythagorean Sentences* (*gnōmai*), which were circulating at the time.[69] Some sentences thus have a Greek equivalent – sometimes also in the *Golden Verses* – while others simply do not have any identified equivalents as of today.

63 On this literature, see Gutas 1975.

64 For the Arabic text, see al-Mubashshir Ibn Fātik 1958. The relevant passages are translated into English by Cottrell 2016, 490–505.

65 Another series of Christian maxims is attributed to "Thesileus" (= probably Basil). For a brief description of the content of these chapters, see Rodriguez Adrados 2009, 280–303.

66 See Anna Izdebska's chapter in this volume.

67 On the difficulty of finding sources for this text, see Cottrell 2012 and Hasse 2002 (particularly 45–52).

68 On this point, see Rosenthal 1937 and, following him, Segonds 1982; Sodano in Porphyry 1998, 198–216; Cottrell 2008; Hugonnard-Roche 2012.

69 On the *Pythagorean Sentences*, see Prochenko 2018a.

Beyond its rich content, this compilation constitutes an account of major importance in helping to understand the spread of Pythagorean ethics in European culture. In fact, it was translated into several languages in the 13th century. Before 1257, an initial translation into Castilian was made by a translator from the court of Alfonso the Wise and entitled *Bocados de oro* (*Morsels of Gold*).[70] This Spanish version was then translated into Latin under the title *Liber philosophorum moralium antiquorum*.[71] This Latin version was in turn translated into French by William of Tignonville (d. 1414). There are also translations into Occitan and English.[72] As indicated by the great number of manuscripts containing its various versions, this text very quickly became a huge success throughout Europe since the 13th century. We know, for example, that Petrarch and Boccaccio had copies of it,[73] but a precise, in-depth investigation should be undertaken on the European reception of the *Kitāb mukhtār al-ḥikam wa-maḥāsin al-kilam*, something we are obviously unable to do here given the scope of this chapter. We will thus content ourselves with making a few observations on the chapter dedicated to Pythagoras in the Latin version.

On the whole, the *Kitāb mukhtār al-ḥikam wa-maḥāsin al-kilam* and its translations focus on predominantly religious considerations, on God and the obedience owed to him,[74] as well as on ascetic, purist morality. This involved rendering a part of Pythagoras' pagan wisdom compatible with monotheistic religion, that is to say Islam, by showing that Greek ethics had been able to conceive the notion of obedience to moral law and had not only developed ethics of virtues related to the immanence of action. In any case, though they were developed in an Islamic context, some of the maxims quoted by al-Mubashshir Ibn Fātik also evoke Christianity, most likely due to their dependence on the Christian Hunayn ibn Ishaq.[75] As a result, Western translators had no difficulty transcribing these maxims into a Christian language and we can thus understand why they were so successful in the West.

The chapter on Pythagoras begins with a presentation of the general principles of his ethics:

70 Crombach 1971.

71 Franceschini 1931–32.

72 For the French and Occitan versions, see Brunel 1939. For the English one, see Bühler 1941.

73 Billanovich 1948 and 2004.

74 As an example, the Latin text has more than 200 occurrences of the word 'Deus' (singular form with all its declensions) and 22 in the chapter on Pythagoras.

75 For instance, one of Pythagoras' sentences (*decet scire horam in qua sit conveniens loqui et in qua tacere*) alludes to the Ecclesiastes (3:7) ("there is a time to keep silent and a time to speak").

> Pythagoras saw that it was good to neglect the world and to serve God, and recommended sanctifying the senses (*sanctificari sensus*), practicing justice and other good things, doing one's utmost to know the truth about everything, loving men, using beautiful things, fasting many times, pursuing studies, ensure that mothers teach mothers and women teach women, speaking with elegance and preaching. He also said that the soul is eternal, able to receive what it merits and its penance (*apta recipere meritum et penam*). He ate with moderation, so that he never was overweight or skinny. Moreover, he had a very discerning soul, he liked to treat his friends well before treating himself well, saying that property should be shared among friends and that they should express their science in an allegorical way (*dicere per allegoriam*). He put together 280 books. On his seal was written: it is better to choose an evil that does not last than a good that does not last. And on his belt was written: silence reassures the penitent's reason.[76]

This preliminary summary takes well-known themes from Pythagorean ethics, as it is summarized in Diogenes Laertius' doxography, in Porphyry or in the *Golden Verses*. We first of all notice the importance given to the theme of obedience to God, and more precisely to a single God who is provident and rewarding. For, as the text indicates, the immortal soul will be judged based on its merits (a point which, incidentally, poses some problems for the Pythagorean theory of the transmigration of souls). Naturally, these few items make all of Pythagoras' theories acceptable in the eyes of a believer, as it is the minimum required to be a respectable pagan in the minds of many medieval thinkers.[77]

The next part of the text adds that this closeness sought with God is possible only under several conditions. The first is a radical form of asceticism that basically involves turning to purely intellectual knowledge, directed towards God, if possible in silence. For only a pure soul (*anima pura*) can get close to God. The second condition concerns social life, which the individual cannot totally avoid. Unable to live as a pure, silent spirit completely focused on the higher sciences, the individuals must love their peers, and share their belongings and their knowledge with their friends. Within this community of friends, knowledge must be shared in allegorical form, in other words using enigmas and symbols (*symbolae*). Outside of this community, knowledge must be spread through the society in the form of a sermon. The third condition concerns the

76 *Liber philosophorum moralium antiquorum*, in Franceschini 1931–1932, 427–428 (my translation).

77 See Marenbon 2015.

body and more specifically the diet. We can thus see a clear hierarchization of the precepts, from the highest ones, concerning the relationship between the pure soul and God, to the lowest ones, which are related to the body, and in between the ones that govern social life, in other words that of the embodied soul.

Each maxim presented after this prologue illustrates one of these conditions, though some of them evoke a sort of wisdom that is not specific to Pythagoreanism. Concerning the relationship with God, the first maxims that open this long collection quite clearly indicate the main components of Pythagorean theology.

> And he said: just as the beginning of our creation comes from God, our souls should return to God.
>
> And he said: when you wish to know God, no longer think of knowing men.
>
> And he said: the sage does not tell himself that God is honored by his eloquence, but by his deeds.
>
> And he said: wisdom consists in loving God, and in loving God, taking actions that God loves, and in taking actions that God loves, acting as if He was an end; for he who is good for Him is close to Him.
>
> And he said: God is not honored by what He is given in sacrifices, but by our wills, which He can accept.
>
> And he said: he who speaks much about <God> proves that he knows little about Him.
>
> And he said: each time you do a good deed, of the body or the soul, remember that God sees you in all your actions and thoughts; so you must feel shame in His eyes.
>
> And he said: God knows the man who is wise and fears God, which is why you will not be troubled if men do not know you.
>
> And he said: God has no place on Earth that suits him better than a pure, clean soul.[78]

Further on, he also says that "it is better for a man to be thrown into a bed of fire while believing in God than to be thrown into a bed of gold while doubting his existence."[79] Clearly, the principles expressed in these maxims are compatible with the three monotheistic religions. As well, we find in them certain items that have already been seen in the other Latin gnomological sources,

78 *Liber philosophorum moralium antiquorum*, in Franceschini 1931–32, 428 (my translation).

79 Franceschini 1931–32, 429.

such as separation from the world, the primacy of the soul over the body, of will over knowledge, of deeds over words, as well as the need to love God and to obey His law.

According to several maxims, ethical judgement must be directly linked to action, and even more to the will that governs our actions and, above all, to knowledge of the cause. For the all-knowing God watches our actions, and is above all the guidance of our will. It would, however, be erroneous to see in these prescriptions a form of pure ethical voluntarism, insofar as intellectual knowledge also plays a fundamental role in certain maxims. However, language is of little or no use in this respect[80] and this is the reason why Pythagoras, ideally, advocates giving up speaking in order to get as close to God as possible. If we must speak, we should stick to noble topics, otherwise we should keep quiet and listen to those who have noble things to say.[81]

Alongside this ideal theoretical life entirely focused on God, other maxims intend to govern the daily life of men in society. For example, several of them teach readers not to trust feelings when solving everyday matters, and thus to stick to intellectual thought, to temper the pleasures of alcohol and sex, and never to lie. As regards friendship, which is the foundation of Pythagorean sociability, we learn that friends must not only follow the above-mentioned rules but must also observe others that pertain directly to this singular relationship. For example, in friendship, one must always accuse oneself rather than one's friends. We also learn how to choose friends: they must be chosen for what they are, and not for what they have and, in addition to this, they must faithfully follow the truth.

In conclusion to this brief overview of the contents of the *Liber philosophorum moralium antiquorum*, it can be said that the ethics and way of life of Pythagoras described in it are very similar to what was presented in the Latin gnomological collections. However, in the theological maxims found in this collection, the Pythagorean God appears even closer to the God of monotheistic religions. And, indeed, the community of friends and life of silence inevitably evoke monastic life.

Before closing this chapter on Arabic sources, we must mention another text, the *Nawādir al-falāsifa* (*Anecdotes of Philosophers*) by Hunayn ibn Ishaq, which was translated into Castilian at the beginning of the 13th century with the title

80 One sentence says (Franceschini 1931–32, 433): "Priusquam alloquaris Deo facias opera grata sibi." (Before you speak to God, do something to show your gratitude.)

81 One sentence says (Franceschini 1931–32, 428): "decet hominem de rebus nobilibus loqui, et si non sit ei possibile, audiat loquentes de eis." (It is appropriate for a man to talk about noble things, but if this is not possible for him, let him listen to those who can talk about them.)

El libro de los buenos proverbios.[82] Though this text was not as successful as the *Bocados de oro* – it was not translated into other languages – it is an interesting compilation in terms of its content, as among the sentences attributed to Pythagoras, some are identical to those brought together by al-Mubashshir Ibn Fātik, but others are specific to this collection and have, for the most part, an equivalent in the *Golden Verses*. Apart from these few differences, we find the same insistence on obedience to God. In the first maxim quoted, we can read that the first of our duties is to obey God and his angels. We also find considerations on friendship and on the role of truth in ethics. The existence of this other source translated into a vernacular language encourages us to consider the importance that Pythagoreanism had in secular milieux, outside of universities and humanist circles. Here again, a more in-depth investigation of the traces of these Pythagorean sentences in the vernacular literature of the Middle Ages and the Renaissance should be undertaken.

6 Pythagorean Asceticism and the Monastic Ideal: The Early Reception of the *Sentences of Sextus*

To these collections of apothegms attributed to Pythagoras must be added the collection of 451 sentences known as the *Sentences of Sextus* (*Sextou gnōmai, Sententiae Sexti*), also called "the Pythagorean," whose Greek version, dating back to the 2nd century AD, was translated into Latin between the end of the 4th and beginning of the 5th centuries by Rufinus of Aquileia, who was Jerome's opponent on the use of pagan philosophers, as we have seen above.[83] This collection of sentences is a Christian adaptation of Pythagorean material, which can be found in part in other collections under the titles *Pythagorean Sentences* and *Sentences of Clitarchus*, widely known in Antiquity, as Porphyry seems to use them in his *Letter to Marcella*.[84] As Henry Chadwick underlined in his edition of the *Sentences of Sextus*, the content of these maxims is in accordance with several themes of importance to Christian authors of the Late Antiquity, and parallels with Biblical texts can even be found.[85] The attribution of these *Sentences* to Pope Sixtus in some texts probably played a role in the spreading of this collection of *dicta* among the Christians. Origen also quotes

82 Knust 1879 (61–64 for references to Pythagoras).

83 For the Latin text with the English translation of these sentences, see Wilson 2012. There are also Syriac, Armenian and other versions of this text. For a substantial overview, see Bouffartigue 2016 and Prochenko 2018b.

84 See Rocca-Serra 1971.

85 Chadwick 1959.

one of these sentences in his *Contra Celsum*,[86] and we know that Jerome's attacks against Rufinus were motivated in part by the latter's Origenism. Indeed, Jerome feared that this false attribution might mislead Christians and that they would draw inspiration from these sentences thinking that they were Christian when in fact they were believed to have been written by a pagan named Xystus Pythagoreus (*Letters*, 133.3).[87]

Jerome attempts to show the differences that remain between the *Sentences of Sextus* and the Christian doctrine. They present man as a divine being, just like God himself: "In this book much is said of perfection in accordance with the doctrine of the Pythagoreans who make man equal to God and maintain that he is of God's substance."[88] This collection is thus at the very heart of the dispute between Christians about orthodoxy and heterodoxy. What is more, while it is true that these sentences were used in Pelagian circles,[89] traces of them – or, in any case of their model – can also be found in Porphyry's *Letter to Marcella*.[90] Jerome was so worried about the influence of these sentences because they were not only popular, but their content, despite slight changes by the Christian compiler, showed the deep resemblances between pagan and Christian asceticism.[91] Some commentators, for example Irini Fotini Viltanioti, feel that "there is nothing spectacularly Christian or pagan in Sextus' collection."[92] This is entirely true when looking at the themes addressed, which are shared by Christians and pagans alike, whatever their school. Nonetheless, there is a particular resemblance between Pythagoreanism and Christian asceticism in this collection.[93] As Pevarello showed, this link can be seen in the radical nature of certain maxims concerning the ideal of voluntary poverty and the silent life. Likewise, the sexual morality of the *Sententiae Sexti* is particularly radical. Beyond celibacy and sexual abstinence, they go so far as to champion self-castration. It is thus not surprising to find some of these maxims in the rule of Saint Benedict adopted by the Benedictines.[94]

86 Origen 2001, *Contra Celsum*, VIII, 545.

87 Pevarello 2013, 1.

88 Quoted in Pevarello 2013, 21.

89 Evans 1968, 48.

90 Viltanioti 2016. Other commentators, including Chadwick (1959), think that this is implausible, and that Poprhyry had another collection of sentences in front of him, which could be the common source of the *Sentences of Sextus*, the *Pythagorean Sentences* and the *Sentences of Clitarchus*.

91 On this, see the comments in Ramelli 2016.

92 Viltanioti 2016, 167.

93 See Jordan 1961.

94 See the comments in Joost-Gaugier 2009, 117.

Although several medieval manuscripts of the *Sentences of Sextus* have been preserved, produced between the 9th and 15th centuries, the use of these maxims seems to have been relatively limited during this period. Some scattered quotations of them can be found in Peter Abelard's *Theologia christiana* and John of Salisbury's *Policraticus*, and sentence 231 is found in Peter Lombard's *Sentences*, each time through Jerome's text.[95] Despite the absence of explicit use of these sentences, they were known and confirmed the image of Pythagoras' school that had been conveyed by the other collections of Pythagorean sentences.

7 The Lives of Pythagoras

Starting in the 13th century, the literary genre of collective biographies gradually regained its former glory. Alongside the lives of saints, popes, famous men or heroes of the Antiquity, we find several collections of biographies of philosophers, well before the Latin translation of Diogenes Laertius' *Lives and Opinions of Eminent Philosophers* by Ambrogio Traversari in 1433[96] and the rediscovery of Iamblichus' or Porphyry's biographies of Pythagoras. These were sometimes inserted in large encyclopedias or universal chronicles, such as those by Helinandus of Froidmont or Vincent of Beauvais, and in some cases brought together in collections exclusively dedicated to philosophers, such as those of John of Wales, Benzo of Alessandria or the Pseudo-Walter Burley.[97] Pythagoras holds an important place in these works, in particular when they are entirely dedicated to philosophers. For example, in his *Compendiloquium de vita et dictis illustrium philosophorum*, John of Wales (d. 1285) distinguishes a sub-category of philosophers, more important than the others – i.e., nearer to the Christian truth – in which Pythagoras plays a leading role.[98] As well, the anonymous 13th-century author of *Fiori e vita di filosafi e d'altri savi e d'imperadori* begins his work with a profile of Pythagoras.[99] The same is true of Benzo of Alessandria's (*ca* 1250 – *ca* 1329) *Liber dictorum et factorum memorabilium philosophorum et aliorum virorum illustrium Grecie*.[100] While these Lives are in most cases simple compilations of older sources, we can nonetheless discern

95 Evans 1983; Pevarello 2013, 35–36.
96 On this translation see most recently Kaiser 2019.
97 For an overview of this literature, see Piaia 1983, Ricklin 2005 and Petoletti 2006.
98 On this division among ancient philosophers, see Ricklin 2006.
99 *Fiori e vita* 1979.
100 Benzo of Alessandria 2000.

in them a new will to present as a true model of virtue, the secular equivalent of the saints of Christian hagiography.

Among the models for this literary genre is Jerome's *On Illustrious Men*, whose style was endlessly copied, by writers ranging from Isidore of Seville (6th–7th centuries) to Sigebert of Gembloux (11th century). It is clear, however, that neither Jerome nor Isidore of Seville in his *Book on Illustrious Men* mention philosophers. The biographical indications related to ancient philosophers are found elsewhere, namely in the *Chronicles* that recall the main historical events since the beginning of the world. For example, in his *Universal Chronicle*, Isidore of Seville mentions a few philosophers – Thales, Pythagoras, Hippocrates, Plato, Aristotle, Xenocrates, Theophrastus and Cato – in most cases describing them in a single sentence. Thus, Pythagoras is simply presented as the inventor of arithmetic.[101] The same type of erratic glances can be found in several medieval encyclopedias such as Rabanus Maurus' *De universo* in the 9th century, in the chapter entitled *De philosophis*,[102] or in Otto of Freising's *Chronicon* in the 12th century.[103]

A first change can be observed at the beginning of the 13th century with Helinandus of Froidmont's *Chronicon* (compiled between 1211 and 1223), of which only the first books have come down to us.[104] Very fortunately, the preserved parts include notes on the lives and works of several ancient philosophers, among them a Life of Pythagoras. This text is particularly important, as it is the main source for the later tradition. Indeed, most of the information brought together by Helinandus of Froidmont will be copied by the Dominican Vincent of Beauvais in his *Speculum historiale* (see below), a fact that led to it being circulated more widely.[105] Later biographies only rearranged this initial material, taking away some items or adding new information.

The Life of Pythagoras in Helinandus of Froidmont's *Chronicon* is a patchwork of quotations taken from classical literature (Solinus, Valerius Maximus, Justin, Cicero and Seneca), the writings of the Church Fathers (Tertullian, Jerome, Augustine and Isidore of Seville) as well as some of his own contemporaries (Hugh of Saint Victor). After having situated Pythagoras in the universal timeline, the first part of the Life is devoted to Pythagoras' exile from his

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101 Isidore of Seville 2003, 83.

102 Rabanus Maurus 1852, 413B–419A.

103 Otto of Freising 1912, II, VIII, 75.

104 This text has not been published yet, but fortunately one can find a description of its chapters in Paulmier-Foucart 1986 (see page 215 for the life of Pythagoras).

105 For example, the anonymous 13th-century *Fiori e vita di filosafi e d'altri savi e d'imperadori* (see the introduction in *Fiori e vita* 1979) is an adaptation of Vincent of Beauvais' *Speculum*.

home island, Samos, until his arrival in Italy, in Magna Graecia. This account, quite similar to what we can read in Diogenes Laertius, is in large part taken from the summary of Pompeius Trogus' *Philippic Histories* made by the Roman historian Justin.[106] In the context of the *Chronicon*, its main role is to describe Pythagoras' personality. In Samos, he was by far the most noble of the citizens; in Croton, he spurred the pleasure-seeking population to embrace frugality, always singing the praises of virtue and tirelessly teaching the love of study; in Metapontum, he was so beloved that a temple was built in his honor. His history continued with the learning of magic in Egypt and Persia, then in Greece, with the well-known episode in which, when asked if he was a wise man, Pythagoras is said to have chosen to answer that he was only a friend of wisdom, thus inventing the word "philosophy" to describe his activity. In Italy, finally, he founded his school, which drew a number of students, deeply transforming thought and practices in the Magna Graecia.

After describing the character of Pythagoras through his peregrinations, Helinandus quotes the precepts and *aenigmata* reported by Jerome in *Contra Rufinum*. However, unlike the textual tradition of the *aenigmata Aristotelis* studied above, Jerome's entire passage is quoted, including the passage originally quoted in Greek by Jerome, which our author attempts to transliterate into Latin, unsuccessfully, showing that he knew nothing at all about this language. Whatever his linguistic skills, this long quotation enables Helinandus to briefly present Pythagoras' philosophy:

> Pythagoras is the first among the Greeks who discovered that souls are immortal and pass from bodies to bodies. He said that he had himself been first Euphorbus, and then Callides,[107] thirdly Hermotimus, fourthly Pyrrhus, and lastly Pythagoras; and that after certain revolutions of time, those things which had existed came into being again; that nothing in the world should be thought of as new; that philosophy is a meditation on death; that its daily struggle is to draw forth the soul from the prison of the body into liberty; *mathêseis anamnêseis*, that is: our knowledge is recollection, and many other things which Plato works out in his books, especially in the *Pheado* and *Timaeus*. For Plato, after having formed the Academy and gained innumerable disciples, felt that his philosophy was largely deficient, and went to Magna Graecia, and there he learned the doctrines of Pythagoras from Archytas of Tarentum and Timaeus

106 Justin 1853, XX, 166–171. Cf. Macris 2018, 733–734.

107 *Sic* for the "Aethalides" of the Greek sources.

> of Locris, and combined his teachings with the elegance and irony of Socrates.[108]

While the transmigration of souls could seem contrary to Christian doctrine, Jerome's quotation helps to present Pythagoras as the first Greek philosopher who conceived of the soul as immortal, which should appear less harmful to Christian dogma than the assertion that the soul is mortal.[109]

The text ends with two *exempla*. The first one comes from Seneca's *De beneficiis* and tells how a disciple of Pythagoras who had to pay a debt to a recently deceased shopkeeper decided, driven by his guilty conscience, to pay off the debt despite the advantageous situation he was in. The second is reported by Valerius Maximus and tells the well-known story of the friendship between two followers of Pythagoras, Damon and Pythias,[110] who together fought off Denys the Tyrant. It clearly appears that these two *exempla* are used here to illustrate two fundamental Pythagorean virtues: examination of conscience – here with regard to money – and friendship as a political lifestyle.

It is striking to see how the portrait of Pythagoras is positive from beginning to end and mainly underscores the beauty and greatness of his moral virtues. His life, as told to us, gives us the account of a conversion to philosophy, in other words to a wisdom that gives precedence to the soul above the body, to moral conscience above material advantages, to virtue over pleasures and suffering. The esoteric reputation of his teachings is linked with a time during his intellectual training spent with the Egyptians and the Persians; however, Pythagoras was able to move beyond their hermetic or magical teachings in order to found philosophy. Likewise, when the transmigration of souls is mentioned, it is in order to underline the positive side of this concept, namely the assertion that the soul is immortal. Pythagoras' morality, like his philosophy, thus represents both the cutting edge of pagan thought, as it were, every time it closely resembles the principles of Christianity, and its limits, by virtue of its ignorance – for philosophy is not wisdom, but merely love of wisdom.

Shortly afterwards, the Dominican Vincent of Beauvais (d. *ca* 1264) copied Helinandus of Froidmont's *Chronicon* nearly word for word in his *Speculum historiale* (*The Mirror of History*), simply leaving out a bit of information and

108 Helinandus of Froidmont, *Chronicon*, MS Vatican, Reg. Lat. 535. My translation is partly inspired by the English translation of Jerome 1886, 538 (the first sentence is not found in this translation but it is present in the original Latin text; see Jerome 1982, 318).

109 See Irene Caiazzo's paper in this volume for medieval discussions of the Pythagorean metempsychosis.

110 *Sic* for the "Phintias" of the Greek sources, on whom see above, n. 19.

adding sparse details.[111] For example, he added two quotations taken from Jerome, one indicating that the successors of Pythagoras had gotten into the habit of living alone in the desert (*Contra Jovinianum*), and the other that they had to remain silent for five years during their initiation to the Pythagorean doctrine (*Super Ecclesiastem*), thus drawing clear parallels with Christian monasticism. Unlike Helinandus of Froidmont, Vincent of Beauvais specifies the use that can be made of these ancient models in his *Libellus totius operis apologeticus*, which is used as a preface to the *Speculum maius*.[112] The Christian, he explains, has nothing to fear from these pagan *exempla*, as they show the extent to which philosophers contradict one another, a sign that they have not attained the truth, which is necessarily one. This is why Vincent leaves the reader the freedom to choose what seems to them to be the most similar, or at least the most adaptable, to Christian morality. It is not about borrowing a truth from the pagans, but merely imitating and even surpassing their virtues.

The true change in this biographical and doxographical literature came in the 1260s, with the Franciscan John of Wales' *Compendiloquium de vita et dictis illustrium philosophorum*.[113] Here, for the first time, the biographies of philosophers were dealt with separately, as in Antiquity, and no longer scattered throughout the long course of universal history. As mentioned above, John of Wales ranked Greek philosophers in two categories, the *majores* and the *minores*. The philosophers in the first group, which Pythagoras was part of, were described in long, detailed texts, whereas those of the second group were mentioned in shorter notes. As Thomas Ricklin has shown, the structure of the Lives of the great philosophers echoes the canon of the lives of saints, in particular Bonaventure's *Legenda maior*, and more broadly the Lives of Saint Francis.[114] Even more so than in Helinandus of Froidmont and Vincent of Beauvais, this is thus a form of philosophical hagiography, the goal of which is quite similar to that of the Lives of saints, namely providing *exempla* to the Christians.

Nonetheless, John of Wales did not set out to make these ancient philosophers into true saints. For, although pagan virtues are genuine virtues, they will always be missing the decisive contribution of the Revelation and the faith that goes along with it. Using, in the prologue, an image dear to Seneca in his *Letters to Lucilius*, John of Wales encourages the reader to make honey of these Lives of ancient philosophers.

111 Vincent of Beauvais 2018.
112 Vincent of Beauvais 1979.
113 John of Wales 1496. On this text see Ricklin 2006.
114 Ricklin 2006.

> On the model of a bee picking flowers, we have collected the notable words of the famous pagan philosophers, the material we can preach, and the examples we can imitate, by protecting ourselves in this collection from harmful errors. For this reason, we can call it without incongruity *Compendiloquium de vita et dictis illustrium philosophorum*.[115]

The Christian can thus gather the nectar from these pagan *flores*, not so much to accumulate new knowledge or attain a hidden metaphysical truth as to transform and enhance their own morality. The deeper justification is found in a passage from *The City of God* in which Augustine asserts that Christians should blush with shame for behaving less well than pagans who have not received the Revelation.[116] It is thus not, strictly speaking, a collection of introductions to the philosophers of the past, but a pool of examples and words to be imitated in order to better oneself.

Roger Bacon (d. *ca* 1292), a Franciscan who was almost the contemporary of John of Wales, shows in an even clearer manner this argument known as the *quanto maius* in his *Compendium studii philosophiae*. In his view, not only was the life of pagan philosophers often better than that of many Christians, but, even more seriously, "we cannot understand the wisdom of the philosophers, for we do not have their moral standards."[117] If I correctly understand the meaning of this sentence, imitating pagan philosophers not only helps us to act virtuously, but also gives us access to their wisdom which is practical and not only purely theoretical.

In reality, the so-called *quanto maius* argument runs through the backdrop of this entire body of literature, from Helinandus of Froidmont in the 13th century until the Renaissance. For that matter, Ambrogio Traversari in the 15th century writes exactly this in his letter of introduction sent to Cosimo de' Medici, attached to his translation of Diogenes Laertius' *Lives and Opinions of Eminent Philosophers*:

> In these biographies you will find powerful words and actions matching them, so that thanks to these books not only will the indisputable truth gain credit, but, what is more, our religion will discover examples that encourage virtue. Indeed, it is shameful to see a Christian, who has counted on his God and can have infallible hope in his salvation, nonetheless refuse to practice virtue and restraint, and to learn at the same

115 John of Wales 1496, f. 167ra.
116 John of Wales 1496, f. 170ra.
117 Roger Bacon 1859, 401.

> time that the pagans, so remote from the religious respect of the true God, sought with greater zeal integrity, moderation, frugality and other adornments of the human spirit. Most of the examples are near this perfection – which I should say is evangelical – and thus bring the greatest shame upon us and make us blush, if it is true that the philosopher of Christ shows less perfection than the philosopher of the world and the love of vain glory can do more in the heart of a pagan than the religious feeling in a Christian soul.[118]

To what extent can Pythagoras be a model for the Christian? In the prologue to his *Communiloquium*, a work that precedes the *Compendiloquium* in the 15th-century edition, John of Wales reminds us that

> it is not only the apostles and the Doctors of the Church who teach virtue, but also the pagan philosophers, as say Trogus Pompeius, in the book on Pythagoras, and [John of Salisbury] in *Policraticus*, book VII, chapter 4: when he [*sc*. Pythagoras] heard that the city of Croton had totally fallen apart and lacked virtuous works, he came and taught women separately from men, and children separately from parents; he taught them reserve and obedience to men, and to these latter he taught modesty and the study of letters [...] and thus, thanks to the assiduousness of the debates, he led the people back to cultivating a better life.[119]

Pythagoras and his tireless preaching thus constitute the very example of the pagan philosopher that Christians could take as a role model.

This framework having been laid, the Life of Pythagoras contained in John of Wales' *Compendiloquium* includes most of the information already found in Vincent of Beauvais, but rounded out by new sources such as Aulus Gellius, Macrobius, Virgil, Ambrose of Milan, Cicero, Seneca and John of Salisbury's *Policraticus*. Once again, Pythagoras' exile is the opportunity to recall his *auctoritas*, his *honor* and his *honestas*. John of Wales thus adds a few details. He reminds readers that some people believed that Numa, the king of Rome, was a hearer of Pythagoras, as Cicero reports in the *Tusculan Disputations* (IV, 1), which shows the prestige Pythagoras enjoyed during Roman times. Moreover,

118 Diogenes Laertius 1566, 4–5 (quoted in Domański 1996, 101). The *quanto maius* argument was abandoned by some humanists from the 15th century onward. See Hankins 2006 for the case of Socrates.

119 John of Wales 1496, f. 1vb-2ra. – On the Greek sources for Pythagoras' "preaching" in Croton, see Macris 2018, 704–705, 722–724, 727–728, 733–734, 799–800, 812, 824–825.

John of Wales insists on the nature of the Pythagorean community, on the role of friendship, on Pythagoras' charisma and on the importance of silence in the learning of the school's principles. Based on Macrobius and Cicero, he also adds a long passage on the role of music in Pythagorean thought. Despite these clarifications, the image of Pythagoras conveyed by this text remains similar to that found in earlier collections. Thanks to the Samian sage, Greek philosophy had arrived in Italy even before the founding of the great schools of Athens and had continued to exert a strong influence in Rome.

Among the noticeable differences of the *Compendiloquium* compared to earlier texts, we can first notice the absence of the *aenigmata*. While John of Wales does quote the first precepts of Jerome's text, he leaves out the enigmatic statements, either because he deemed them too obscure or because he wished to stand apart from the Dominican tradition. In any case, when he quotes the end of the passage from *Against Rufinus* on the transmigration of souls, he adds two quotations, one drawn from Virgil's *Aeneid*, the other from Seneca's *Letter* 90, and confirms that it is a philosophical mistake (*error*). Immediately after having presented this doctrine as an error, he adds a new paragraph entitled "The contempt for the gods" (*De contemptu deorum*) in which he defends Pythagoras against some accusations. Indeed, he recalls that the anecdote according to which "Pythagoras sacrificed an ox to the Muses when he had made a new discovery in geometry" is false according to Cicero.[120] He adds to the passage that God has undoubtedly revealed many truths to Pythagoras, but that without the grace of faith, he was mistaken regarding certain things, in particular metempsychosis.[121] In other words, Pythagoras made a mistake – which is understandable from the Christian point of view, insofar as he had not received the Revelation –, but at least he did not scorn God and did not make sacrifices to the pagan gods. In addition, his life illustrates many virtues that some Christians are incapable of attaining, despite their faith.

John of Wales' text greatly influenced the later tradition, and although few manuscripts of the *Compendiloquium* are extant, the 15th-century edition earned him a certain level of success during the Renaissance. After him, the genre of Lives of philosophers became extremely popular in the 14th century, particularly in Italy. Among the most renowned collections, we must mention Benzo of Alessandria's *Liber dictorum et factorum memorabilium philosophorum et aliorum virorum illustrium Grecie*, which is book 24 of his *Chronicon*.[122] Though largely unknown, it provides an interesting account, insofar as, unlike

120 Cicero 1933, 372–375.

121 John of Wales 1496, f. 206ra–rb.

122 Benzo of Alessandria 2000.

the usages of the 13th-century *Chronicles*, the succession of biographies of these eminent men is handled separately, as in John of Wales, who was the main source of inspiration for Benzo of Alessandria. The particularity of this text is that it sticks to illustrious men of ancient Greece, whether they are philosophers, doctors or poets. Benzo had initially planned to continue this book with the Roman philosophers, but this work never saw the light of day.

In the prologue, Benzo of Alessandria emphasizes above all the importance of having all of these Greek models together, in a single book, as if it were necessary to hoard the entire body of pagan wisdom. Yet the argument that underpins his undertaking is once again that of the *quanto maius* and the philosophers are there to provide a pool of examples. As for the content, his Life of Pythagoras reincorporates items from Vincent of Beauvais' *Speculum* that had been left out by John of Wales and adds a few new items compared to the latter's *Compendiloquium*. One new item concerns Pythagoras' youth, which he is believed to have spent in poverty, forced to perform manual work before being able to study the liberal arts. In fact, this anecdote, taken from Aulus Gellius' *Attic Nights*, concerns Protagoras and not Pythagoras.[123] Yet this erroneous reading clearly indicates the will to make Pythagoras a more popular figure. This is why Benzo attempts to confirm this information using a quote from Apuleius suggesting that Pythagoras' father was not rich, contrary to what other biographical accounts claim about him.[124] He also strengthens the part of his narrative concerning the reverence towards Pythagoras after his death by adding a passage from Flavius Josephus' *Against Apion* relating something that Hermippus had written in his book on Pythagoras (a book that Diogenes Laertius also quotes): after the death of one of his disciples named Calliphon, Pythagoras is said to have pretended to remain in contact with his soul.[125] While this account was originally intended to substantiate the theory of metempsychosis, or more precisely the possibility of a communication with the souls of the deceased, the purpose of Benzo of Alessandria's *Chronicon* seems to have been to emphasize the relationship that Pythagoras maintained with his followers.

With regard to Pythagoras' thinking, Benzo of Alessandria first of all reincorporates the *aenigmata* left out by John of Wales, and by way of a true innovation he inserts several sentences excerpted from the *Liber philosophorum*

123 Benzo of Alessandria 2000, 251–252. Aulus Gellius 1968, 192 (V, III, 1–7).

124 Benzo of Alessandria 2000, 252. Apuleius 2017, 285 (§15). On Pythagoras' father and his social background according to the ancient sources, see Macris 2018, 781–782; on Apuleius as a source, ibid., 751.

125 Benzo of Alessandria 2000, 257. Flavius Josephus 1961, I, 228–229. On this episode see Bar-Kochva 2010, 164–205, esp. 165–166, 184.

moralium antiquorum (which he here calls *De vita philosophorum*).[126] The main point he takes from this long doxography is that Pythagoras put his friends before himself when encouraging the sharing of property between friends. He also keeps the information indicating that Pythagoras is believed to have written 280 books, contrary to the frequent assertion that he had written nothing.[127] Next, he recalls that on Pythagoras' seal was written "it is better to choose an evil that doesn't last than a good that doesn't last" and on his belt "silence enables us to regret vice."[128] Benzo also recalls the words attributed to Pythagoras according to which in order to be free, one must make oneself a slave to honesty, in other words to virtue. On the topic of death, the Samian is also believed to have claimed that we can die anywhere, since our soul will in any case go to another world. Finally, to a young man who had difficulties learning, he is believed to have said: "Oh young man, if you do not suffer from the travails of learning, you shall suffer from the travails of ignorance." This more complete portrait of Pythagoras clearly reveals the foundations of his ethics: study, teaching, practicing virtue and strictly obeying its principles, and community life.

We finally arrive at the most well-known text of this tradition, the *Liber de vita et moribus philosophorum illiustrium*, which has long been attributed to the English philosopher Walter Burley (*ca* 1275–*post* 1344), but was in fact written in Italy, by an anonymous author, in the first decades of the 14th century. This book, transmitted by over 270 manuscripts and translated into several languages since the 14th century, enjoyed phenomenal success until the 16th century.[129] Particularly rich, it is one of the few texts from this era that quotes Diogenes Laertius' *Lives*, a century before Ambrogio Traversari's translation, thanks to a partial translation by Henry Aristippus (d. 1162) that was circulated on a very small scale and which we have no manuscripts of.[130] As Tiziano Dorandi has shown, this translation did not include the chapter on

126 Benzo of Alessandria 2000, 258.

127 On this assertion found in the ancient sources see Riedweg 1997. On the pseudepigraphic works attributed to Pythagoras in Antiquity, Macris 2018, 843–850. On the 280 books, Izdebska 2018, 861 (bibliography).

128 One may note that the sentence written on Pythagoras' belt as reported by Benzo is different from the one transmitted by John of Wales (quoted above). It is Benzo who is a bit closer to the Arabic original ("Silence protects against regrets", according to Cottrell's translation [2016, 497]).

129 On this text see Stigall 1956, Prelog 1983 and 1985, Grignaschi 1990, Dorandi 1999 and Ricklin 2005.

130 On the circulation of Diogenes Laertius' *Lives* in Latin (the so-called *versio latina antiqua* or *versio Aristippi* and the *versio Ambrosiana*), see Dorandi 2009, 201–228.

Pythagoras.[131] It was rather inspired by John of Wales. Indeed, Thomas Ricklin has shown the similarity of the Lives of Socrates contained in John of Wales' *Compendiloquium* and in the pseudo-Burley's *Liber*.[132] Fortunately, the Life of Pythagoras is the only one that today has a scholarly edition[133] and we can observe the same likeness with John of Wales, even though Pseudo-Burley's work retains a greater number of items from the tradition described above. Like Benzo of Alessandria, he reincorporates the items found in Vincent of Beauvais, in particular the *aenigmata*, yet also quotes certain sentences from the *Liber philosophorum moralium antiquorum* and even adds several maxims quoted in the Pseudo-Caecilius Balbus and other medieval anthologies. The work thus embodies the acme of this literary genre, with precision that remained unparalleled until the translation of Diogenes Laertius' *Lives and Opinions of Eminent Philosophers* in 1433.

A final change occurred during the era of Petrarch (1304–1374). Indeed, the scholar devoted two short notes to Pythagoras in his *Rerum memorandarum libri*, a thematic collection of *exempla* written *ca* 1364 and inspired by the model of Valerius Maximus' *Factorum et dictorum memorabilium libri*.[134] In this book, Petrarch seeks to illustrate the main virtues, distinguishing in every instance the Roman examples, the examples from outside Rome (for example the Greek philosophers) and the contemporary examples. The first passage mentioning Pythagoras is meant to illustrate his teaching (*doctrina*). As in the Lives of Pythagoras we have seen thus far, Petrarch builds on Justin's biographical account in order to claim that Pythagoras' first virtue was to have opted for exile, in other words for freedom as opposed to servitude, without ever having stopped studying and teaching. The second comprises the humility of having been able to break with the vain glory of the elders who claimed to be wise men because they were familiar with occult matters. In other words, Pythagoras' wisdom entailed accepting that he himself was not a wise man, but rather a simple friend of wisdom, thus embodying the humble posture that the philosopher must take. Petrarch concludes: "He was truly a great man and, as Cicero writes, he not only invented the name of philosophy, but also increased its content, adorning Italy with laws and excellent institutions, public and private alike." The second passage specifically concerns wisdom (*sapientia*) and uses virtually the same items as Justin's account, albeit this time underscoring the moral nature of Pythagoras' teachings in the various cities

131 Dorandi 1999.

132 Ricklin 2005.

133 Prelog 1990.

134 Francesco Petrarca 2014, I, §24, 62–67 and III, §70, 296–299.

he journeyed through. As we can see, the choice Petrarch made in the sources he had access to is distinctly narrower than in the biographical collections of the previous generations. In addition, whenever he had the opportunity, he harshly criticized Pythagoras' philosophy, in particular his theories on the reincarnation of the soul.[135]

Some of the writers close to Petrarch will follow his example in their own biographies of Pythagoras, summing up the latter's life in his exile. This is the case, for example, of William of Pastrengo (1290–1362) in his *De viris illustribus et de originibus*.[136] Others, like Giovanni Colonna (1298–1343), in his *De viris illustribus*, provide a longer Life of Pythagoras, albeit with items that can be found in the works of John of Wales or the Pseudo-Burley.[137] What is different in Colonna is the attitude towards pagan philosophers and their lives. In his *De divisione et laude philosophie quae ad mores pertinet*, which serves as a prologue to his *De viris illustribus*, he harshly criticizes the philosophers, their pride and their theoretical pretentions out of touch with any practical considerations, thus the discrepancy between their writings and their life. Taking themes dear to Augustine, Lactantius and Petrarch, Giovanni Colonna espouses the primacy of moral philosophy over the other branches of philosophy and the subordination of philosophy to theology. He thus concludes that the virtues of the pagans are false and imaginary, and that they do not have the same value as the goodness of "true philosophers," in other words those who have chosen to follow only God, i.e., the Christian God.

From the end of the 14th century onwards, the literary genre of collective biographies appears to have become poorer, to such an extent that their authors added no new information and even tended, in most cases, to leave out certain data, sticking to the most factual items regarding the philosophers' lives. More broadly, the humanists favored monographs dedicated to a few major ancient figures, such as Socrates, Plato, Aristotle, Seneca or Cicero, not collective biographies, which they preserved primarily for their contemporaries.[138] One may mention, *exempli gratia*, Leonardo Bruni's (*ca* 1370–1444) *Life of Aristotle* and Giannozzo Manetti's (1396–1459) *Life of Socrates*,[139] yet it is clear that biographies of Pythagoras of this new type are quite rare, with the exception

135 See, for instance, *Rerum Familiarium*, X, 3, quoted in Palmer 2016, 211–212.

136 William of Pastrengo 1991, 171–172.

137 I worked on the copy of the MS Vatican, Barb. Lat. 2351, f. 92r–94r. The prologue as well as the table of contents are published in Garfagnini 1980.

138 On these collective biographies, see Baker 2015 and 2017. Concerning the predominance of single Lives in the humanist tradition, this is also the case for the Lives of poets. See Bartuschat 2007.

139 On Manetti's *Life of Socrates*, see Hankins 2019.

of the very concise profiles sometimes found in the epigraphs of Latin translations of Pythagorean texts.[140] As regards chronicles, they continued to exist, albeit granting less and less importance to the *dicta* and increasingly emphasizing, like Petrarch, the biography *sensu stricto*. In the renowned *Chronica chronicarum* by the German scholar Hartmann Schedel (1440–1514), published in Nuremberg in 1493, the life of Pythagoras is summed up in a few lines: he invented philosophy and art music, used geometry to analyze reality and championed the transmigration of souls.[141] At the end of this brief note, after stating that the books by Pythagoras had been burned, he remarks that Diogenes Laertius quotes several of his sentences, yet he opts to quote Jerome.[142] In his *Supplementum chronicarum*, the Italian humanist Giacomo Filippo Foresti (1434–1520) simply repeats Hartmann Schedel's text.[143]

The advent of Diogenes Laertius' text did not fundamentally revolutionize the entries of these collections. For example, the humanist Giannozzo Manetti, the author of important biographies of Socrates and Seneca, wrote *ca* 1439 a treatise *De illustribus longaevis*, the fifth book of which is dedicated to the philosophers. On the whole, he follows the text of Diogenes Laertius' *Lives and Opinions of Eminent Philosophers*, drawing inspiration from the account yet not simply copying it.[144] Indeed, he makes a drastic selection of the items that he wishes to include in his text. The Life of Pythagoras, which appears as a short folio in the manuscript I was able to read, copies or rewords only a few biographical items from chapters 1 to 7 of book VIII by Diogenes Laertius and leaves out the entire doxographical section (chapters 8 to 49).[145] As regards ethics, the passages selected add absolutely nothing to the contents of the medieval biographies. The same can be said of the very short Life of Pythagoras found in Francesco Filelfo's *Epistula de opinionibus philosophorum* written in 1464.[146]

The situation seems to have changed little in the 16th century, at least in this literary genre. In the *Commentariorum urbanorum libri*, an encyclopedia of 38 volumes published for the first time in Rome in 1506, Raffaele Maffei takes

140 Palmer 2016 and 2017.

141 Hartmann Schedel 1493, f. LXIv.

142 Hartmann Schedel 1493, f. LXIv: "Varie eius feruntur sententie per Laercium, unam hic apponimus: fugienda, inquit, modis omnibus et abscindenda sunt hec omnia scilicet languor a corpore, imperitia ab anima, luxuria a ventre, a civitate seditio, a domo discordia et a cunctis rebus intemperentia."

143 Giacomo Foresti 1492, f. 54r.

144 Kahle 2017.

145 Giannozzo Manetti, *De illustribus longaevis*, MS Vatican, Urb. Lat. 387, f. 139v–140r.

146 The text is published in Hankins 1990, vol. 2, 515–523 (in particular 521–522). Filelfo merely decribes Pythagoras' travels.

virtually nothing from Diogenes Laertius' life of Pythagoras.[147] The information on his life is taken from Justin, as in the Middle Ages, and the author adds only the numerous reincarnations of Pythagoras, already presented in the Lives we have seen here. He quotes a single apophthegm, already known in the Middle Ages: Friends hold all things in common.[148] At the other end of the spectrum of this encyclopedic literature, the German humanist Hermann von dem Busche (1468–1534) writes almost nothing on Pythagoras' life in his *Spicilegium XXXV illustrium philosophorum auctoritates* published in 1506,[149] contenting himself with quoting a selection of *symbola* taken from the works of Jerome, Diogenes Laertius and Plutarch.

8 Concluding Remarks

To conclude this survey of the biographical and gnomological sources concerning Pythagoras' ethics in the Middle Ages, it can be said that there were many and that they enabled readers to gain a fairly clear sense of the main trends of his practical philosophy. It is true that from the Church Fathers to the end of the Middle Ages, choices were made in the older sources in order to make Pythagoras acceptable to Christians and these vary from one collection to another, from one philosophical biography to another. The image conveyed by the large body of gnomological material handed down in Latin, from Roman times or in medieval Latin translations, most likely does not faithfully reflect the teachings of Pythagoras. However, it is striking to note that in most of the anthologies, including when they are in the form of a biography, Pythagoras is described as a great wise man. Newly found sources and those translated during the Renaissance certainly rounded out this portrait yet did not fundamentally change the overall representation. Only few circles, such as those of Marsilio Ficino or Johannes Reuchlin, would take a closer look at Pythagoras' teachings and, in particular, his *symbola*. Elsewhere, the information disseminated remained quite similar to that which people had already been reading for several centuries.

From the 12th to 15th centuries, collections of *exempla* and Lives of ancient philosophers aimed to underscore, albeit implicitly, the moral weakness of Christians who were not even able to practice virtues as did the pagans, who were in fact unaware of the Christian God. In these pantheons of respectable

147 Raffaele Maffei 1506, f. CCLIVr-v.

148 Eden 1998 and Macris 2018, 841.

149 Hermann von dem Busche 1513, *sub voce* (no page number).

ancient philosophers, Pythagoras embodied not only an ethics of obedience to the law and to God, similar in certain respects to one part of the Christian tradition, but also the epitome of the monk and, particularly in the Lives, the very image of the preacher bringing moral teachings from city to city. Thus, the collections of Pythagoras' *exempla* and *dicta* served not only to provide material for scholarly works, but also provided clerics with a pagan mirror in which to contemplate, with shame, their own shortcomings.

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PART 3

Theology, Metaphysics and the Soul

CHAPTER 8

Pythagoras' Philosophy of Unity as a Precursor of Islamic Monotheism. Pseudo-Ammonius and Related Sources

Daniel De Smet

Doxographies reporting the opinions and sayings of the most famous sages from Antiquity were a popular genre in medieval Arabic literature.[1] However, with the exception of the *Placita philosophorum* attributed to Plutarch or to Aetius, the Arabic doxographies are generally not translations of known Greek texts, but rather genuine compilations, based on ancient sources, readapting and transforming the borrowed material in a Neoplatonic and, above all, Islamic perspective. At first glance, one gets the impression that the (often anonymous) authors of these doxographies had only a superficial knowledge of ancient philosophy and ascribed at random their own doctrines to renowned authorities of the past. A closer study however, reveals that their interpretations are far from arbitrary as they join the general trend of previous Christian and Neoplatonic authors in presenting the main philosophers of ancient Greece as true monotheists, believing in the one and unique God who is the ultimate cause of the production of the world.[2] This is in particular the case with the Arabic Pythagoras, whose alleged doctrine of *tawḥīd*, the profession of God's oneness, is deeply rooted in pre-Islamic (neo)-Pythagoreanism. In the following pages, we will try to follow the subsequent steps of the islamisation of Pythagoras' theory of numbers in the Arabic doxographical tradition.

1 The Doxography of Pseudo-Ammonius and the *Refutatio omnium haeresium*

The *Book of Ammonius Concerning the Opinions of the Philosophers and Their Different Doctrines about the Principles and the Creator* is an Arabic doxography whose purpose is to show that the elite among the ancient philosophers, although disagreeing about details, shared a common belief in the oneness

1 For a general overview, see Daiber 1990 and 1994.
2 On this general trend, see Wakelnig 2015, 205–214.

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of God, who created the world ex nihilo.[3] Compared with his colleagues, philosophers as famous as Thales, Xenophanes, Empedocles, Democritus, Anaximenes, Anaxagoras, Heraclitus or Parmenides, Pythagoras holds a most prominent position: "the doctrine of Pythagoras, the sage from Samos, is not based on personal opinion (*raʾy*), but is the result of spiritual contemplation (*iṭṭilāʿ rūḥānī*) of the worlds and their contents" (Ps.-Ammonius, XIV, 1). As the Prophet Muhammad will experience much later, by his celestial ascent (*miʿrāj*), Pythagoras rose through the sky and travelled from the world of Nature to the world of Soul and then, upwards, to the world of Intellect, seeing with his own eyes the forms and the lights contained in these worlds (XVI, 5).[4] Thus, he discovered that the universe has a harmonic structure, based on numbers and melodies (*luḥūn*), and that the different realms of reality (Nature, Soul, Intellect) are organized in a hierarchical order, increasing in perfection, beauty and brightness (XIV, 7–14; XVI, 7–10). But his ascension stopped at the level of the Intellect, which is, however, not the ultimate cause of the existence of the universe. He realized then that the path leading to God remains blocked forever.

In Italy (*Īṭāliya*), where he died, Pythagoras proclaimed that "the Creator (*al-bāriʾ*) is one (*wāḥid*) and that He can be perceived neither by the Intellect nor by the Soul; as He transcends the highest spiritual attributes, His essence (*huwiyya*) remains imperceptible." In the realms of Intellect, Soul and Nature, God can only be perceived through the traces (*āthār*) He leaves in each of them (XIV, 2–6). In other words, God is hidden in His essence, but reveals Himself through His actions.

Pythagoras' doctrine of *tawḥīd* – the attestation of God's oneness – is the result of his speculations about the numbers of the decade. The first number, one, is male, whereas two is female; subsequently, three is male and four female. These two pairs, coupled together [(1 + 2) + (3 + 4)], generate the number ten, the completion and achievement of all numbers. Similarly, the numbers one to four correspond respectively to the elements of fire (male), air (female), water

3 Transmitted in a single manuscript (Istanbul, *Aya Sofya* 2450), the text was edited, with an annotated German translation, by Rudolph 1989. The identity of its author "Ammonius," the existence of a (Christian) Greek or Syriac model used by the Muslim Arabic compiler and the date of composition remain open questions (Rudolph 1989, 11–15). It is likely that the work, in its actual form, dates from the second half of the 9th century; the first author who quotes from it was the Ismaʿili Abū Ḥātim al-Rāzī (d. 934); see De Smet 2014 and 2019.

4 According to the Ikhwān al-Ṣafāʾ 1957, I, 137–138, such an experience is quite common, as Ptolemy, Hermes, Aristotle (in fact Plotinus, as the author is referring to the Arabic *Theology* based on the *Enneads* but transmitted in the Arabic tradition under the name of Aristotle), Pythagoras, Jesus and Muhammad were all granted a celestial ascension.

(male) and earth (female); mixed together, they generate the world of nature in its perfection, reflecting the perfect number ten. In his books, Pythagoras clearly explained that the first two numbers are created (*mubdaʿa*), their Creator being the One, "the ultimate first above whom there is no first" (*awwal al-awāʾil wa lā awwal fawqahu*) (XV, 6–18, 20–21, 29–30). This means that God, as Ultimate reality, represents a unity transcending the number one.

A disciple of Pythagoras went to Persia and taught this subtle doctrine to the Magi, who misunderstood its meaning completely. A certain FLNJS, also called MRĪŪS,[5] identified the male principle, the monad, with light and the female dyad with obscurity; mixed together, they generated the universe. Another Magus, Zardusht (Zarathustra), claimed that the first two principles are two creating Gods, thus perverting Pythagoras' monotheism into dualism (XV, 1–5, 19, 25–28, 31–38). A second disciple of Pythagoras, an Italian named Kalanos, was more successful in India. He found in Brahman an intelligent and noble pupil, preaching the purification of soul and body. Unfortunately, after Brahman's death, the doctrine was corrupted and gave rise to two competing sects, preaching an anthropomorphic conception of God and an excessive form of asceticism, even promoting suicide (XVI, 1–3, 13–59).

A main source for Ps.-Ammonius is the *Refutation of All Heresies*, traditionally attributed to the bishop Hippolytus of Rome (first half of the 3rd century).[6] According to the Christian author, Pythagoras, a Samian exiled in Italy, identified God with the monad (μονάς). This first monad – the number one, which has no limit and cannot be comprehended[7] – is not generated (ἀγέννητον) and acts as the principle (ἀρχή) of all numbers; being male, he generates them as a father. The second number, the dyad generated by the monad, is female and functions as the mother of all subsequent beings. The first four numbers produce the perfect number ten (*Ref.*, I, 2, 1–2, 6–8; VI, 23, 1–3). The author of the *Refutatio* further explains that Pythagoras met Zaratas (Zarathustra) and learned from him the distinction between the monad and the dyad; he once mentions in his chapter on the Brahmans the gymnosophist Kalanos as

5 These names appear in a corrupted form in the manuscript. The passage is quoted in al-Shahrastānī's *Milal*, where the names could be read as Philonikos and Marzinūsh (Marinus?); see al-Shahrastānī 1961, t. II, 82; Id. 1993, 211.

6 New edition of the Greek text with English translation by Litwa in [Hippolytus of Rome] 2016; for the question of authorship, see ibid., XXXII–XL. Cf. the studies edited by Aragione and Norelli 2011. The parallels with Ps.-Ammonius are indicated in Rudolph 1989, 91–94, 98.

7 It seems here that the author of the *Refutatio* makes no distinction between the monad (μονάς) and the number one (ἕν); cf. Litwa in [Hippolytus of Rome] 2016, 13, note 21. However, this distinction is clearly present in Ps.-Ammonius, when he has Pythagoras declare that the unity of God transcends the first of numbers. On this distinction, see below.

a contemporary of Alexander the Great (*Ref.*, I, 2, 12–13; 24,7). The meeting between Pythagoras and Zarathustra, and the figure of Kalanos as an Indian Pythagorean are common *topoi* in ancient literature.[8] However, the idea that Zarathustra and Kalanos were direct disciples of Pythagoras may be an invention of the Arabic Ps.-Ammonius, in order to show that Persian dualism and Indian anthropomorphism are perversions of Pythagoras' sound monotheism.

2 Aetius Arabus

The account concerning Pythagoras in the *Refutatio* shares a number of elements with the *Placita philosophorum* attributed to Plutarch or Aetius.[9] The Arabic translation of the *Placita* by Qusṭā b. Lūqā (d. *ca* 912) is a further contribution to the islamisation of Pythagoras' thought. Here we learn that the Samian, settled in Italy, was with Thales one of the founders of philosophy, and even the "inventor" of the term *falsafa*.[10] He described the monad (*al-wāḥida*) as "the special efficient cause (*al-ʿilla al-fāʿila al-khāṣṣa*), which is God Most High and the Intellect" (*wa hiya Allāh ʿazza wa jalla wa-l-ʿaql*).[11] The dyad is a receptive principle, identified with matter (*ʿunṣur*), receiving the influences from the monad and producing the sensible world (*Aetius Arabus*, I, 3, 8).[12] As God and Intellect, the monad is Goodness (*al-khayr*), whereas the dyad, called "daemon" (*dhāmun*), represents evil (*al-sharr*) and contains the plurality of the material world (*Aetius Ar.*, I, 7, 18). This dualism reminds us of the erroneous interpretation of Pythagoras' theory of numbers by the Magi according to Ps.-Ammonius. Moreover, Pythagoras is said to believe in the generation of the universe: "the world is generated and it is God Most High who generated it" (*al-ʿālam mukawwan wa Allāh ʿazza wa jalla kawwanahu*). Being corporeal, the world is subject to corruption, but thanks to God's governance (*siyāsa*) and protection (*ḥifẓ*) it is kept in existence (*Aetius Ar.*, II, 4, 1–2).

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8 Rudolph 1989, 170–173, 176; Id. 1994.

9 Mansfeld 1992, 22–25 enumerates the similarities and concludes that both texts depend on a common source.

10 Same idea in Iamblichus' *Vita Pythagorica*, § 58.

11 By contrast, we have seen that in Ps.-Ammonius *Allāh* and the Intellect are clearly distinguished, as are the monad and the number one, the first of numbers.

12 Arabic text and German translation in Daiber 1980. The Arabic version gives the impression that the Dyad is an inferior principle generated by the Monad. In the Greek original (I, 3, 7), however, both principles seem to be on the same level; see the Greek text in Mansfeld and Runia 2020, 203–204, and the commentary 253–256. See also Mansfeld 2019.

3 Pythagoras, Leader of the Alchemists: The *Turba philosophorum*

The *Refutation of All Heresies* also served as a source for the author of the *Turba philosophorum*, an alchemical treatise composed around the year 900, and thus roughly contemporary with Ps.-Ammonius. The work, transmitted in a Medieval Latin translation, opens with 9 *sermones* in which Presocratic philosophers are discussing, under the guidance of Pythagoras, God and the creation of the universe. Ulrich Rudolph has discovered that the lecture given by Pythagoras himself (*sermo* 8) has only a remote connection with Pythagorean doctrine, as it is mainly based on a passage from the *Refutatio* (X, 32–33) which gives a short summary of Christian theology according to the author's interpretation.[13] This means that the *Turba* presents us with a Christianized, monotheistic Pythagoras, emphasizing the creation of the world by God. Thus, Pythagoras declares before the assembly of philosophers and alchemists that God existed before everything, and that no being is coexistent with Him. He created (*creavit*) first the four elements (fire, air, water and earth), and then, He created from them all things, both in the higher and the lower worlds.[14]

A fragment of the Arabic original of the *Turba*, which is, however, lacking in the Latin version, gives an Islamic adaptation, including references to the Qur'ān, of Pythagoras' theory of numbers as exposed in the *Refutatio* (I, 2, 6–8):

> Pythagoras said: I say to you, oh sons of learning, that God is one and alone; He has not generated and is not generated, and nothing is with Him (*Allāh wāḥid fard lam yalid wa lam yūlad wa lam yakun lahu shay'un*) [cf. Qur'ān, sura 112]. When He created [the imperative] 'be' (*kun*),[15] He created as first thing the one (*al-wāḥid*). This one is alone (*fard*) and masculine, being the beginning of calculation; the root (*aṣl*) of all things is one and nobody is able to attain knowledge about this one or about its root. Here, the erring nations are confused. But you, assembly of natural philosophers, you have to profess God's unity (*tawḥīd Allāh*) and to avoid doubts [about it]. Know that the root and beginning of calculation is the one, male and alone, and that all creatures proceed from this one. As for

13 Rudolph 1990, 108–114, 121; on the *Turba*, see Rudolph 2005, 164–167, and most recently Lacaze 2018.

14 Latin text with German translation in Plessner 1975, 72–73, 75.

15 According to the Qur'ān (for instance sura 2: 117), if God wants to create something, He says *kun* (the imperative of the verb *kāna*, "to be") and the thing comes into existence (*fayakūnu*, "and it is"). Hence, God's order (*amr*) is often considered in the Islamic tradition as the first created being.

> the two, it comes after the one and it is female. Both [the one and the two] form a composed pair.[16]

The Arabic *Turba*, following Ps.-Ammonius, makes a clear distinction between the uncreated unity of God and the created unity of the number one, which is the principle of all further numbers and beings in the universe.

4 The Oxford Manuscript Marsh 539 and the Tehran Doxography

A similar distinction is implied in a passage ascribed to Pythagoras in the anonymous doxography transmitted in the Oxford manuscript Marsh 539, which Elvira Wakelnig qualifies as a "philosophy reader" compiled in the circle of Miskawayh (d. *ca* 1030).[17] Pythagoras mentions here four divine numbers (*aʿdād ilāhiyya*), which are all good and devoid of evil; they are identified as the intellect, the soul, the [first] form, and "the beneficent power which brings all beings to perfection." They are opposed to the four numbers of nature (*aʿdād al-ṭabīʿa*), which are devoid of good, and correspond to nature, matter (*hayūlā*), the second form, and time with its motions. Their ultimate principle is

> [the] One, which is self-subsistent (*qāʾim bi-dhātihi*). It is the first goodness from which appear the four numbers; from their movements, [the One] generates (*kawwana*) the other four [numbers]. The first One is the head (*raʾs*) of the eight numbers, their mover (*muḥarrik*), their organizer (*mudabbir*), their sustainer (*muqawwim*), their protector (*ḥāfiẓ*) and their creator (*bāriʾ*).[18]

A second passage attributed to Pythagoras in the same collection confirms the distinction between the First (*al-awwal*), which is simple, one, pure, different from all things and devoid of every form of duality, and the intellect in this world, which is pure, separate, and acting as the agent (*al-fāʿil*) for all that is contained in it.[19]

Wakelnig discovered recently in a Tehran manuscript a doxography entirely devoted to the sayings of the ancient sages concerning God's unity and

16 The Arabic text was published, with a German translation, in Ruska 1931, 297, 300–301; cf. Rudolph 1990, 116.
17 Wakelnig 2014, 1–7.
18 Arabic text and English translation in Wakelnig 2014, 66–67.
19 Wakelnig 2014, 78–79.

uniqueness, entitled *The Most Precious Words of the Philosophers Professing God's Oneness and of the Authorities of the Past* (*Nawādir min kalām al-falāsifa al-muwaḥḥidīn wa-l-aʿlām al-māḍiyyīn*).[20] The author of the doxography claims that ancient sages such as Hermes, Pythagoras, Empedocles, Socrates and Plato have professed God's oneness (*tawḥīd*); they accepted *Allāh* as the cause (*ʿilla*) of the seen and the unseen, the creator of all things (*mubdiʿ al-kull*) and their organizer (*mudabbir*).[21] In particular, Pythagoras developed an apophatic theology, as according to him God's essence cannot be perceived by the intellect – an idea we found already ascribed to Pythagoras in Ps.-Ammonius. His argument is that "God Most High is the Creator and the Maker of the intellect (*mubdiʿ al-ʿaql wa ṣāniʿuhu*); if the intellect would be able to obtain knowledge of God's essence, then the knower would be superior to the known object, which is impossible in the case of God. Hence, the intellect can only grasp the traces (*āthār*) of His wisdom in His creatures; God can only be known through His actions."[22] Nevertheless, Pythagoras, by a sound reflection of his intellect on his own essence, came to the insight of the existence and oneness of God, his Creator. God's essence being out of reach for the intellect, he concluded that the only thing we can state about Him is *al-huwiyya* (the he-ness or ipseity), "as we say: 'he'" (*ka-qawlinā huwa*).[23]

5 The Prophetic Origin of Pythagoras' Monotheism According to al-ʿĀmirī and the Ikhwān al-Ṣafāʾ

In his *Book on the Afterlife* (*Kitāb al-Amad ʿala l-abad*), al-ʿĀmirī (d. 992) used the doxography of Ps.-Ammonius while presenting the teachings of the ancient sages. According to al-ʿĀmirī, Empedocles, Pythagoras, Socrates, Plato and Aristotle all agree on "the affirmation of the Creator, the unity of His essence and the negation of rivals and antagonists (*andād wa aḍdād*) to Him" (*Amad*, III, 14).[24] He also reveals how those sages acquired such a sound conception of

20 On this work, its contents, its possible sources, along with the edition of the Arabic text and an English translation, see Wakelnig 2015, in particular 223–245.

21 Wakelnig 2015, 234–235.

22 Ibid., 240–241.

23 Ibid., 234–237.

24 Rowson 1988, 76–77. Al-ʿĀmirī attributes to Pythagoras a cosmology based on a hierarchy of twelve ranks: the four elements compose the sublunary world; the seven planetary spheres and the Throne enveloping them form the celestial world; at the top, there is a luminous world imperceptible to the intellects. The purified human souls can gradually ascend these ranks and finally contemplate what proceeds from the divine Wisdom

monotheism, an explanation lacking in Ps.-Ammonius, at least in the version that came down to us. Empedocles was a companion of the sage Luqmān in Syria, a contemporary of the prophet Solomon mentioned in the Qurʾān (sura 31:12) (*Amad*, III, 1–2).[25] Pythagoras, travelling in Egypt, learned geometry from the local priests, whereas the companions of Solomon, son of David, who lived there as exiles from Syria, initiated him in the sciences of nature and metaphysics (*al-ʿulūm al-ilāhiyya*). He brought these three sciences – geometry, physics and "the science of religion" (*ʿilm al-dīn*) – to Greece, where he discovered by his own thinking the science of melodies and numbers. "He claimed that he acquired these sciences from the niche of prophecy" (*mishkāt al-nubuwwa*) (*Amad*, III, 3).[26]

The puzzling affirmation of the Brethren of Purity (Ikhwān al-Ṣafāʾ), claiming that "Pythagoras was a monotheist sage (*ḥakīm muwaḥḥid*), one of the people of Ḥarrān in Syria,"[27] is probably not alien to the supposed prophetic origin of his philosophy, as Ḥarrān is mentioned in the Bible as a place where Abraham stayed.[28] And indeed, the text continues with the exposition of Pythagoras' theory of numbers, in particular his speculations about the number one in relation to God's oneness.[29] For our present purpose, it is important to note that the Ikhwān's Pythagoras associates the Creator with the number one, the Intellect with the number two, the Soul with the number three, and so on.[30] Apparently, he makes no distinction between an uncreated monad, corresponding to the Creator, and the created number one, corresponding to the first created being, the Intellect.

(*Amad*, IV, 6–7; Rowson 1988, 80–83). This reminds Pythagoras' spiritual ascension described in Ps.-Ammonius.

25 Rowson 1988, 70–71; cf. De Smet 1998, 38–45.

26 Rowson 1988, 70–71. Travels in Egypt are a commonplace in ancient biographies of Pythagoras; Christian authors, such as Eusebius (*Praeparatio evangelica*, in Eusebius 1903, 410–411, 470–471, 523, 664–665), claim that Pythagoras met Jewish theologians who had come to Egypt after the fall of Jerusalem; see Rowson 1988, 204–205, 208–209; De Smet 1998, 40–41.

27 Walker *et al.* 2015, 27 (translation), 17 (text in Arabic).

28 Genesis 11: 31, 12: 4–5. Of course, in the Islamic tradition, Ḥarrān was very famous for its pagan "Sabeans" and their esoteric, Hermetic religion; see De Smet 2011, 119–126.

29 See Carmela Baffioni's chapter in this volume.

30 Walker *et al.* 2015, 17–19 (transl.), 5–10 (Arabic); 28–30 (transl.), 17–22 (Arabic).

6 Al-Shahrastānī's Synthesis

In the long chapter about Pythagoras in his famous *Book of Religions and Sects* (*Kitāb al-milal wa-l-niḥal*), al-Shahrastānī (d. 1153) compiled the different accounts he could find in Arabic doxographical sources, in the first place Ps.-Ammonius, which he used extensively.[31] About the oneness of the Creator, he wrote:

> The Most-High Creator is one, but not as [the number] one, as He does not fall under the numbers. He can be perceived neither by the intellect nor by the soul, as intellectual thought is not able to grasp Him and the language of the soul is unable to describe Him, being elevated above the spiritual attributes. He is not perceptible through His essence, but can only be grasped through His traces, operations and actions. Hence, each of the worlds perceives Him by way of the traces left in it by His operation, so that He is qualified and described according to the operation that is specific to Him […].[32]

Thus far, al-Shahrastānī followed Ps.-Ammonius' elaborations about God's incomprehensiveness and unity, transcending the first of the numbers. Next comes a long reflection about different kinds of unity, also attributed to Pythagoras:

> He [*sc*. Pythagoras] further said: unity (*al-waḥda*) can be divided into unity which is not acquired from something else – it is the unity of the Most-High Creator, a unity encompassing everything, a unity governing everything, a unity from which proceed the unities and the plurality in the existents – and unity received from something else, which is the unity of the creatures. Sometimes he said: unity in general can be divided into unity prior to eternity (*dahr*), unity simultaneous with eternity, unity posterior to eternity but prior to time, and unity simultaneous with time. The unity prior to eternity is the unity of the Most-High Creator; the unity simultaneous with eternity is the unity of the first Intellect; the unity posterior to eternity and prior to time is the unity of the Soul; the unity simultaneous with time is the unity of the elements (*al-ʿanāṣir*) and

31 Al-Shahrastānī 1961, t. II, 74–83. For a survey of the sources, see the notes to the French translation of Jolivet and Monnot (al-Shahrastānī 1993, 201–212); for the use of Ps.-Ammonius in particular, see Rudolph 1989, 24, 29.

32 Al-Shahrastānī 1961, t. II, 74; cf. al-Shahrastānī 1993, 201.

> the composed beings. Sometimes he had still another division of unity, saying: unity can be divided into unity by essence and unity by accident. Unity by essence only belongs to the Originator (*mubdiʿ*) of the universe; from Him the unities proceed, which are in the numbers and in the numerable things. Unity by accident can be divided into what is the principle of the number without being comprised in the number, and what is the principle of the number, being comprised in it. The first [member of this division] is like the unity of the Agent Intellect (*al-ʿaql al-faʿāl*), as it is neither comprised in the number, nor in the numerable thing. The second [member of the division] can in turn be divided into what is comprised in [the number], as a part of it – for instance, two is composed of two ones; similarly, every number is necessarily composed of ones; as far as the number increases, its relation with the unity decreases –, and into what is comprised in it, as something that is inherent in it, and not simply as a part of it. This means that every number or numerable thing always has a unity, which is inherent in it; for instance, two and three, in so far as they are two and three, are one. In the same way, the numerable things, be they composed or simple, are one, either by genus or species or individually; for instance, substance, in so far as it is substance in general, man in so far as he is man, and a given individual, such as Zayd, in so far as he is this specific individual, [all these are] one. Thus, unity is never dissociated from the existents. This unity is acquired from the unity of the Most-High Creator, being inherent in all beings, even if they are multiple in their essences. The nobility of every existent depends on the predominance of unity in it; all that is more removed from plurality is nobler and more perfect.[33]

Starting from Ps.-Ammonius' distinction between the unity of the Creator and the unity of the numbers, al-Shahrastānī's Pythagoras develops a complex theory about unity, which can be summarized as follows. Unity can be divided in three ways, which are partly overlapping:

(1) Unity not acquired from outside = the unity of the Creator; unity acquired from outside = the unity of His creatures.

(2) Unity prior to eternity = the unity of the Creator; unity in eternity = the unity of the Intellect; unity between eternity and time = the unity of the Soul; unity in time = the unity of the material beings.[34]

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33 Al-Shahrastānī 1961, t. II, 74–75; cf. al-Shahrastānī 1993, 202.

34 The distinction of these four levels (above eternity, eternity, between eternity and time, and time) and their association with respectively the First Cause, the Intellect, the Soul

(3) Unity by essence = the unity of the Creator; unity by accident (a) as the principle of number exterior to number and numerable realities = the unity of the Intellect; (b) as the principle of number inside number and numerable realities: [i] the unity as part of every number (2 = 1 + 1); [ii] the unity that unifies the essence of every genus, species or individual.

In other terms, the unity of the Creator, which is the ultimate principle of all unity in the universe, remains hidden and inaccessible; it is dissociated from the unity of His creatures by the intermediate level of the Intellect, the first created being whose unity is the direct cause of all numbers in the corporeal world, beginning with the number one. However, further on in the chapter, Pythagoras associates the Intellect with the number two, the Soul with three and Nature with four, implying that the Creator corresponds to the first of numbers.[35] This inconsistency with what precedes is the result of al-Shahrastānī's compilation of different sources with conflicting views.

7 The Transcendent One and the First of Numbers

In later Antiquity, a certain consensus arose, both in pagan and in Christian sources, about the fact that Pythagoras' theory of numbers has to be understood in a monotheistic and even creationist way.[36] As the principle of numbers, the one or unity produces all subsequent numbers; numbers being the principles of what exists in the universe, all existents ultimately proceed from the first principle, which is one and unique. Here, two conflicting opinions seem to have coexisted: (1) the ultimate unity and first principle is the first number, the number one; (2) the ultimate unity, as principle or creator of the first number, transcends numerical unity.[37] Both positions are attested in the Arabic Pythagoras.

The first position is common in Christian apologetic literature, trying to convince the pagan reader that Pythagoras' philosophy is only a pale imitation

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and the corporeal world derives from the Arabic paraphrase of Proclus' *Elementatio Theologica*; see Badawi 1977, 4–5.

35 Al-Shahrastānī 1961, t. II, 77; cf. al-Shahrastānī 1993, 204–205.

36 Nevertheless, there was still a living tradition which presented Pythagoras as a dualist. For instance, according to Ps.-Hippolytus (I, 2, 12–13) Pythagoras borrowed his dualism from his teacher Zaratas (Zarathustra). As we have seen above, the Arabic Ps.-Ammonius inverted their relationship and considered Zarathustra's dualism as a perversion of Pythagoras' strict monotheism. Manifestly, the dualistic trends in Greek Pythagoreanism were eliminated in the Arabic tradition.

37 On the sources of this distinction in the Greek tradition, see Festugière 1954, 18–53, 307–315; Kalvesmaki 2013, 175–182.

of the revealed truth and, hence, that it is better to convert to Christianity. Cyril of Alexandria (d. 444), for instance (following Clement of Alexandria, *Protrepticus*, 6.72.4), claimed that Pythagoras openly professed the unity of God, the ultimate principle and mover of the universe, and thus that he is far removed from the rough polytheism of Cyril's adversary, the Emperor Julian (*Contra Julianum*, I, 42).[38] This passage was adapted in Syriac in the 6th century by an anonymous Christian author aiming at converting the pagan community in Ḥarrān.[39] Similarly, we have already met with the idea that the one and unique God, creator of the universe, corresponds to the number one, according to the account about Pythagoras in the *Refutatio omnium haeresium*, whose aim is to show that Christian heretics are Christian only in appearance, as they in fact rely on pagan philosophers. The *Refutatio* is a major source for the Arabic Pythagoras, in particular Ps.-Ammonius, but here, as we have seen, there is a shift from the first to the second position (the One transcending the number one).

However, the first position is also ascribed to Pythagoras in pagan sources. We found it in the Arabic *Placita philosophorum*, but it is also attested, for instance, in Diogenes Laertius.[40] By far the most remarkable text in Arabic reflecting this position is a commentary on Pythagoras' *Golden Verses*, attributed to Proclus and supposedly translated by Ibn al-Ṭayyib (11th century). This translation is quite unique, as it makes no attempt to eliminate the references to pagan religion and polytheism. Here we learn that, according to Pythagoras, "the first of numbers refers to God the Most High (*awwal al-ʿadad ishāra ilā Allāh taʿālā*), because He possesses pure unity (*al-waḥdāniyya al-khāliṣa*)," whereas the subsequent numbers of the decade, composed by the addition of one, refer to the secondary divinities (*al-āliha al-thawānī*), as the unity is only reserved to Zeus (Zāʾūs). In fact there are four types of numbers: divine (the units), intellectual (dyads), psychic (triads) and natural (tetrads), Zeus being the one unique father (*Zāʾūs al-ab al-mutawaḥḥid*) of all.[41]

Similarly, we have seen that, according to the Ikhwān al-Ṣafāʾ, Pythagoras identified God with the first of all numbers. However, this position appears rather scarcely in the Arabic Pythagoras. Most of the sources we have examined make a clear distinction between the unity of God and the unity of the

38 Cf. Arcari 2016, 188–191.

39 Brock 1983, 229–230.

40 In Diogenes Laertius VIII, 25 (in a passage of the so-called *Pythagorean Notebooks* quoted by the Greek polymath Alexander Polyhistor) it is stated that, according to Pythagoras, the monad is the principle of all things, which generates the dyad, and so on.

41 *Ibn aṭ-Ṭayyib* 1984, 78–79; 92–93; on this text, see O'Meara 1989, 231–232; Izdebska 2019. See also Anna Izdebska's chapter in this volume.

number one: Ps.-Ammonius, the *Turba*, the Oxford manuscript Marsh 539, the Tehran doxography, al-ʿĀmirī and al-Shahrastānī, all depending in some way or another on Ps.-Ammonius.

And indeed, Ps.-Ammonius seems to be the great promoter of this doctrine, which he explicitly attributes to Anaximenes, who would have stated:

> [The Creator] is the One, which is not the one of the numbers, because the one of the numbers can be multiplied, whereas He cannot be multiplied [Ps.-Ammonius, XI, 6].[42]

Clearly under the influence of Ps.-Ammonius, al-ʿĀmirī ascribes a similar teaching to Empedocles:

> [The Creator's] unity is not like the unity of any of the existents in the world, as the worldly existents are subjected to multiplication, either in their parts or in their qualities or in their equivalents, whereas His essence is exalted above this [*Amad*, IV, 2].[43]

At the same time, he acknowledges that Pythagoras agreed with Empedocles on the matter of the Creator's attributes (*Amad*, IV, 5).[44]

In fact, the thesis about the superiority of the Creator's unity is widespread in the Arabic Neoplatonic tradition, where it often appears without any direct connection to Pythagoras. Following Plotinus' claim that the One, the first principle of the emanation of the universe, is distinct from the numerical one (for instance in *Enneads*, V, 5, 4; V, 6, 4; VI, 9, 5–6), the *Plotiniana arabica* develop this position, identifying the One with the Creator of monotheistic religions. In the so-called *Theology of Aristotle*, which contains, attributed to Aristotle, the Arabic version of selected passages drawn from the *Enneads*, we read that "the true, pure and simple One, which encompasses all things, be they simple or composed, which is prior to all plurality, and is the cause of the existence (*āniyya*) of things and their plurality, is what produces the numbers (*fāʿil al-ʿadad*)." Contrary to a widespread idea, the author explains, the number one is not the principle of all things.[45] This is virtually impossible, because

42 Rudolph 1989, 45, 86.

43 Rowson 1988, 78–79.

44 Ibid., 80–81; cf. De Smet 1998, 63.

45 Badawi 1977b, 112; Lewis 1959, VIII, § 130, p. 271.

> the First Creator (*al-bāriʾ al-awwal*) is only one (*wāḥid faqaṭ*) in all respects, and His essence is an originating essence (*dhāt mubdiʿa*) […]. He originated (*abdaʿa*) the world as one; it is necessary that the oneness of the originated (*waḥdāniyyat al-mubdaʿ*) should not be like the oneness of the originator (*waḥdāniyyat al-mubdiʿ*), otherwise the originator and the originated, the cause and the effect, would be one thing.[46]

This distinction between the uncreated creative unity of God and the created unity of the first number, which we have found attributed to the Arabic Pythagoras, also occurs in the *Treatise on Divine Science* (*Risāla fil-ʿilm al-ilāhī*), still another Arabic Plotinus text. The ultimate first Principle cannot be described, as it is devoid of any form of multiplicity:

> If the First is not simple (*mabsūṭ*) and truly one, and outside any description and composition, it is not first at all […]. The pure, true One is not one in respect of anything, but by itself alone. Apart from the First, there is nothing purely simple.[47]

Hence, the One remains outside all numbers:

> Although we mention Him and count Him with the number of all things, yet He is above all number and is outside all categories of number. For substantial number (*al-ʿadad al-jawharī*) is not applicable to Him, nor is quantitative number (*al-ʿadad al-kammī*). Substantial number is that which applies to the essence of a thing and quantitative number is what applies to the amount of the essence of a thing: how much is it?[48]

8 Neoplatonism versus Pythagoreanism?

From these samples it is clear that the Arabic Pythagoras, as depicted by Ps.-Ammonius and al-Shahrastānī, takes the form of a Neoplatonic philosopher. Could we conclude then that the Arabic authors assigned almost at random Neoplatonic doctrines, which they found in sources such as the *Theology of Aristotle*, to ancient sages? In the case of Pythagoras, this is quite unlikely, as the debate about the relationship between the divine unity and the number

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46 Badawi 1977b, 148; Lewis 1959, x, § 93, p. 455.
47 Badawi 1977b, 178; Lewis 1959, § 162–165, p. 333.
48 Badawi 1977b, 180; Lewis 1959, § 182–183, p. 345.

one is attested in the Pythagorean School long before Plotinus. Already in the *Corpus Hermeticum*, Hermes advised his son in the following way: "Oh my son Tat, pray first the Lord, the Father, the Unique, who is not the one but source of the one" (*Corp. Herm.*, V,2). Festugière compiled from the works of Philo (d. *ca* 50 CE) conflicting Pythagorean passages, sometimes opposing, sometimes identifying the transcendent monad and the first of numbers.[49] In a long doxographical part of his *Mathematics Useful for Reading Plato*, Theon of Smyrna (2nd century CE) enumerates no less than six different opinions about this question, showing the lack of consensus among different Pythagorean fractions or individual thinkers. Some of the arguments also appear in our Arabic sources, for instance that the transcendent monad is not subject to multiplication, being inalterably one, whereas the numerical one can be multiplied and divided, a position attributed by Ps.-Ammonius to Anaximenes (VI, 1) and by al-ʿĀmirī to Empedocles.[50] The same lack of consensus among the Pythagoreans is manifest in the chapter "On the Monad" of the *Theology of Arithmetic* attributed to Iamblichus.[51] It seems thus that the different versions of the doctrine of *tawḥīd* attributed to the Arabic Pythagoras rely on this kind of sources, sharing authentic but conflicting Pythagorean elements, where dualism was also represented, often integrated into an overall Neoplatonic background.

9 The Creation of a Shiʿi Pythagoras

The philosophy of unity, supposedly taught by the Arabic Pythagoras, was not an exotic curiosum fancied by the authors of doxographies for a learned public fond of "alternative" forms of thought. It really influenced Islamic philosophers in their endeavor to give a philosophical legitimation to Qurʾānic monotheism. This happened mainly in the Shiʿi tradition, as is clear from the Ikhwān al-Ṣafāʾ, who explicitly claim the heritage of Pythagoras. In the same period, the Ismaʿili theologian Ḥamīd al-Dīn al-Kirmānī (d. *ca* 1021), without mentioning the name of Pythagoras, elaborates in his *Brilliant Epistle on the Meaning of Unification, the Unifier and the Unified* (*al-Risāla al-Durriyya fī maʿnā al-tawḥīd wa-l-muwaḥḥid wa-l-muwaḥḥad*) a complex reflection about the concept of

49 Festugière 1954, 19–25.

50 This interesting passage from Theon of Smyrna's *Expositio rerum mathematicarum ad legendum Platonem utilium* is edited, translated and commented in Kalvesmaki 2013, 175–180.

51 [Iamblichus], *The Theology of Arithmetic*, in Waterfield 1988, 36–39.

"one," which is highly reminiscent of what we found in the Arabic sources about the philosopher from Samos. According to al-Kirmānī, God cannot be associated with the number one, as this latter is always generated (*muḥdath*). He then adopts a fourfold division of the term "one," which is very close to the division later ascribed to Pythagoras by al-Shahrastānī: the one in relation to the individual, the one in relation to the species, the one in relation to the genus and, finally, the "absolute one" (*al-wāḥid al-muṭlaq*). The latter is the numerical one, the first of numbers, whose essence is twofold, being composed of unity and what bears this unity. In its four forms, the one is generated as the result of a preceding cause. The most perfect form of unity is realized in the first Intellect, the first originated being. As God is the Originator of the perfect unity of the Intellect, He transcends unity and oneness: God is neither *aḥad* ("one" as opposed to multiplicity) nor *wāḥid* ("one" in a numerical sense), but *fard* ("single," "unique"), His uniqueness being unknowable even to the Intellect.[52]

In the second half of the 17th century, Quṭb al-Dīn Ashkevarī presented in his *Beloved of the Hearts* (*Maḥbūb al-qulūb*) an ultimate synthesis, piling together most of what had been written in Arabic during previous centuries about the sage Pythagoras. He introduced into his chapter long quotations from the famous Sunni mystic Ibn al-ʿArabī, along with traditions attributed to the first Shiʿi Imams, thus showing how the Samian philosopher was perfectly integrated into the complex network of Iranian thought in the Safavid period. Here, at the eve of modern times, Pythagoras was still a guide to a true and profound understanding of the fundamental tenet of Islam: the unity and oneness of God.[53]

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52 See the English translation of Kirmānī's treatise in Hunzai 2008, 200–207; cf. De Smet 1995, 85–86.

53 Terrier 2016, 373–420, and Id. 2019.

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CHAPTER 9

The "Brethren of Purity" and the Pythagorean Tradition

Carmela Baffioni

After the fundamental work by Yves Marquet published shortly before his death,[1] it might seem pointless to reconsider the real or presumed links between the Ikhwān al-Ṣafāʾ and Pythagoreanism, or to re-examine the texts devoted to Pythagoras or Pythagoreanism in the Brethren's encyclopaedia.[2] Marquet discussed all these references, detected various Pythagorean doctrines and even considered the epistles that make no reference to Pythagoras and/or the Pythagoreans.

Though, of course, not every arithmologist is a Pythagorean and vice versa, if one takes a system based on an arithmological vision of reality to be "Pythagorean," then the Ikhwān are clearly Pythagoreans because the arithmological approach is one of the keys of their work.[3] Indeed, in Epistles 32[4] and 33 in the printed versions of the *Rasāʾil*[5] the Ikhwān successively expound the Pythagoreans' doctrine and their own, as reflected in the titles (see below).

1 Marquet 2006. Cf. the reviews by Lory 2007, 27–28 and De Smet 2007, 491–500.

2 The Ikhwān al-Ṣafāʾ – the "Brethren of Purity" – are the authors of the first medieval encyclopaedia of sciences, which precedes by at least two centuries the best known ones in the Latin world (by Alexander Neckham, Thomas de Cantimpré, Vincent de Beauvais, Bartholomaeus Anglicus, all dating back to the 13th century). The encyclopaedia is a collection of 52 epistles, divided into four sections: i) propaedeutic sciences; ii) natural sciences; iii) sciences of the soul; and iv) theological sciences. In opposition to the older hypothesis that situated the encyclopaedia in the second half of the tenth century, the most recent researches consider that the various epistles were written at different times around the end of the ninth century and the first half of the tenth century. The epistles are extremely heterogeneous, reflecting Babylonian, Indian, Persian, Jewish and Gnostic influences, with a number of Biblical quotations. The core source is, however, Greek thought. Foreign sciences are reworked to represent the whole religious education intended for an *élite*. Many scholars hypothesise that the Ikhwān al-Ṣafāʾ were committed to Ismāʿīlī thought, but the question is still open. Further details in Baffioni 2020 online and 2021[3].

3 I use this term in the sense clarified by El-Bizri 2018, 17.

4 On this epistle cf. Izdebska 2016b, 362ff.

5 The three versions of the encyclopaedia published in Bombay (ed. Wilāyat Ḥusayn 1887–1889), Cairo (ed. Khayr al-Dīn al-Ziriklī 1928) and Beirut (ed. Buṭrus al-Bustānī 1957) provide no details of the manuscripts used by their editors and do not differ substantially from each

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In view of the new edition of the two epistles published in the *Rasāʾil* collection sponsored by the Institute of Ismaili Studies, however, this may be the time to reconsider the Pythagorean commitment of the Ikhwān al-Ṣafāʾ. This new edition by the renowned scholar of Ismāʿīlism Paul E. Walker provides different versions of the two epistles, and throws new light on the interpretation of the Pythagoreanism of the Ikhwān al-Ṣafāʾ.[6]

We have to accept that in the printed versions the two epistles leave the reader disappointed, because their contents almost overlap. Ep. 33, which was expected to expound the doctrine of the Ikhwān, opens with the most general reference to Pythagoras in the two epistles, and indeed in the whole Encyclopaedia.[7]

Walker has identified two versions of Ep. 32,[8] based on MSS [ع], [د], [ل], and [ق], and on MSS [غ], [أ], [ع], [ح], and [ن] respectively. With regard to the Atif manuscript, Walker claims that it contains, after a section similar to Ep. 32 (ed. Bustānī), a text that the copyist considers to belong to Ep. 51 (ed. Bustānī),

other. However, a careful comparison between these editions and the new one (still in progress) is being drawn by my colleague and friend Omar Ali-de-Unzaga (Institute of Ismaili Studies), and interesting results are expected by his research.

6 Walker *et al.* 2015. Apart from a few exceptions, the new editions of the various epistles are based on the Istanbul MS Atif (ʿĀtif) Efendi 1681 [ع], the oldest MS of the encyclopaedia now available, dating back to 1182 CE.

7 "[…] in BCB, Epistle 33 contained some discussion of the views of the Pythagoreans that seemed out of place. Those are now confined to Epistle 32. But it is difficult to determine who exactly adheres to the doctrines advocated in the newer Epistle 33, and may thus remain a mystery." (Walker *et al.* 2015, 6; the siglum BCB – Bombay, Cairo, Beirut – indicates the three printed versions of the *Rasāʾil* mentioned above, n. 5).

8 Walker claims that: "We can […] see the two versions side by side […]. Having both allows us to trace the existence of each in the various manuscripts not influenced by Atif. And it helps determine what constitutes the actual text of a separate epistle, number 33, the one that follows 32 […] it is necessary […] that two or more versions of Epistle 32 existed from the earliest period and that some scribes had access to one, which they copied, and yet others to another version, which is the one they copied without realizing that there was any difference" (Walker *et al.* 2015, 5). "[…] it is clear that the material presented belongs to two epistles. That determination must have been set, not by the nature of the actual text available, but by a preordered plan for section three of the *Rasāʾil*. Moreover, the first epistle needed to begin with some reference to Pythagoras and to explain the doctrine of his school in regard to the essential importance of numbers and their correspondence with the natural world and the order of existing things. A second, subsequent epistle may have simply continued more or less with the same topic, the separation of one from the other having little significance or meaning. Alternatively, as the title given by Atif for the second implies, and which is confirmed by the printed text BCB in a different way, this latter treatise possibly intends to present the viewpoint of the Brethren and their contemporaries on these same issues" (ibid., 7–8).

On the Arrangement of the World. The new version of Ep. 33 is based on MSS [ع], [غ], [خ], [ل] and [ن].[9]

The editor titled both versions of Ep. 32 *On the Intellectual Principles According to the View of Pythagoras*,[10] whereas Ep. 33 is called *On the Intellectual Principles According to the View of the Ikhwān al-Ṣafāʾ*.[11] My comparisons ascertained that version (a) corresponds to the first part of Ep. 32;[12] that version (b) corresponds to Ep. 33;[13] and that Ep. 33 in Walker's edition corresponds to the second part of Ep. 32.[14] Let us give now a brief summary of the contents of these epistles in Walker's edition.[15]

1 The Epistle 32 in Its New Edition

Chapter 1 of Ep. 32 (a) opens with a reference to Pythagoras:

> Pythagoras the sage was the first to speak about the science of numbers and their natures. He maintained that the nature of existing beings

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9 *Conspectus siglorum*: [أ] MS Esad Efendi 3638 (*ca* 1287 CE); [ن] MS Esad Efendi 3637 (*ca* thirteenth century CE); [ق] MS Feyzullah 2131 (AH 704); [ل] MS Köprülü 871 (AH 820/1417); [د] MS BNF 6.647 (AH 695); [ح] MS Laud Or. 255 (n.d.); [خ] MS Laud Or. 260 (1560 CE); [غ] MS Marsh 189 (n.d.).

10 Walker remarks that Pythagoreans are not always mentioned in the titles of Ep. 32 in the MSS consulted (Walker *et al.* 2015, 6).

11 Walker notes that the title in the printed text has support in MS [ن] only. MSS [ق] and [أ] indicate that this epistle offers "the view of the Pythagoreans." MS [غ] has "Book on the Intellectual Principles According to the Views of the Philosophers and Theoretical Considerations (*al-ishārāt al-naẓariyya*)." The title in the Atif MS and MSS [ح] and [ج] shows that this epistle expresses the viewpoint of the "Aḥdāth" – possibly "the Youth" – rather than that of the Ikhwān. Wilferd Madelung, however, who has carefully studied the table of contents of the *Rasāʾil* as a whole, is convinced that the original title is the one in MS [ن] and that changing "Ikhwān al-Ṣafāʾ" to "al-Aḥdāth" is a later innovation. Walker *et al.* 2015, 40.

12 III, 178.14–186.13 ed. Bustānī.

13 III, 199.8–211.14 ed. Bustānī.

14 III, 187.6–198.7 ed. Bustānī. – My comparisons have also ascertained that the last lines of the Bustānī edition (III, 211.11–14; cf. 33.1–4 Walker) differ from the version of MSS [خ] and [غ] (see Madelung *et al.* 2019, 334). Between the first and the second part of Ep. 32 Bustānī, there is a series of "Questions about principles" (III, 186.14–19), reported in MSS [ق], [د], [خ], and [ل] only, and not included in the new edition (Walker *et al.* 2015, 11–12). The same for the poem after the "philosophical questions" in III, 186.20–187.3 Bustānī, which appears in MSS [د], [ق], and [ر] only (ibid., 12–13).

15 A comparison between the two versions is made ibid., 9.

> accords with the nature of numbers, such that he who knows the numbers and their natures, their species, types, and special properties, that allows him to know the quantity of existing beings and their various kinds. He also knows what rule governs their quantities at present and for what reason they are not more or less.[16]

This statement is related to the uniqueness of the Creator and the multiplicity of created beings that are made of the same matter but come to be in pairs or in groups of three, four, five, six, and so on. In Walker's translation:

> That is because the Originator, since He was the Cause of existing beings and the Creator of created things, and since He is One in true reality, there is no wisdom in things in their entirety being one thing in all respects. It is necessary that they be only one in material even though [they are] multiple in form. It is also not allowed that things in their entirety be pairs, triples, quadruples, quintuples, or sextuples, or anything at a point beyond these. Instead the most evident and most certain [situation] is that they are as they are now, in accord with these numbers and amounts, and that that is at the furthest extent of wisdom and certainty, and that there are some things that come in pairs, some tripled, quadrupled, quintupled, sextupled, septupled, octupled, nonupled, decupled, and any beyond these.[17]

Several examples of pairs and of groups of three and four are given, after which criticism is levelled at those who emphasised groups of seven,[18] Dualists, Christians who emphasised the trinity, Naturalists who exaggerated the importance of the four natures, the *Khurramiyya*,[19] who exaggerated the importance of things in fives, and the "inhabitants of India" who exaggerated the importance of nine in numbers and things. The Pythagoreans, on the contrary, "give everything its proper due when they claim that existing beings accord with the

16 Cf. Walker *et al.* 2015, 5.10–6.3 (Arabic text) and 17 (English transl.). This text corresponds to III, 178.13–179.2 Bustānī, where Pythagoras' doctrine is explicitly linked to the Ikhwān al-Ṣafāʾ (*fa-naqūlu ʿalā raʾy Fīthāghūras al-ḥakīm …*).

17 Walker *et al.* 2015, 17.

18 "[…] they came up with marvelous things among them. They were infatuated with them, bragged at length about them, and neglected those [existents] that might be enumerated by other numbers" (ibid., 18).

19 In the Islamic sources, this name indicates the religious movement founded by Mazdak in the late 5th century. Further details in Madelung 1986, 63–65.

nature of numbers, meaning here that there are things that are sets of twos, sets of threes, sets of fours, sets of fives, and so on beyond these."[20]

Authentic Pythagorean doctrine is identified in the fact that "one is the origin of numbers and their beginning. From it all numbers are composed, both the small and the large, pairs and singles, whole and fractions."[21] Afterwards, an analogy between the one and the Creator is advanced:

> Just as the Creator is the cause of existing things, [...]. Just as one has no parts to it, nor [does it have] a like, similarly the Creator is unique without likeness, or similar, or partner. Just as one exists in all numbers enveloping them, similarly God is present in every existent enveloping it. Just as one gives its name to every number and amount, similarly the Creator gives existence to every existent. Just as one maintains the permanence of number, similarly by the permanence of the Creator the endurance and permanence of existing beings is sustained. Just as one determines the value of every number and thing enumerated and measured, so too, similarly, the knowledge of the Creator encompasses all things, both the visible and the invisible.[22]

The Pythagorean theory of number, mainly developed in Ep. 1, is later linked to the Neoplatonic emanation process.[23] Hence the Active Intellect corresponds to number two, the Universal Soul to number three and Nature to number four. The whole hierarchy of beings portrayed in the encyclopaedia is described:

> Prime Matter – five
> Body – six
> Spheres – seven
> Elements – eight
> the Three Natural Kingdoms – nine.[24]

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20 Walker *et al.* 2015, 18.

21 Ibid.

22 Ibid., 18–19.

23 See El-Bizri 2018, and the following statement in particular: "The *Ikhwān al-Ṣafāʾ* were aware that eschewing the Pythagorean pathway in favour of strict Neo-Platonism carries its own epistemic risks in mathematics, given that Plotinus' ontology of numbers lacked the technical aspects of arithmetic as practiced by the Pythagoreans." (ibid., 20).

24 Minerals are assimilated to the tens, plants to the hundreds, and animals to the thousands; the mixture is like one.

The heavenly beings – the spheres – correspond to the nature of whole numbers, whereas the beings subject to generation and corruption correspond to the nature of fractional numbers.

Chapter 2 establishes an ascending hierarchy of existence, permanence, completion and perfection. Interestingly, the authors add: "In the epistle in which we mentioned the special properties of numbers, we have explained the difference between completion and perfection."[25]

To know the principles of existing beings, one must first know "the principles of bodily matters subject to sensation" and then the "exact realities" (*ḥaqāʾiq*) of existing beings themselves.[26] The epistle then speaks of the "Body" as a compound of Prime Matter and Form, but states that not every kind of matter accepts every form.[27] Prime Matter has four aspects: i) materials for production such as wood, iron and cotton; ii) natural materials – fire, air, water and earth; iii) the matter of all, that is, the absolute matter; and iv) the first Prime Matter receptive to forms – the first source of intelligible objects. In fact, "this matter is the first effect of soul, and soul is the first effect of intellect, and intellect is the first effect of the Creator, and the Creator is the cause of all existing beings."[28] Just as existing beings follow each other like numbers, the succession of the Intellect originated by God without intermediary, the Universal Soul and Prime Matter is now repeated in terms of their relationship with perfection, completion, and permanence:

> [...] existence precedes permanence, and permanence precedes completion, and completion precedes perfection, because every perfect being is complete, and every complete being is permanent, and every permanent being is an existing being. However, not every existing being is permanent, and not every permanent being is complete, and not every complete being is a perfect being. That is because the Creator, who is the cause of existing beings and their maintainer, completer, and perfecter,

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25 Walker *et al.* 2015, 20. The reference should be to Ep. 1, and the distinction between *tāmm* ("perfect") and *kāmil* ("complete") referred to the numbers six and seven. Six is called "perfect" because it is equal to the sum of its proper positive divisors. Seven is called "complete" because it combines in itself all sorts (*maʿānī*) of numbers – even and odd, and the first and the second of them both.

26 Walker *et al.* 2015, 20. This is because, according to the Aristotelian principle, sensible objects are easier to understand than intelligible objects.

27 When matter is the same, form differentiates objects, which are therefore referred to by different names. See also above, the passage cited at p. 299.

28 Walker *et al.* 2015, 21. The text obviously refers to heavenly hypostases in spite of the lower-case initial letters used by Walker.

> first emanates existence, then next permanence, then completion, then perfection.[29]

The Active Intellect is permanent, complete and perfect.[30] The Universal Soul is permanent and complete, but not perfect. Prime Matter is permanent but neither complete nor perfect.[31]

In Ep. 32 (b), the "introductory" Chapter opens with a statement that "scholars and sages among the monotheists" exaggerated the importance of single numbers.[32] The Pythagorean view is then given as the "correct" interpretation and "the doctrine of our brethren": that existing beings accord with the nature of numbers.[33] This is strange, because the *muwaḥḥidūn* ("monotheists") are normally opposed to those who exaggerate the importance of single numbers, notwithstanding that the Ikhwān consider themselves to be both monotheists *and* Pythagoreans. Walker gives no alternative reading in the critical apparatus *ad loc.*, so we can only note that the corresponding passage in Bustānī's edition merely has *al-falāsifa wa-l-ʿulamāʾ wa-l-ḥukamāʾ* ("philosophers, learned and sages"). In what Walker called "Chapter 1" there is another reference to Pythagoras as "a monotheist sage, one of the people of Ḥarrān in Syria" that is clearly intended to explain the correctness of the Brethren's positions.

> He was notably intensive in his attention to the investigation of the science of numbers and how they developed, devoting much research to them and to their special properties, their ranks and organic structure. He used to say that, in the understanding of numbers and how they develop from the one that precedes two, there is the understanding of God's unique oneness.[34] In understanding the special properties of numbers and how they are ordered and structured, there is the understanding of the Creator's existing beings and knowledge of His having brought them into being, and of the manner of their structure and organization. The science of numbers is embedded in every soul, needing the smallest

29 Walker *et al.* 2015, 20.

30 Ibid., 51.

31 Ibid., 22.

32 Ibid., 27. In terms of form and content this passage is quite similar to the first version. Those who emphasised the seven are here called "the esotericists."

33 Ibid.

34 Ibid. Cf. Walker *et al.* 2015, 17.4–10 (Arabic text) and 27 (English transl.). This passage is quite similar to the one that opens Ep. 33 in the printed versions (III, 200.6–12 ed. Bustānī).

consideration and the slightest remembrance in order to be clear and understood without proof or demonstration from elsewhere.[35]

In this passage, as with the one reported above (at p. 300), it is difficult to separate what the Ikhwān attribute to Pythagoras from their comments on Pythagoras' view. Chapter 2 of version (b) also sets out Ikhwānian ideas: the Brethren repeat that created beings are ordered like the hierarchy of numbers, whose multiplicity should attest God's oneness and uniqueness. It is impossible for originated beings to be one – like God –, but they are at least "in pairs."[36]

Another distinction is established between universal and particular existing beings.[37] Universal beings exist in nine ranks "like the nine of the single digits":[38] i) the Creator, the absolute one; ii) the Intellect, endowed with two powers;[39] iii) the Soul, "possessing three features";[40] iv) Prime Matter, "the possessor of four attributions";[41] v) Nature, called with five names;[42] vi) Body, "the possessor of six directions";[43] vii) the spheres, "having seven planet managers"; viii) the four elements, "possessing eight mixtures";[44] and ix) generated beings, "of which there are nine species."[45] A long list of groups of three and four is given with regard to particular things that "are within the universals mentioned previously."[46] Following an allusion to things in fours, fives, sixes, sevens, eights, nines, tens and so on until thousands of thousands,[47] examples of things "in sixes" are given and the Ikhwān reiterate their censure of groups

35 Ibid.

36 Ibid., 28; cf. 29.

37 Cf. on this division Ep. 40 in Baffioni and Poonawala 2017, 217–218.

38 Walker *et al.* 2015, 31.

39 As clarified later they are the innate and the acquired intellect.

40 Or "kinds" – vegetal, animal and rational soul.

41 The material of the arts, the material of nature, the material of the all, and the first Prime Matter (see above, p. 301).

42 The nature of the spheres, and four beneath the spheres.

43 These are the only ones mentioned in this passage and not later: "namely up, down, right, left, in front, and behind."

44 Earth is cold and dry; water is cold and wet; air is hot and moist; and fire is hot and dry.

45 "Minerals are of three types: earthly that does not melt or burn [...]; rocks that melt but do not burn [...]; essences that melt and burn [...]. Plants are of three kinds: [...] those that are planted [...]; those that are cultivated [...]; those that are sprouted [...]. Animals are of three kinds: those that give birth and suckle; those that lay eggs and incubate; those that are formed out of decomposing rot" (Walker *et al.* 2015, 30–31).

46 Ibid., 31.

47 Ibid., 31–32.

of seven.[48] Ep. 5, *On Music* is mentioned with regard to groups of eight, and the authors refer to "Indians" and al-Kayyāl with regard to nines.[49] Chapter 3 explains "the layout of the world and that it is spherical in shape."

With regard to the new arrangement of Ep. 32, Walker remarks that the epistle is "limited solely to the Pythagorean-type discussion of the correspondence between the nature of numbers and the world at large. Other topics then belong to the second epistle (number 33)."[50]

In version (a), the Ikhwān's presentation of the spiritual hypostases of their system is based on the succession of the first four units, and the whole of created reality is understood according to the nine single digits. Instead of continuing with Pythagorean doctrines, Chapter 2 echoes doctrines expounded in other parts of the encyclopaedia and seems to develop ideas more proper to Islamic thought. The groups of five demonstrate in particular that the authors have Shīʿī esoteric milieux in mind rather than ancient Pythagorean doctrines. In my opinion, therefore, the ideas of the Ikhwān play an important role in this first epistle, despite their declared intention to expound Pythagorean ideas. This is especially evident in Chapter 2 in version (b), where "the Tablet and the Pen" are introduced among the pairs invented by philosophers, and the Arabic letters *kāf* and *nūn* are mentioned. These are typically Ismāʿīlī,[51] just as the vision of creation "in couples," also mentioned in the Quran,[52] is widely developed in esoteric writings of early Arabic literature.[53]

48 "[…] we will leave off mentioning them, since a group of those devoted to the sciences were passionate about these and went to excessive lengths in discussing them. They are understood as existing in the hands of these scholars" (ibid., 32).

49 "[…] he was especially keen on them, and there are many citations of them in his book and they are known and exist in the possession of scholars" (ibid.). Al-Kayyāl was a Shīʿī gnostic active in the 9th century, and a missionary of some Ismāʿīlī imams. Further details about him in Madelung 1990.

50 Walker *et al.* 2015, 7.

51 Cf. Daftary 2007^2, 134.

52 Cf., e.g., 13:3; 51:49; 53:45.

53 I refer to Balīnūs – the Arabic Apollonius of Tyana (inspired by the famous Neopythagorean sage of the 1st cent. CE; cf. Plessner 1986) –, whose *Sirr al-Khalīqa* (*The Secret of Creation*) probably dates back to the beginning of the 9th century. Cf. Balīnūs al-ḥakīm/pseudo-Apollonius of Tyana 1979. Having emphasised the uniqueness of the Creator as opposed to the duality of His creation (100.11–101.4 ed. Weisser), Balīnūs says that the first thing that was created was His speech, and the rest of creation is an effect of this. This was the beginning of couples – creation itself (101.8–9). The author adds that God cannot be the cause of anything because a cause always resembles its effect (102.1–6). The Intellect and the Soul constituted the first couple, from which all creatures came. "The principle of creation was from the one. Then came the two – heat and cold, the three – the combination (*ijtimāʿ*) of the two, and the four – heat, cold, humid (*līn*) and dryness. Then the process of generation (*wilāda*) halted (*istaqarrat*) and nothing more was added

In the new edition the mention of Pythagoras is correctly placed at the beginning of the two versions of Ep. 32, introducing the whole. The Ikhwān clearly consider the Pythagorean views on the one and on numbers as the best doctrine for interpreting the nature of God and His relationship with creatures – as far as this is permitted for human beings. It is equally clear that "modern" (or self-proclamed) Pythagoreans – i.e., individuals or schools who base their doctrines on number – are often in error in deeming a single number more important than the others. Instead, the correct original doctrine claims that reality exemplifies *all* numbers: its value consists in the fact that it adopts a quantitative rather than qualitative vision of reality. This is the approach proper to the Brethren, who demonstrate this assumption through examples drawn from well-known philosophical doctrines of the Greeks and also from doctrines proper to Islam.[54]

With regard to Chapter 3 of version (b), Walker remarks that it is very similar to Ep. 51, *On the Arrangement of the World*, in the printed text of the encyclopaedia,[55] which according to many scholars was a later addition. Accordingly, scholars hypothesise that it was once part of Ep. 32.[56] The question of the content and position of Chapter 3 exceeds the scope of this article, but spherical shape is mentioned again in Ep. 33 (in Walker's edition).

So the Pythagorean nature of this chapter seems to consist largely in the arithmological approach that characterizes the first part of the epistle. Nonetheless ideas occurring in various parts of the encyclopaedia are repeated here, some of which have also a distinctively Pythagorean pedigree:

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or separated and added (again) because the perfection of all contraries (comes) from the four" (113.3–6).

54 Cf. Walker's conclusion: "Quite unlike 32a, 32b ends with a discussion of fives in Islamic religious law and ritual, and the role of angels, a curious departure from Pythagoras. However, it closes that discussion with the following comment: 'We have thus, with what we have explained, made clear the meaning of the Pythagorean sages saying that the beings that exist accord with the nature of the numbers.' That statement appears to bring to an end the epistle on the views of the Pythagoreans" (Walker *et al.* 2015, 9).

55 See now the new edition by Nuha Alshaar in Madelung *et al.* 2019, 365–432.

56 On Ep. 51 Walker says: "Scholars have speculated that, in fact, Epistle 51 is an addition and does not belong in the collection. The editor of Atif would have it be authentic. However, the main point of his comment for our present purposes is the evidence it provides about his editorial methods, notably his use of several manuscripts of the text in order to form a single authentic version. Furthermore, and most importantly, he continues after this comment with a second version of Epistle 32" (Walker *et al.* 2015, 5). This is also linked to establishing the real total number of epistles, a debated issue since the Middle Ages. See ibid., 10.

i) existing beings exist inside a single sphere, echoing Qur'an 36:40;[57]
ii) all other spheres are inside it, according to the Ptolemaic system;
iii) the circle is the noblest shape;
iv) the sphere is divided into twelve parts because twelve is the first "abundant" number;[58]
v) the motion of the planets and spheres is circular, and
vi) the Earth is a globe, and what it contains is segments of a sphere.

All this proves that God "is uniquely one," that "there is no end to His acts," that "He is one, the Almighty," and that he has no partner.[59]

2 The Epistle 33 in Its New Edition

The new version of Ep. 33[60] also refers to the spiritual hypostases as a decreasing hierarchy in terms of completeness, perfection and excellence. The text speaks of Prime Matter, and then of Absolute Body conceived as a "second Prime Matter" endowed with the three dimensions, and with the spherical shape – the most excellent of shapes –, from which spheres and planets are derived. The Ikhwān again refer to ideas developed in the encyclopaedia:

i) the rotation of the spheres around the four elements determines the alternation of night and day and winter and summer;
ii) the three natural realms derive from the mixture of the four basic qualities, and are briefly described;
iii) the ascending hierarchy culminates in human beings;
iv) because a human being is composed of a physical body and an immaterial soul, it is a "microcosm" and the universe is a "macroanthropos";
v) the distinguishing feature of humans is intellectual comprehension; and
vi) having understood itself, humanity can proceed to the knowledge of all other beings.

Chapters 2–4 are centred on the Universal Soul that emanates from the Active Intellect. The Soul has "two powers that run through all the bodies, from the proximity of the enveloping sphere to the lowest limit at the centre of the Earth, similar to the diffusion of the light of the Sun through all parts of the air. One of its powers is knowing and the other is acting."[61]

57 Walker considers the spherical form of the world to be a Pythagorean concept (ibid.).
58 Namely, the sum of its parts exceeds its total.
59 Cf. Walker *et al.* 2015, 38.
60 The epistle is summarised ibid., 10.
61 Ibid., 43.

The Universal Soul completes bodies through its power of action, and perfects the human essence in terms of the sciences, beliefs and actions through its power of knowing. The Universal Soul does not perish because its matter is eternal and it endlessly receives the emanation of the Intellect: this is because the excellence of the Creator never ceases, and His gifts never end. The Universal Soul ranks above the all-encompassing sphere, and its powers are sent out to all parts of the sphere and all beings. The power of action particular to each individual is made manifest as its "particular" soul.[62]

> This is the true reality of what is cited in the divine books which say that they are angels, the highest host and army of God, never rebelling against His command and always doing what they are ordered to do. This is likewise the true reality of what the sages and philosophers mean in discussing the specifics of the particular souls termed spiritual in the realm of the spheres and the elements and who are charged with the preservation of the world, the governing of creatures, the ruling of the spheres and the coursing of the planets, the unfolding of the ages, the alternations of the seasons, the care of the elements, the cultivation of plants and animals, and the preservation of both.[63]

Chapter 5 continues with the beings that exist beneath the sphere of the Moon. According to philosophers the guiding principle is "the nature of generation and corruption," which religious Law calls "angels." Because of the Universal Soul, each natural individual has a particular soul – starting with the four elements in cooperation with the influence of the stars. Chapter 6 explains that the first action of these powers – heat, cold, wet, and dry – in the generation of minerals produced mercury and sulphur, after which the other minerals were formed. Chapter 7 clarifies the seven powers of the vegetative soul, which cooperate in the formation of plants. The process of nourishment in animals is also described, culminating with the generative capability of humanity. The development of the embryo and the physical and mental growth of the new-born are considered. The religious Law consigns men to death after fifty years of life through progressive separation from material goods. If a soul is perfected during life in this world, the person is ready to ascend to the highest place. Otherwise, the soul "reverts to the lowest of the low" and "it thereby

62 Hence all the planets have souls.

63 Walker *et al.* 2015, 45.

becomes accustomed to the regimen from the beginning."[64] Qur'an 95:4–5, 21:104, 40:67, 16:70, and 22:5 are referred to.

The question is then asked, whether things were "created at the utmost limit of completeness and perfection, and then they subsequently deteriorate," or "were they all brought into being at the limit of defectiveness, then they subsequently increase and become perfect and complete […]. Or are some like this and others like that?" but no answer is provided.

In Chapter 8 the authors return to the perfection of the Creator, Who is "retained exclusively in His essence and not shared or emanated." Goodness and excellences emanate from Him, just as light and brilliance continuously emanate from the Sun. The first result of divine emanation is the Active Intellect – the extreme of completeness and perfection, which contains the forms of all things, just as the forms of known things are in the thought of the knower. The "passive intellect" emanates from the Active Intellect – the Universal Soul, which is lower in rank: it receives forms and excellences from the Active Intellect as a student receives instruction from his teacher. Prime Matter emanates from the Soul and receives forms "over time, thing by thing."[65] By receiving the three dimensions it becomes an Absolute Body (a "second matter"). No other substance emanates from it because of its low rank and its distance from the first cause. As a result of the eternal emanation of the Intellect, the Soul is "favourably disposed to body, wanting to fashion in it forms, shapes, and colours, in order to complete it with excellences and good qualities, commensurate with what is possible for the body and the relative purity of its substance to receive."[66] The first form fashioned in body is the spherical shape, which the Soul causes to move with circular motion. The heavenly spheres are then arranged as explained before. These beings decrease in perfection according to their distance from the Creator. God has no resemblance to any being, just as "The agent does not resemble the effect in any way, nor for any reason."[67] The epistle concludes by echoing Qur'an 3:30.

Chapter 1 of Ep. 33 recalls the well-known hierarchy of beings, and introduces another topic of the encyclopaedia – the microcosmos-macroanthropos pairing. Because of its attitude to knowledge, humanity marks the culmination of perfection in the sublunary world. In the same way, the Universal Soul is represented in Chapter 2 as the junction of elements between the Intellect and the lower beings, in whom it is "incarnated" in the form of particular souls.

64 Ibid., 50.
65 Ibid., 51.
66 Ibid.
67 Ibid., 52.

The Qur'anic verses quoted in Chapter 7 (for which see above, p. 308) can be variously interpreted, even as an allusion to the Pythagorean doctrine of metempsychosis. In Chapter 8 the "positive" role of the Universal Soul in the formation of the lower world is further clarified – again in line with Ep. 40, *On Causes and Effects*.[68] The references to the spherical shape and the heavenly world are the same as in Ep. 32.

Despite all the common elements indicated above, Walker remarks:

> This makes a neat arrangement, one epistle delineating the opinion of Pythagoras about intellectual principles, and the following one, the view held by the Brethren themselves, which thus promises to be especially important for determining the exact doctrines espoused by the Brethren, as opposed to those they report from others. As such, it appears to constitute one of the few direct references in the corpus as a whole to the ideas of the society that composed it.[69]

In my opinion, however, this *summa* of Ikhwānian doctrines has little in common with Pythagorean ideas. The parallels between the so-called Pythagorean contents outlined in the preceding epistle and ancient Pythagorean literature are actually weak, and Ikhwānian doctrines in fact permeate the "Pythagorean" descriptions in both versions of Ep. 32. Even so we must be fair to Walker's edition, in which the various contents are much more clearly set out than in the former printed versions.

3 The "Fragments" and "Testimonia" of Pythagoras in the Encyclopaedia

The lax use of Pythagorean doctrines seems to be confirmed by the passages of the encyclopaedia in which Pythagoras is explicitly mentioned.[70] There are eleven of these, which can be divided into four groups:

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68 The final chapter on the emanation of beings is considered to be a later addition to Ep. 33. See Walker *et al.* 2015, 11. – On the parallel with Ep. 40, see Baffioni and Poonawala 2017, 193–194.

69 Walker *et al.* 2015, 2. "In the new version of Ep. 33, the Pythagorean elements in the former epistle do not reappear. Instead we have material that might well represent a later tradition, quite possibly that of the Brethren […]" (ibid., 10).

70 The new editions of these passages do not differ substantially from the printed versions. I indicate the most relevant discrepancies below.

i) The definition of arithmetic as knowledge of the properties of number and of existing beings that conform to them.[71]

ii) A quotation from Pythagoras' *Golden verses* (corresponding to *Χρυσᾶ ἔπη*, v. 70–71)[72] and occurring no less than four times in the corpus: in Ep. 3, *On Astronomy*;[73] Ep. 44, *On the Explanation of the Belief of the Brethren of Purity and the School of the Spiritual*;[74] Ep. 45, *On the Way of the Ikhwān al-Ṣafāʾ's Mutual Help and Their Reciprocal Cooperation, and on the Truth of Solicitude and Affection both in Religion and in this World*;[75] and Ep. 48, *On the Modality of the Invitation* (daʿwa) *to God*.[76] The various passages are similar, and for the sake of brevity I translate the longest, from Ep. 44:

Among what indicates that Pythagoras the mathematician[77] and one of the virtuous sages held this opinion and believed in it [is] his saying in the Golden Epistle – his testament to Diogenes[78] – and his word at its end: "When you are separated from this body, so to [float] free in the air, then you will travel safe and quiet, without returning to the human [condition] and no [longer] subject to death."

71 Ep. 1, *On Number*, I, 49.6–8 ed. Bustānī = El-Bizri 2012, 11.1–3 (Arabic text) and 66 (English transl.). At p. 66, note 4, the author refers to Nicomachus (on whom see the final note of the present chapter).

72 Cf. Baffioni 1994; Izdebska 2016b, 362 and 2018, 863. Izdebska 2016a examines with a different focus four commentaries on the *Golden Verses* – Iamblichus' *Protrepticus* and Hierocles' *Commentary on the "Golden Verses,"* as well as two commentaries preserved in Arabic, attributed to Iamblichus and Proclus. Cf. also Izdebska's contribution to the present volume.

73 I, 138.14–16 ed. Bustānī = Ragep and Mimura 2015, 89.9–90.4 (Arabic text) and 63 (English transl.). For further references see ibid., p. 63, note 124. The most relevant difference is the authors' reading مخلا ("released") instead of نحلا (this is a ductus with mistaken diacritics that has no sense here – perhaps "grown thin?" – but should correspond to the Greek ἐλεύθερος; see Baffioni 1994, 117).

74 IV, 35.22–36.4 ed. Bustānī = Traboulsi, Mayer and Netton 2016, 81.14–82.4 (Arabic text) and 106 (English transl.).

75 IV, 58.15–17 ed. Bustānī = Traboulsi, Mayer and Netton 2016, 134.2–4 (Arabic text) and 137 (English transl.). The new edition has أوصيتك به ("[what] I have advised you [to do]") instead of أوصيك ("[what] it has been advised").

76 IV, 175.12–13 ed. Bustānī = Hamdani and Soufan 2019, 74.6–75.1 (Arabic text) and 86 (English transl.).

77 Netton translates the Arabic "ṣāḥib al-ʿadad" as "author of works on numbers."

78 The new edition has لروجانس (another name for the recipient of the testament) instead of لديوجانس ("to Diogenes"). As for the mention of the "testament," which is of course a spiritual testament, Netton (in Traboulsi, Mayer and Netton 2016, 106, n. 153) seems to consider it to be different from the *Golden Verses*.

iii) Mention of the fact that Pythagoras was able to hear the music produced by the movements of the spheres because of the purity of his nature and the sharpness of his mind (Ep. 5, *On Music*;[79] Ep. 31, *On the Causes of the Diversity of Languages*[80]).

iv) Statements of cosmological/astronomical content that are probably related to a "pseudo"-Pythagoras (Ep. 52, *On Magic*[81]).

The theories in (iii) are narrative rather than explaining aspects of the Pythagorean doctrine. Those in (iv) belong to the complex texture of Ep. 51 (or 52),[82] *On Magic*, and the relationship with alchemy, which are not discussed here.[83]

In most of the other passages reported above the name of Pythagoras is not mentioned alone: in the text of the first group (i), Pythagoras is associated with Nicomachus. In those of (ii) he is variously and repeatedly associated with Hermes, Aristotle, Jesus, Socrates, Plato, the sage Bilawhar[84] and the Qur'anic and prophetical tradition. Of the texts under (iii) the first and third ones associate Pythagoras with "Nicomachus, Ptolemy, Euclid and other sages."

79 I, 208.16–20 ed. Bustānī = Wright 2010, 82.4–9 (Arabic text) and 121 (English transl.). The most relevant difference is the author's reading فكره ("of his thought") instead of فطرته ("of his natural disposition"), for which see note ١٠٩٦. Cf. also I, 226.2–4 ed. Bustānī = Wright 2010, 140.4–7 (Arabic text) and 148 (English transl.). The most relevant difference is the author's reading الروحانية ("spiritual") instead of الدائمة ("eternal"), for which see note ١٩٢٨.

80 III, 94.1–5 and 125.3–8 ed. Bustānī, where this capability is associated to Pythagoras' invention of the lute.

81 IV, 414.20–415.1 ed. Bustānī: "Pythagoras said: 'The seven [planets] act on the twelve [constellations], and likewise the faculties [of the Universal Soul] on the body. The Sun is the soul, the Moon the spirit. The soul is hot and dry, the spirit cold and humid. Dry is mixed with humid and heat is balanced with cold, and the power of the Intellect in the marrow set in the brain [is] like a king at the head of prominent people'." – Ibid., 418.10–16: "Because of that Pythagoras said: 'The Sun is the king of every substance, its nature is the most balanced among natures. Neither the earth corrupts it, nor the things that burn bodies burn it, because its mixture in heat, dry, cold and humid [here: *nadāwa* instead of the usual *ruṭūba*] has equal parts, and in its nature there is nothing exceeding another thing, decreasing [in front of it] or subject to corruption. For this [reason people] extolled and honoured it, called it "sun," and kings moulded crowns and diadem[s] from it [...] because it is the most beautiful mineral existing in the world of generation and corruption' [...]."

82 Depending on whether one considers Epistle 51 belonging to the corpus or not.

83 The statements attributed to Pythagoras may have been drawn from one of the numerous works attributed to him in Antiquity; for lists of them, see most recently Macris 2018, 843–850 (Greek and Latin sources) and Izdebska 2018, 860–869 (Syriac and Arabic sources). See also Baffioni 2018.

84 Bilawhar, usually associated with Yūdāsaf (Būdāsaf), is one of the two main heroes of an Arabic work deriving from the traditional biography of Gautama Buddha, which subsequently provided the prototype for the Christian legend of Barlaam and Josaphat. See Lang 1986.

Note that the texts of (ii) are intended to endorse belief in the immortality of the soul, and the passage from Ep. 45 hints at the "angelic degree" – the highest spiritual level of the initiate in the encyclopaedia. Only the text under (i) can be linked to the Pythagorean doctrines expounded in Ep. 32.

All this tends to dilute the relationship of the Ikhwān al-Ṣafāʾ with Pythagoras and Pythagoreanism. My conclusions are therefore different from those of Walker, who says:

> The essential connection between the doctrine of the Brethren and that of Pythagoras is too well known to require much explanation, and it was moreover evidently quite obvious from the beginning, a fact observed with disapproval by, among other medieval authorities, al-Ghazālī, who comments in his *al-Munqidh min al-Ḍalāl* [*Deliverance from Error*] that not only was Pythagoras a weak thinker, but the Ikhwān al-Ṣafāʾ, who have followed him and his teachings, promote poor and substandard philosophy as well for that reason.[85]

4 The Testimony of al-Shahrastānī

The source of the doctrines outlined in Ep. 32 and Ep. 33 seems to be Neoplatonism. An *a contrario* proof in the context of this debate can be found in a work by the Arabist and logician Albino Nagy.[86] In the Albino Nagy Fund in the Fondo Leone Caetani at the Accademia Nazionale dei Lincei in Rome I discovered two versions of an article by this scholar – I believe unpublished – entitled "Archytas in the Arabic tradition." Despite the mention of Archytas – of whom, as Nagy remarks, the Arabs hardly speak[87] –, the main source of the article is the long chapter devoted to Pythagoras by the theologian, philosopher and historian of religions Muḥammad ibn ʿAbd al-Karīm al-Shahrastānī

85 Walker *et al.* 2015, 8, n. 5. The scholar refers to Watt 1995, 53.

86 Nagy was the first professor of mathematical logic in Italy. He was born in Traù on 2 October 1866 and died in Rome on 26 March 1901 (or according to other sources, 1900) at the early age of 35. He graduated in Wien in philosophy and mathematics and taught for many years in Italian secondary schools. He received an award from the Accademia dei Lincei for his researches in mathematical logic. The most complete survey on Albino Nagy's life and works is Nasti De Vincentis 1986. Alessandro Padoa seems to have written a short note on Nagy in *Rivista filosofica* 1901. I was not able to find additional references.

87 For an exception, see Ibn Abī Usaybiʿa, 42.20–43.4 ed. Müller; cf. Cottrell 2008, 533, 534 n. 42 and 536; Horky 2020 (forthcoming).

(m. 1153) in his *Kitāb al-milal wa-l-niḥal* (*The Book of Religions and Sects*).[88] Let us examine what Nagy considers the "central" doctrine of Archytas in the light of the information in Ep. 32 and Ep. 33 of the Ikhwānian encyclopaedia.

Nagy argues that even though the extant sources do not show that Arabs had any direct or indirect knowledge of Archytas' doctrines, we can obtain some idea of what they thought about the relationship between such doctrines and Pythagorean philosophy. Nagy refers to Shahrastānī's testimony, according to which Pythagoras distinguished four kinds of unity (*al-waḥda ʿalā al-iṭlāq*):

i) a unity before eternity (*waḥda qabla al-dahr*);
ii) a unity within eternity (*waḥda maʿa al-dahr*);
iii) a unity after eternity and before time (*waḥda baʿda al-dahr wa-qabla al-zamān*); and
iv) a unity inside time (*waḥda maʿa al-zamān*).

These are identified with i) the unity of the Creator, ii) the First Intellect, iii) the Soul, and iv) the elements and compound things respectively; and the Creator, the First Intellect, the Soul, and the elements are symbolised by the first four numbers.[89] This identification is a weak recollection of the passages quoted above from Ep. 32 (a). Interestingly, Nagy remarks that these doctrines are "Alexandrian" rather than Pythagorean, and refers to passages by Proclus in the *Elements of Theology* (*Στοιχείωσις θεολογική*) – edited by Otto Bardenhewer in 1882[90] – and of the *Fons vitae* by the Spanish Jewish philosopher Ibn Gabirol (d. 1070).[91] The scholar also notes that the so-called "Pythagorean" doctrines known to the Arabs were in fact more likely to be Neo-Pythagorean or Neoplatonic, as demonstrated by another passage in Shahrastānī in which Pythagoras considers ecstasy as a means to rise to the higher world of simple substances. Nagy does not quote this passage because it had already been translated into French by Munk. But it is worth reading:[92]

> [...] After extensive exercise, I [*sc.* Pythagoras] [was able to] view by the senses these higher worlds; I have ascended from the world of natures

88 I have translated this chapter into Italian with an extensive commentary in Baffioni 1983. See also Izdebska 2016b, 365ff.

89 The passage corresponds to al-Shahrastānī 1842–1846, Part II, 266.7–11. French translation by J. Jolivet in Shahrastani 1986–1993, vol. 2, 202.

90 See now the standard edition and translation of the text by Dodds 1963.

91 Nagy refers to the pioneering work of Munk 1859, which broadly deals with Ibn Gabirol's work, and to Clemens Baeumker's edition of the *Fons vitae* (Ibn Gabirol 1895). See now Ibn Gabirol 2007.

92 It is found in Munk 1859, 246–247, where it is quoted from Shahrastānī's *Milal* in Cureton's edition (al-Shahrastānī 1842–1846, Part II, 278.2–11).

> to the world of the Soul and the world of the Intellect; I have perceived with my eyes (*fa-naẓartu*) the abstract forms that are in them and the beauty, splendour and light they have; I have heard the noble melodies and the touching spiritual sounds. [...] What is in this world contains a small extent of beauty, as it is an effect of nature; the worlds above it are more splendid, nobler and more beautiful, until description arrives to the world of the Soul and of the Intellect, and stops, because language (*al-manṭiq*) is not capable to describe the nobility, munificence, beauty and splendour that are in them. So be your endeavour and effort towards the union with that world, until you remain in permanence and duration after the corruption and extinction that [affected] you, and you [may] arrive to the world that is all beauty, all splendour, all joy, all might and truth, and your joy and pleasure are eternal without interruption.[93]

This text recalls to some extent those in groups (ii) and (iii) above. With regard to the contents of Ep. 32, the following statement by Shahrastānī is also noteworthy:

> (Pythagoras) said: the Laws that brought the measures of prayers, alms and other acts of worship are [such that] these relationships do not stand as an equivalent (*innamā hiya lā yuqā*ʿ [...] *fī muqābala*) of those spiritual harmonies.[94]

But we must return to Shahrastānī's text, quoted by Nagy.[95] If we look into the wider context of this fourfold division, we see that Shahrastānī had just spoken of two kinds of unity:

i) a unity not received by anything (or anybody) else – the unity of the Creator; and
ii) a unity received by something (or someone) else – the unity of the created beings.[96]

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93 My translation.

94 al-Shahrastānī 1842–1846, Part II, 276.3–5. Cf. Jolivet's translation in Shahrastani 1986–1993, vol. 2, 209: "Il dit: les lois qui ont fixé la mesure des prières, des aumônes et des autres pratiques cultuelles, ont eu pour but de faire correspondre ces rapports avec ces harmonies spirituelles." The French translation used the Badran edition.

95 See above, p. 313 and n. 89.

96 al-Shahrastānī 1842–1846, Part II, 266.4–7. French translation by J. Jolivet in Shahrastani 1986–1993, vol. 2, 202. This unity is also called *waḥdat al-iḥāṭa bi-kull shay'*, *waḥdat al-ḥikma ʿalā kull shay'*, *waḥda taṣduru ʿanhu* (sic) *al-āḥād al-mawjūdāt wa-l-kathra fīhā* ("unity encompassing [Jolivet: "de la connaissance"] everything, unity of the reason underlying [Jolivet: "du statut imposé"] everything, unity from which the unities of existing beings and the multiplicity in them proceed").

After the passage referred to by Nagy, Shahrastānī adds that unity is also divided into "unity by essence" (*waḥda bi l-dhāt*) and "unity by accident" (*waḥda bi l-ʿaraḍ*).[97]

This distinction is similar to that found in a passage in Ep. 40, *On the Causes and Effects*[98] explaining the meaning of the sages' saying that "substance is substance in itself, and accident is accident in itself." The sages found that some existing beings were attributes and others were qualified objects, and realised that the cause of diversity in qualified objects resulted from the diversity of attributes. Attributes differ intrinsically (*li-anfusihā*) because God created them as different in themselves (*bi-aʿyānihā*), not on account of any cause affecting them: black and white, for example, are different because of the essential (*fī dhātayhimā*) difference of blackness and whiteness, not on account of some other cause – which would imply an infinite sequence of positing one cause after another. *Qua* subject of the attribute, a black thing is only black because blackness is in it; similarly with a white thing. Blackness and whiteness are intrinsically different from one another, not because of any other attribute in them but as two things differing from each other in essence because the Creator formed them in that way.

We now return to Shahrastānī. The first unity is said to be proper to the Creator (*mubdiʿ*) only, from Whom the unities (*al-waḥdāniyyāt*) that are in numbers and in numbered objects come. The second is divided into "what is the principle of number (*mabdaʾ al-ʿadad*) without being in number" and "what is the principle of number being in number."[99]

The former is compared to the unity of the Active Intellect, whereas the latter is divided into "what is in number as a part of it" – the number two, for example, is composed of two units and the relationship of unity with the number diminishes as it increases to a higher value –, and "what is in it as necessarily linked to it" (*k-al-lāzim lahu*) – there is, for example, an inseparable unity in the numbers two and three as with all compound and simple objects that are one in terms of genus, species or individual.

A similar passage is found in Ep. 39, *On the Kinds of Motion*,[100] where the Ikhwān explain how the world came to be made by the Creator:

> [...] the existence of the world from the Creator [...] is not like the existence of a house due to a builder or like the existence of a book due to

97 al-Shahrastānī 1842–1846, Part II, 266.11–12.
98 Cf. Baffioni and Poonawala 2017, 199–200.
99 al-Shahrastānī 1842–1846, Part II, 266.13–15.
100 Cf. Baffioni and Poonawala 2017, 163–164.

> a writer, fixed in itself, independent in its essence, which can dispense with the writer after he has finished writing, and with the builder after he has finished building the house, but [it is] like the existence of speech that is due to a speaker, so that were he to fall silent, the existence of the speech would come to an end. So speech [is] what is existent as long as the speaker speaks, and when he falls silent, it has no existence. Or like the existence in the air of the light from a torch: as long as the torch remains, the light continues to exist [...]. Or like the existence of number from the [unit] one that precedes [the number] two [...].[101] Know that the speech of a speaker is not a part of him, but an act he accomplishes and a deed he makes manifest where there was no act before. The same rule applies to the light from the mass (*jirm*) of the Sun, which is visible in the air: it is not a part of [the Sun], but is free flowing from it and a surplus of emanation. The same rule applies to the heat [that] spreads from and around [a fire]: it is not a part of its essence, but a surplus it confers, the emanation of a goodness that it emanates, and an act it accomplishes where there was no act before, just as the speaker makes [his] speech manifest where before he had not been speaking, and the speech is not a part of the speaker, but an act he accomplishes and a work he makes manifest.[102]

Shahrastānī says the inseparable unit is the one received by the Creator's unity that exists in the essences of beings, despite their plurality. The further a thing is from plurality, the nobler it is.[103]

5 Conclusions

Though some ancient sources express ideas similar to those referred to in Ep. 32 and Ep. 33,[104] what we have seen so far suggests that the core of the

101 This is a common statement in the encyclopaedia. See, e.g., Ep. 1, I, 50.2, 54.15 Bustānī = El-Bizri 2012, 12.14, 23.3 (Arabic text) and 68, 73 (English transl.).

102 This concept of "part" is also found in a passage of Ep. 40. See Baffioni and Poonawala 2017, 189–190.

103 al-Shahrastānī 1842–1846, Part II, 266.11–267.6. French transl. in Shahrastani 1986–1993, vol. 2, 202.

104 E.g., for the resemblance between things and numbers cf. Alexander of Aphrodisias, *ad Arist. Metaph.* 985b23 ff., part. 38.10 ed. Hayduck; for number one as the origin of numbers: Aristotle, *Metaphysics*, 986a15 ff.; for the one that gives its name to every number and numbered: Theon of Smyrna, 22.5 ed. Hiller; for the distinction between odd and even numbers: Schol. 7 in Euclid. *Elem.* VII, in Heiberg V, 364.6; on the universe as a sphere and

Pythagoreanism of the Ikhwān al-Ṣafāʾ is the conception of the number one as the root of numbers, which makes it wholly different from all other numbers.

This endorses the absolute monotheism of Islam. Substantial parts of Shahrastānī's chapter on Pythagoras are echoed in passages of the encyclopaedia that are in no way connected with Pythagoreanism and that have almost nothing in common with those in Ep. 32 and Ep. 33. I have tried to connect another passage in Ep. 11, *On the Meaning of the* Isagoge – similar to the one above describing an ecstatic experience – with an "*ishrāqī*" milieu.[105] I give it here in abridged form:

> As for the tools of pure disincarnate souls, they do not need phrases or statements to make […] concepts […] reciprocally understood; these are the souls of angels […] purified from the filth of bodily passions […] attached to the shining substances and to the diaphanous globes, a part of which appears in the whole while the whole appears in the part, such as the surfaces of polished mirrors appear one inside the other […] They are within the illumination of the lights that are the place of origin of the best and the pious.[106]

The desired agreement between divine books and sages and philosophers, repeatedly affirmed in Ep. 33, is another basic tenet of the Brethren's invitation to science that is also an esoteric knowledge and lies outside Pythagoreanism. And the Ikhwān would never have accepted the Active Intellect to be considered "the principle of number without being in number" because God is the principle of number and the Intellect is compared to the number 2 – the first number.

The teaching in Shahrastānī that lies closest to the Ikhwān's "Pythagorean" epistles is that the relationship of unity in a number diminishes as the number rises to a higher value, and consequently the closer a creature is to the One the more perfect it is. This explains the primacy assigned by the Ikhwān to number one, which is considered a perfect image of God. But if the origin of this idea is Pythagorean, it has been adopted and discussed by numerous Islamic onto-cosmologies as in the later Ismāʿīlī thinker and missionary Ḥamīd

on the earth at the centre of the universe: Diogenes Laertius, VIII 25. See in Timpanaro Cardini 1962 [t. 2], 342–343, and 1964 [t. 3], 68–69, 78–79, 120–127, 220–221.

105 Cf. Baffioni 2008, 175 ff. *Ishrāq* (*illumination*) *is* the name given by Shihāb al-Dīn Suhrawardī (d. 1191) to his system based on illuminative wisdom. Greek elements are recognizable in it along with Eastern elements deriving from the traditions of classical Persia and ancient Egypt.

106 I, 402.13–403.3 ed. Bustānī = Baffioni 2010, 41.1–42.9 (Arabic text) and 83 (English transl.).

al-Dīn al-Kirmānī (d. after 1020),[107] which actually brings the Ikhwān closer to proto-Ismāʿīlī ideas.[108]

The Brethren use the idea of the number one as the root of numbers to clarify the difference between God and created things. Apart from some arithmological approaches, the Pythagorean core of the encyclopaedia is to be found in Ep. 1 and related passages and in Ep. 6, *On the Arithmetical, Geometrical and Musical Proportions*, which contains several textual quotations from Nicomachus of Gerasa.[109]

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107 On Kirmānī's main work, *Repose of the Intellect*, see De Smet 1995.

108 In Straface 1987, the Ismāʿīlism of Ep. 32 and Ep. 33 is emphasised over Pythagoreanism.

109 Cf. Baffioni and Baffioni 1995; Baffioni 1997.

Baffioni, C. 2018. “L’Epistola 52 degli Iḫwān al-Ṣafāʾ ‘Sulla magia’.” In L. Bianchi and A. Sannino (ed.), *La magia naturale tra Medioevo e prima età moderna*. Firenze: SISMEL-Edizioni del Galluzzo, 15–37.

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CHAPTER 10

"Pythagoras' Mistake": The Transmigration of Souls in the Latin Middle Ages and Beyond

Irene Caiazzo

In the *Disputatio de anima rationali secundum substantiam* (the *reportatio* of a course given at the *Collegio Romano* in 1609–10), the Jesuit Nicola Baldelli, Professor of Metaphysics, affirms that a rational soul cannot be present in several human beings, either simultaneously or successively.[1] Baldelli has in mind both Averroes, whose doctrine of the unity of the intellect he criticizes throughout his course, and the tenants of the transmigration of the soul, viz., among others, Pythagoras and Plato, as well as certain heretics, such as the Albigensians and the *Albanenses* (i.e., the Italian Cathar community based at Desenzano on Lake Garda). According to them, the soul could transmigrate, sometimes into the body of another human being, sometimes into that of an animal. Baldelli gets his information from the commentaries of the Conimbricenses, viz., Coimbra Jesuit Aristotelian course, whom he cites explicitly and who had previously juxtaposed the doctrines of the unique intellect and of transmigration. So far, then, there is nothing surprising here. However, Baldelli goes further: he rejects transmigration by appealing to the doctrine on the soul of the Roman Catholic Church, as established by the bull *Apostolici Regiminis* in 1513. He says that because of the multiplicity of the bodies into which it is infused, the rational soul is singularly multipliable, that it has always been multiplied, and that it will be multiplied in the future.[2] We should specify that Baldelli does not cite the definition of the *Apostolici Regiminis* literally, and that the latter says not a word about transmigration, the two doctrines

* English translation by Michael Chase (CNRS, Centre Jean Pépin – UMR 8230). I am grateful to Luca Bianchi (Università degli Studi di Milano), Barbara Obrist (CNRS, SPHERE – UMR 7219), and Stéphane Toussaint (CNRS, Centre André Chastel) for their valuable and helpful remarks.

1 Nicola Baldelli 2000, 119: "Dico tertio, anima rationalis neque simul neque successive potest esse una in pluribus hominibus numero differentibus."

2 Ibid., 120: "Nostra tamen conclusio expresse definitur in Concilio Lateranensi sub Leone decimo sessione 8a saepius citato, ubi dicitur anima rationalis pro multitudine corporum, quibus infunditur, non solum esse singulariter multiplicabilis, sed etiam de facto semper fuisse multiplicata et in posterum esse multiplicanda."

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expressly condemned being mortalism and the unicity of the intellect.[3] In other words, Baldelli uses the definition of the *Apostolici Regiminis* in order to reject the transmigration of the rational soul from one human being to another human being. He then uses a passage from the *De anima* to reject the transmigration of the soul into brute animals, viz., because Aristotle "denies that any soul can enter any body whatsoever" (*De anima* I, 3, 407b20–23). What is more, presumably relying on a gloss taken from the commentary on the *De anima* by the Conimbricenses,[4] Baldelli rejects the transmigration of the soul of one human being into another human being, using the argument that it is impossible to uphold the distinction between individuals who share the same soul. Finally, he concludes that there is no proof attesting transmigration. Baldelli uses several types of arguments – of faith, philosophical, rational – all of which converge among themselves. The doctrine of transmigration, like that of the unicity of the intellect, entails the very considerable defect of undermining the singular individuality of human beings.

It may seem surprising that the transmigration of the soul should be combated at the beginning of the 17th century. This doctrine has a long, and in a sense negative, history, for it is criticized more or less severely by almost all the authors of the Middle Ages – who will be the subject of the present study – but also of the Renaissance. In fact, the history of the doctrine of transmigration is the history of its condemnation, and Baldelli is merely one of the last links in a long chain. His arguments are to be found, in part, in the commentary by the Conimbricenses and in the writings of medieval authors: the association of the doctrine of the unicity of the intellect with the transmigration of the soul occurs as early as the 13th century, as does the appeal to Aristotle's *De anima*, and the attribution of belief in transmigration to the Cathars. Let us begin with the latter.

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3 On Baldelli and the *Apostolici Regiminis*, see De Libera 2014. De Libera does not mention the transmigration of the soul. See the definition of the soul in the *Apostolici Regiminis* in Alberigo 1973, 605: "Hoc sacro approbante concilio damnamus et reprobamus omnes asserentes animam intellectivam mortalem esse, aut unicam in cunctis hominibus […], et pro corporum quibus infunditur multitudine singulariter multiplicabilis, et multiplicata, et multiplicanda sit." On the medieval background of the *Apostolici Regiminis*, see Bianchi 2008.

4 Nicola Baldelli 2000, 121: "Quod vero neque etiam detur huiusmodi tranmigratio in corpora hominum, probatur ex dictis conclusione secunda, quia alioquin non potest salvari distinctio illorum individuorum, quae habuissent eandem animam, et praeterea, quia nullam habemus indicium, ex quo huiusmodi tranmigratio colligi possit." Wels, the editor of the text, does not identify the source of this passage. Baldelli seems to be citing the commentary on the *De anima* by the Conimbricenses published by Horace Cardon (Conimbricenses 1604, 124, note "p"). The subdivision of the *De anima* he uses is, moreover, the same as that of the Conimbricenses 1604. On the Conimbricenses, see Carvalho 2018.

The belief in the transmigration of the soul is imputed to the Cathars by inquisitors and polemicists as early as the end of the 12th century. The French theologian Alan of Lille is probably the only one who connects their belief to Pythagoras.[5] In the chapter of his *Dialogus miraculorum* (*ca* 1220) devoted to the Albigensian heresy, Caesarius of Heisterbach tells the story of a Cathar knight who, since he believed in the transmigration of souls, wished to make a good impression by giving alms to the poor, for he was afraid of ending up, after his death, in the body of a bad man, or worse, in that of an animal.[6] The Dominican polemicist Moneta of Cremona also attributes belief in the transmigration of souls to the Cathars, although he does not dwell on this point (1241).[7] The Dominican Stephen of Bourbon attributes to the Albigensians the belief that souls can migrate into trees and plants (1256–1261).[8] Moreover, several accounts of the interrogations transcribed in the books of the inquisitors mention the transmigration of the soul. For instance, the record of Jacques Fournier includes a deposition by Arnaud Sicre d'Ax (recorded in 1321–1322), who attributes belief in transmigration to the "heretics." In particular, he recounts at second hand the story of a man who remembered having been a horse in a previous life when he took the road on which he had once lost his horseshoe, and that he had found it again.[9] Two other testimonies are provided by the interrogations carried during the inquest by the Dominican inquisitors Bernard de Caux and Jean de Saint-Pierre in Southern France during the years 1245–1247: eyewitnesses heard the heretics speak about reincarnation and the soul's peregrinations after death.[10] These testimonies do not mention the

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5 See Alan of Lille 1855, 137: "[…] in errorem pythagoricum cadunt, qui asseruit animam hominis merito peccati post mortem intrare in corpus alterius hominis vel bruti animalis."

6 Caesarius of Heisterbach 1851, 301: "[…]. Credidit stultus, sicut et ceteri Albigenses, quod anima secundum meritum transeat per diversa corpora, etiam animalium atque serpentum." See also Maaz 1998, 387. On the Cathars and reincarnation, see Zander 1999, 199–216.

7 Moneta of Cremona 1743, 61–62.

8 Lecoy de la Marche 1877, 301.

9 Duvernoy 1978, vol. III, 763–764: "[…] He answered that there was no other hell than this visible world, in which spirits move from body to body and from tunic to tunic, as an act of penitence […] 'When I was a horse, one night I lost a shoe between these two rocks, and then I wandered unshod throughout the night.' Thereupon they both began to search for that horseshoe, found it between the two rocks, and preserved it. The heretic then said that the human soul enters the body of a beast and the spirit of the beast into the body of a human being."

10 The testimonies on transmigration were pointed out by Duvernoy 1978. The interrogations are preserved in the MS Toulouse, Bibliothèque d'Étude et du Patrimoine de Toulouse, 609, fol. 120v: "Item dixit quod audivit hereticos dicentes quod quando anima exiebat de corpore hominis, intrabat corpora asinorum"; 125r: "Et audivit eos dicentes

learned authority of Pythagoras or other ancient philosophers, but seem to refer instead to popular beliefs in transmigration or reincarnation, without any particular philosophical certification. It should be emphasized that the two Cathar texts from the 13th century discovered long ago by Antoine Dondaine, viz. the *Liber de duobus principiis*[11] and the *Anonymous Cathar Treatise* (which is known only fragmentarily from the quotations included in the *Liber contra Manicheos* attributed, without any certainty, to Durandus of Huesca),[12] do not contain any explicit reference, or the slightest allusion to the transmigration of the soul. The polemicists' accusations thus reveal themselves to be without foundation, if one relies on the preserved Cathar treatises. These, however, are perhaps not entirely representative of Cathar doctrine, which must have included several different tendencies. Hans Söderberg, in his authoritative study, attributes the belief in transmigration only to "absolute Cathars."[13]

1 The Return of Pythagoras

Let us now focus on the philosophers and theologians active from the end of the 11th century, for this is precisely the period in which the doctrine of the transmigration of the soul began to be discussed, quite probably owing to the increased diffusion of the ancient authors who transmitted it, such as Calcidius and Macrobius.[14] Manegold of Lautenbach wrote the *Liber contra Wolfelmum* in the period of the quarrel over investitures, in which he was on the side of Pope Gregory VII († 1085). He denounces and refutes the doctrines of the pagan philosophers, including "metempsychosis": Pythagoras believed that if a soul does not conduct itself correctly during its first incarnation, it would enter an inferior body, or even several bodies, in order to expiate its sins.[15] Yet

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quod quaelibet anima hominis circuibat tot corpora hominum quousque posset salvari." This document is still unpublished. The Toulouse manuscript dates from the mid-13th century.

11 The *Liber de duobus principiis* (see Thouzellier 1973) is a collection of texts written before 1250 by Jean of Lugio (or by one of his disciples), the leader of the *Albanenses*, an influential figure of the Cathar community of Desenzano.

12 Thouzellier 1961.

13 Söderberg 1949 and 2015, 152–154. See also Greco 2000, 200–205.

14 In the Middle Ages, the sources that mention the Pythagorean doctrine of transmigration are highly scattered and heterogeneous. There is no specific study on this topic: for a first approach, essentially doxographical but well documented, see Maaz 1998, Zander 1999, and Roling 2016. These three studies are complementary; Maaz 1998 concentrates mainly on enumerating the literary texts.

15 Manegold of Lautenbach 1972, 44–47.

according to Manegold, it is impossible for the human soul to deviate from rationality and become irrational. He then addresses his adversary Wolfhelm, with obvious irony, to remind him that, together with his acolytes who defend the Germanic emperor against Pope Gregory, he will be incarnated in several bodies in order to purge his sins.[16]

William of Conches does not take the Pythagorean doctrine of transmigration literally; he mentions it very sporadically, although he commented on Plato's *Timaeus* and Macrobius' *Commentary on the Dream of Scipio*. For him, it is a metaphor (*similitudo*) used for moral purposes, aimed at encouraging virtuous behavior. In fact, there is no question at all of a change of place or essence; the soul of a man will not enter the body of a woman, then that of an animal, and so on as far as earthworms.[17] A moral interpretation is proposed in the *Glosae Colonienses super Macrobium*, an anonymous commentary on the *Commentary on the Dream of Scipio* from the beginning of the 12th century. The commentator first sets forth in a neutral way Pythagoras' theory of metempsychosis, viz. that the soul, once it has exited the body of a just man, goes directly to heaven, whereas the soul of a bad man passes through several bodies, moving into bodies of animals that are situated on increasingly lower levels of being in order to expiate its faults, before being able to turn back, re-traversing the cycle in reverse until reaching the body of a human being once again, and finally, after several centuries of transmigrations, reaching the heavens. Next, the commentator explains that Plato corrected this doctrine, by adducing a moral interpretation: the soul can follow different ways of moral behaviour, and if, as a consequence, it yields to jealousy, it is said to live in the body of a dog, and if it yields to anger, in the body of a lion.[18]

16 Ibid., 46–47.

17 William of Conches 2006, 223: "Et haec mutatio similitudine, non loco uel essentia, est credenda. Non enim credendum est eandem animam prius esse in uiro et post transire in mulierem, deinde in bruta animalia usque in uermes ut affirmant quidam Pitagoram uoluisse. Nec credendum est quod ipsa anima in se aliquem sexum habeat, sed secundum mores haec mutatio est tenenda."

18 Caiazzo 2002: "(*Comm.* I, 9, 4) In pecudem ex homine. Hic vocatur metapsichosis Pitagorae, id est transitio animae: 'meta' est 'transitio,' 'phios' [*sic*] 'animae.' Nam Pitagoras dixit animam iusti exeuntem corpus statim adire caelum; cum vero exiret iniusti, non caelum intraret, sed de malo corpore in peius, ut tandem de homine in bestiam, de bestia etiam in ulteriora et ultra; iam, non invento deteriori, iterum gradatim ascenderet de malo in minus malum, usque ad hominem, sicque transactis multis seculis tandem rediret in caelum. Plato vero hanc sententiam corrigens moraliter intelligi voluit, dixitque animam in humano corpore diversos mores accipere, et secundum diversitatem morum asseruit diversitatem corporum, ut anima quae invide vixit in homine, diceretur vivere in canino corpore, cumve libidinose in hircino, cumve iracunde in leonino."

The concern to exonerate Plato is also manifest in the *Glosae super Platonem* attributed to Bernard of Chartres, who quotes the *Commentary on the Timaeus* by Calcidius: it was Pythagoras who maintained that human souls are really able to move into the bodies of animals, whereas for Plato this was a manner of speaking, as when it is said that a man is behaving like a woman when he lives in an effeminate way, or like a pig if he lives in a filthy way. What is more, according to Plato, animals have no soul, but merely a vital principle.[19] In contrast, Pythagoras considered that all souls were equal, and that the rational part of the soul would only be active in certain bodies in accordance with the latters' capacities; in other words, brute bodies are obstacles to the deployment of reason.[20] (We shall see below that this explanation is also proposed by Albert the Great.) John of Salisbury devotes a chapter of his *Policraticus* (1159) to the life of Pythagoras, a chapter that was re-used by, among others, John of Wales and the pseudo-Walter Burley. John of Salisbury mentions the theory of transmigration, of which he disapproves in no uncertain terms, and which, he claims, Plato was wrong to follow. Like many other medieval authors, John quotes a passage from the speech of Pythagoras from book XV of Ovid's *Metamorphoses*, in which the philosopher remembers having been Euphorbus in a previous life at the time of the Trojan War. John of Salisbury specifies that one should not allow oneself to be blinded by the authority of Pythagoras, for "when one gathers a rose, one sometimes is stung by a nettle."[21] In short, despite the obvious admiration John of Salisbury feels for Pythagoras and the Pythagoreans, whose highly sober lifestyle he appreciates above all, he is not prepared to follow them as far as the transmigration of souls is concerned.

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19 Bernard of Chartres 1991, 199: "Sententia Pythagorae fuit quod realiter animae hominum in alia animalia transirent. Sed Plato, teste Calcidio, hanc mutationem incorporationis noluit, immo quod in eodem corpore diuersa animalia uiuerent, ut si quis molliter uiuat, mulierem uiuat, si immunde, porcum et similia. Et, secundum hanc Platonis sententiam, dicuntur bruta animalia non habere animam, nisi quamdam uitalitatem quae, pereuntibus ipsis, redeat ad uniuersitatem."

20 Ibid., 195: "[…] id est animam UNIUERSI GENERIS, id est omnium hominum, secundum Calcidium, uel omnium animalium, secundum Pythagoram, sed non in omnibus exercentem rationem, quia corpora impediunt."

21 John of Salisbury 1909, vol. 2, 133–134: "Dum innocentiam, frugalitatem, contemptum mundi docent Pitagorici, audiantur; dum animas, quas in celum euexerunt, retrudunt in corpora bestiarum, iuguletur uel Plato. Nam in eo Pitagoram nimis secutus est quod sicut mortuos ex uiuis, ita ex mortuis reciproce uiuos fieri docuit et per uaria corpora pro morum qualitate migrare. Vnde illud: *Ipse ego, nam memini, Troiani tempore belli/ Pantoides Euforbus eram, cui pectore leuo/ haesit ab aduerso grauis hasta minoris Atridae*. Sic ergo legantur ut auctoritas non praeiudicet rationi; nam et urtica, dum rosa legitur, quandoque manum tangentis urit."

In summary, the conception, of Platonic origin, of the soul as an immaterial, immortal substance, which moves and is separable from the body, is dominant until the end of the 12th century. Known primarily thanks to Calcidius and Macrobius, the Pythagorean doctrine of the transmigration of the soul could be considered as a variation, or even a somewhat inflammatory appendix to the Platonic doctrine. It does, however, not represent a challenge to the theoretical foundations of this doctrine.

2 Aristotle Against Pythagoras

Beginning in the 13th century, the diffusion of the Latin translation of Aristotle's *De anima* marked a turning point in the reception of the Pythagorean doctrine of the transmigration of the soul. The most articulated discussion, and, to my knowledge, the longest, is to be found in the *De universo* of William of Auvergne, begun around 1230 and probably completed around 1240.[22] The bishop attacks the Platonic doctrine of the pre-existence and incorporation of the soul, which will however not be discussed here. The critique of Pythagoras concerns several aspects, and arguments *pro et contra* are presented for each of them. Let us try to reconstruct the main stages of his argument.[23] First, William begins by reminding the reader that man is defined as a rational animal precisely because of his rational soul. If the soul of a man passes into the body of a brute animal, there will, as a consequence, be rational brute animals.[24] Those who consider that bodies can prevent or, on the contrary, facilitate the use of the rational soul – which, incidentally, would imply that reason is not connected to human beings by nature, but simply according to the random nature of bodies – are forced to admit that the disposition of the bodies of brute animals is not compatible with human souls: hence the impossibility of their utilizing a rational soul. It follows, according to William, that God has sometimes created animal bodies with brute souls, sometimes animal bodies that are capable of receiving rational souls some day, with the consequence

22 Gauthier 1982, 356; Teske 2006, 9; Sannino 2018, 42.

23 William of Auvergne 1674, vol. 1, 704bG-707aA (ch. XIV: Hic impugnat opinionem Pythagorae de transitu animarum.) The only scholar to have analyzed William of Auvergne's critique of the Pythagorean doctrine of the transmigration of the soul is Roling 2016, 109–113.

24 William of Auvergne 1674, 704bG: "Homo enim nec est, nec dicitur animal rationale, nisi propter animam rationalem, et nullo modo propter corpus. Cum igitur animae rationales, hoc est, animae humanae sint in corporibus brutorum, eadem de causa erunt bruta animalia rationabilia."

that man would be incapable of knowing whether he possesses real animals, real asses, real cattle, and real horses, or whether he has to do with an animal that could be inhabited by a soul that comes from a human being, even the soul of his parents, his brother, or one of his relatives.[25] The second argument concerns the desire which, according to Pythagoras, souls have of uniting with the body. According to William, the soul cannot have a natural desire for incarceration, that is, to enter into a body in order to expiate its faults. On the contrary, the human soul's incorporation is natural, and the separation of the soul from the body is naturally violent (death); it is unthinkable that the human soul should desire to enter the body of an animal.[26] William goes on to add that there is no reason for the human soul to be purified by means of a sojourn in the body of an animal,[27] for why should the soul undergo all these passages, which it will not even remember once it has left the bodies in question? After all, the soul does not retain any memory of its previous lives, with a few exceptions mentioned by the poets. In any case, no one – not even sages, philosophers, or students – recalls the blessed sojourn of his soul in the heavenly sphere, on its star, before and after incorporation. Only madmen claim to have been kings or birds in a previous life.[28] In other words, according to William, we simply have no proof of the transmigration of the soul. Another argument concerns the "traffic jams" that the migrations of souls – of all souls, including those of flies – could cause, sometimes between bodies, sometimes between stars and bodies, and vice versa; not to mention, William adds, that the number of stars would not suffice to lodge the souls exiting the bodies of all animals, including flies and fleas.[29] Other considerations follow; firstly, on the inefficacy of a punishment that is not remembered, and secondly, on the paradox of brute animals possessing a human soul, which leads to the positing

25 Ibid., 704bH–705aA.

26 Ibid., 705aB–705b.

27 Ibid., 705bB: "Quare aptiora expiationi et santificationi animarum humanarum sunt corpora brutorum quam corpora humana."

28 Ibid., 705bD: "Quod si dixerit, quia aliquae aliquando reminiscuntur se fuisse in aliis corporibus, quia et talia narrantur a poetis: respondeo, quia nec illis creditur a sapientibus, nec illi probant hoc, praesertim cum ex narrationibus eorum, nec praesumptio appareat eas vel in stellis fuisse, vel in aliis corporibus, sed quemadmodum mente alienatis interdum videtur, quod fuerint reges, vel etiam aves, ut apparet manifeste desipientia eorum sic dico et de hominibus huiusmodi." Ibid., 706aE: "In mille millibus autem studentium et discentium, vel etiam philosophantium, non est invenire animam unam, quae reminiscatur aliquod eorum, quae dicit se scivisse in coelo, vel apud stellam suam."

29 Ibid., 706aG: "Quotidie igitur revertuntur innumerabilia millia animarum humanarum ad compares stellas de solis corporibus muscarum. Amplius. Manifestum est non esse tot stellas in coelo, quot sunt pulices et muscae."

of the existence of truly brute animals (not endowed with a human soul) and of non-truly brute animals (with a human soul).[30] Further, William attributes the doctrine of transmigration to the Manichaeans, but without dwelling on this particular issue.[31]

Finally, without saying so explicitly, William mobilizes an Aristotelian definition of the soul (*De anima* 412a26–28): the brute soul is the form and perfection of a brute animal; it is the natural inhabitant of an animal body, which is its home. Why would this not be the case for the human soul which, on the contrary, is obliged to enter into a body of which it is not the natural perfection, in which it would merely be a guest, according to the opinion of Plato and Pythagoras?[32] Still referring implicitly to the Aristotelian definition of the soul, William explains that the human soul is just as much naturally organic, that is, instrumental, and adapted to accomplish tasks, as the animal body; it is even far superior to that body. It therefore cannot be envisaged that the Creator did not provide for a natural perfection that was appropriate and specific to the human body.[33] Human souls were not created by God outside of human bodies.[34] He then concludes by affirming that the doctrine of Pythagoras is an aberration, for the human soul cannot have a natural desire to pass from one

30 Ibid., 706aH: "Erunt igitur quaedam animalia brutalia vera, et quaedam falsa, quod est dicere, equi veri et equi quidam non veri, et de leonibus similiter: manifestum autem est, quia translatio animarum humanarum in corpora brutorum, nihil omnino habet utilitatis, plurimum vero habet noxietatis"; 706bE: "Inutilis autem est manifeste poena omnis, quae nec patienti, nec aliis prodesse potest, talis est autem omnis poena incogniscibilis."

31 Ibid., 706bF. According to Alan Bernstein, 1) William of Auvergne makes no distinction between the Manichaeans and the Cathars, and 2) The *De universo* may be considered as refutation of Catharism (cf. Bernstein 2005). I am unable to confirm or invalidate this thesis. See also, in this regard, the brief remarks by Roling 2016, 109.

32 Ibid., 706bG: "[…] cum anima brutalis omnis pars sit ipsius animalis bruti, ut forma ipsius, atque perfectio corporis eiusdem, et ut ita dixerim, naturalis inhabitatrix corporis eiusdem, et corpus brutale habitaculum ipsius, anima vero humana econverso […]." William says the same thing when criticizing the Platonic theory of the pre-existence of the soul in the *De anima*; cf. William of Auvergne 1674, vol. 2/2, 125. Cf. the definition in the *De anima*, William of Auvergne 1674, vol. 2/2, 65b: "Anima igitur est prout diffinit Aristoteles perfectio corporis physici organici potentia vitam habentis." William then explains that this definition is appropriate for humans and animals, but not plants. On the psychology of William of Auvergne, see at least Bazán 1969, 43–48, who however does not discuss transmigration. On the Aristotelian definition, see Lenzi 2011.

33 Ibid., 706bH: "Cum corpus humanum non minus organicum sit naturaliter, nec minoris aptitudinis naturalis, immo evidenter incomparabiliter maioris, quam corpus brutorum ad operationes multiplices, […] quomodo creator non providit huiusmodi corporibus naturalem perfectionem eis congruentem et debitam, ut est perfectio naturalis?"

34 Ibid., 706bH–707aA.

body to another.[35] William's arguments are primarily counterfactual, taken from common sense and logic, while the Aristotle of the *De anima* makes a furtive and anonymous appearance.

The first author to explicitly refer to Aristotle's *De anima* in order to counter Pythagoras is Roland of Cremona in his *Summa theologiae*. Very probably written around 1244, it is presumably the written version of the teachings of Roland, who was the first Dominican regent master in theology at Paris (1229–1230).[36] Before entering the Dominicans in 1219, Roland of Cremona had first been *magister artium* and of medicine in the *Studium* of Bologna. This, no doubt, explains his approach to the doctrine of transmigration, debated in the context of the discussion of the animation of the fetus. Pythagoras, writes Roland, maintained that the human soul is able to pass from one body to another, even into the body of a horse.[37] This would mean that a man's soul can enter, after its death, the body of a horse; that is, it would be fitted to the new body in the womb of the horse's mother. This soul possesses the intellective faculty, and yet the horse is not capable of learning geometry. If the soul cannot make use of its rationality in the body of a horse, then it will not be able to make use of its free will either; hence, it will not be able to be rewarded or reprimanded. This is why it is by no means possible that the passage through many bodies will be able to make the soul better. Roland then makes a striking analogy: there are appropriate receptacles for oil, for nuts, and for wine, and since the body is like a receptacle for the soul, each body is not appropriate for just any soul. According to Roland, this is an argument by Aristotle against Pythagoras. In fact, it is a quotation from the *De anima* (I, 3, 407b20–23;[38] we have already encountered it above with regard to Baldelli). What is more, since the soul is the perfection of the body and the form of the organic body, one body requires a rational soul,

35 Ibid., 707aA: "Pythagorae vero maior evidenter esse apparet insania in translatione animarum humanarum a corporibus in corpora. Impossibile enim est, ut hoc naturali earum desiderio fiat, vel etiam alio ipsarum desiderio, [...]."

36 Gauthier 1982, 330–331.

37 Roland of Cremona 2016, 332: "Videamus si possit esse vera opinio Pithagore qui dicebat quod anima transit de corpore in corpus, et etiam in corpus iumenti. Hanc opinionem bene vocat Aristoteles fabulam in libro de anima."

38 See the Greco-Latin translation of *De an.* I, 3, 407b20–26 by James of Venice, in Albert the Great 1968, 37: "De susceptibili autem corpore nihil adhuc determinant, tamquam contingens sit secundum Pythagoricas fabulas *quamlibet animam quodlibet ingredi corpus*. Videtur autem unumquodque propriam habere speciem et formam. Simile autem aliquid dicunt, sicut si quis dicat tectonicam tibicines indui. Oportet enim artem uti organis, animam autem corpore."

and another body requires a brute soul.[39] Roland adds a supplementary argument that combines medical knowledge, theology, and Aristotelian physics: the soul acts on the body through the vital spirit, which is located in the heart. God prepares the heart and the spirit at the same moment, confers form, and inserts the soul. The latter cannot therefore be external to this preparation, for it cannot enter the heart in an instant, since instantaneous motion is impossible. Finally, Roland completes his tirade by calling Pythagoras stupid and a heretic as far as natural philosophy is concerned, although he was strong in the arts of the quadrivium.[40] The science of the soul, or psychology, is quite clearly considered as a part of natural philosophy, and the shift toward Aristotle has taken place.

The *Sententia super De anima*, a literal commentary by an anonymous Master of Arts which is to be dated around 1246/1247, limits itself to paraphrasing Aristotle's text, explaining that the soul cannot enter a new body after death, because the soul cannot be the act of just any body.[41] The situation is quite different for the anonymous Master of Arts of the *Lectura in Librum de anima*, which its editor R.-A. Gauthier conjecturally dates to the 1246/1247. After excluding the possibility that just any soul can enter just any body,[42] the Master of Arts admits that a human soul can enter into any human body and move it. Moreover, once it is separated from the body, the soul will be able to enter another body of the same species, just as a pilot can steer any boat of the same type.[43] Later on, the Master demonstrates that it is possible by nature that the soul, once separated from the body, can enter another body

39 Roland of Cremona 2016, 332: "Vel ita: aliud vas vult habere vinum, et aliud oleum, et aliud nuces; ergo, cum corpus sit vas anime, non competit cuilibet anime quodlibet corpus. Et hec est probatio Aristotelis contra Pithagoram. Et potest melius explicari, quia anima est perfectio corporis et forma corporis organici; ergo aliud organicum vult habere rationalis anima, et aliud anima bruti." The apparatus of sources contains no reference.

40 Ibid., 333: "[…] ergo in illo instanti in quo facta est preparatio, datur forma, sed in instanti non posset illa anima, que extra est usque ad preparationem, cor intrare, cum non possit fieri motus in instanti, ut probatum est superius. Ergo *novam animam infundit novis corporibus*, sicut dicit Augustinus. Quod concedimus, et quilibet sane mentis. Nec nocet si hoc dicit Pithagoras, quoniam ipse non fuit philosophus in naturalibus, sed potius stultus et hereticus, etsi valuerit in quadruvialibus."

41 Anonymus Magister Artium 1998, 60. According to the Greco-Latin translation of the *De anima*, the soul is the act of the body (*actus corporis*).

42 Anonymus Magister Artium 1985, 111.

43 Anonymus Magister Artium 1985, 126: "[…] ut nauta qui mouet hanc nauem quamlibet nauem eiusdem speciei potest mouere; ergo, cum anima humana sit ad mouendum corpus humanum, quodlibet corpus humanum potest mouere; set non mouet nisi corpus quod ingreditur; ergo poterit ingredi quodlibet corpus humanum; et ita, separata ab uno corpore, poterit intrare quodlibet eiusdem speciei."

predisposed to receive an intellective soul. However, the author concludes that by divine disposition, it is not appropriate that a soul that has committed good or evil in a given body should subsequently enter another body without being punished or rewarded for what it has accomplished previously; since the end of the lecture on the first book is lacking in the manuscript, we will, unfortunately, not learn further details.[44] At any event, here this Master of Arts unexpectedly reverses the Pythagorean doctrine: entering a new body is not a punishment, but there is a risk that the soul may not be judged for its actions as it should. Further on, the Master once more evokes the Pythagoreans and Aristotle. He rejects the thesis that one and the same soul could act in several bodies at the same time (*simul*) and successively (*successive*), since this would lead to many objections.[45] All in all, the Master of the *Lectura* concedes the possibility of transmigration according to nature, no doubt because he is an Aristotelian and a Platonist at the same time: the soul, as the motor and form of the body, is immortal and separable from the body.

Albert the Great dwells several times on transmigration, which he rejects without hesitation. In the *De animalibus*, he affirms that the behavior of some animals – such as ants and bees, who store their food, for instance – seems hard to explain without supposing some assistance by reason: this is precisely why Pythagoras had maintained that the rational soul was present in men just as much as in animals.[46]

In the *De natura et origine animae*, Albert proposes to compare the doctrines of Pythagoras and Plato on transmigration. According to Pythagoras, the soul is immortal, and after death it passes into a new body, for he made no distinction between souls, and considered them all equal and of the same nature, their operations being conditional upon the bodies in which they were present. It would therefore be the instrument, that is, the body that makes the difference: a defective instrument does not allow the rational soul to exercise its functions. What is more, the alleged equality between the souls of human beings and of animals is supposed to have led Pythagoras to posit, in his laws, the prohibition against killing and eating animals, and Albert adds a quote

44 Anonymus Magister Artium 1985, 127: "Et dico quod secundum naturam non est incoueniens animam separatam ab uno corpore ingredi corpus aliud dispositum ad susceptionem anime intellectiue. […] Tamen secundum ordinem et dispositionem diuinam esset inconueniens."

45 Anonymus Magister Artium 1985, 457–458.

46 Albert the Great 1916, vol. 1, 672–673. See also Albert the Great 1890 (*In Aristotelis libros Elenchorum*), 664b: "Verbi gratia in exemplo: quia utrum corruptibilis vel immortalis sive incorruptibilis sit anima animalium, non determinatum est multis Philosophis. Pythagoras enim dixit omnem animam animalium immortalem esse."

from Ovid's *Metamorphoses* (Book XV) on vegetarianism.[47] This is a reference to the political regime that Pythagoras had established at Crotona in *Magna Graecia*. Albert then evokes a passage from the *De anima* (407b24–26), according to which every art, like every soul, requires its own instruments. A very brief remark by Aristotle is then transformed into a development that proceeds through the following stages. Every artisan (*artifex*) has his own instruments. A flute player cannot carry out his art by means of the instruments of the art of architecture, for instance. Likewise, the diversity of bodies is based on the diversity of the souls that move the bodies in question, and not *vice versa*. Similarly, it cannot be said that architecture passes into another body (*transcorporetur*) and builds by means of flutes or flute-playing, since it has neither understanding nor knowledge of the playing. Nor can it be said that souls go into other bodies (*transcorporentur*).[48] Albert then explains that *transcorporatio* (a term he uses for transmigration) was merely a metaphor for Plato (who had distinguished several types of souls, and had located them in the human body), whereas for Pythagoras, for whom souls actually do enter different bodies, it corresponded to the truth.[49]

In the *De anima*, a periphrastic commentary on the Aristotelian treatise, Albert had already set forth these same arguments on the arts and the instruments that are peculiar to them.[50] In addition, no doubt following Averroes' *Long commentary* of Aristotle's *De anima*, Albert had written that Pythagoras had affirmed in his laws that the souls of bad citizens pass into the inferior body of a lion or an ass, in order to make the citizens better. These fables, completely invented, are used for political goals: once established as true, they serve to found and maintain the social and political order.[51] Be that as it may,

47 Albert the Great 2006, 174–176; 176: "Et ideo in legibus suis dixit sacrilegium esse animalia occidere et carnem comedere. *Quid*, enim, *meruere boves, animal sine fraude dolisve*? *O quantum scelus est in viscera viscera mergi!*"

48 Ibid., 176: "Sed nos videmus, quod quilibet artifex habet diversa ab alio artifice organa artis, et diversitas organorum artificum est propter diversitates artium et non e converso. [...] Similiter igitur omnis diversitas quae est in organis naturalium corporum, est propter diversitates animarum moventium illa corpora. Et sicut non potest dici, quod ars tectonica transcorporetur et aedificet fistulis aut etiam canat fistulis, eo quod non habet rationem et scientiam canendi neque ex talibus principiis est ars aedificandi, ita non potest dici, quod transcorporentur animae."

49 Ibid., 174–178; 178: "Et hunc modum metaphorice (metaphoricae *ed.*) transcorporationis Plato intellexit, Pythagoras autem secundum veritatem in diversa et naturalia corpora transcorporari dixit animas."

50 Albert the Great 1968, 38, and 60.

51 Ibid., 38: "[...] ac si ipsi arbitrentur fabulas Pythagorae in legibus suis dictas esse veras. Volens enim Pythagoras facere cives colere pietatem et iustitiam, fabulatus est animas malorum civium exire de uno corpore in aliud peioris conditionis, ita quod anima

Albert adds (as does Averroes), that transmigration is impossible, for the diversity of the contours and organs of the body is due to the diversity of souls, since the diversity of matter derives from the diversity of form.[52]

The passage from the *De anima* on the soul which cannot enter just any body (*quaelibet anima in quolibet corpore*) becomes almost a *Leitmotiv* in the writings of 13th century authors. Siger of Brabant discusses it briefly in the *De anima intellectiva*, which is probably indebted to Averroes' *Long Commentary*. The soul makes use of a determinate body that is prepared to receive its works; it is the act of a specific body (and not of just any body) that possesses life potentially. The soul is the form of the body, and each form has its own matter: it is precisely matter that differentiates a lion from a deer. This example is used, in particular, by Averroes. The *fabulae pythagoricae* are rejected by Siger of Brabant, who, however, does not examine in detail the question of the transmigration of the soul.[53] To confirm this, let us consider the *Quaestiones in ter-*

hominis ingredietur corpus leonis vel asini. Quod nullo modo fieri potest secundum naturam, quoniam non est diversitas in figuris et organis corporis nisi propter diversitatem animarum, quia diversitas materiae est propter diversitatem formae." Cf. Averroes 1953, 74–75: "Et quia hoc dimiserunt, videtur esse possibile apud eos ut quelibet anima existat in quolibet corpore et transferatur de corpore in corpus, sicut dixit Pitagoras in Apologo, quem posuit ad corrigendum animas civium. Et ista opinio est falsa; videmus enim quod quidlibet habet formam propriam et corpus proprium, idest animam propriam et corpus animalis proprium. Et hoc quod dicit est manifestum in speciebus valde. Membra enim leonis non differunt a membris cervi nisi propter diversitatem anime cervi ab anima leonis. Et si esset possibile ut anima leonis existeret in corpore cervi, tunc Natura ociose ageret. Et hoc etiam manifestum est in individuis eiusdem speciei; et ideo diversi sunt mores. Et ex hoc destruximus opinionem Pitagoricam." In Averroes' text there is the word "apologus" (which Richard Taylor [Averroes 2009, 67] translates by "myth"), whereas Albert uses the word "fabula," which was more frequent in the Latin translation (*fabulae pythagoricae* are mentioned in the Greco-Latin version of Aristotle's *De anima*). On the political use of fables and religions, see Bianchi 2018. This passage from Albert is repeated explicitly by Humbert of Prouilly; on this subject, see once again Bianchi 2018, 337.

52 It is not without interest to point out that this passage from Albert's *De anima* is subsequently quoted in the *De intellectu* of Agostino Nifo; see Nifo 2011, 187. The editor of the text erroneously points to Albert's *De natura et origine animae*, but Nifo is obviously citing the *De anima*.

53 Siger of Brabant 1972, 75: "Nunc autem anima utitur corpore ad sua opera, et est ut ars insita, ut ars domificandi si intraret ligna. Anima ergo requirit corpus determinatum, dispositum et ad sua opera praeparatum. Bene ergo dictum est quod anima est actus corporis vivere potentis, hoc est, non cuiuscumque corporis, sed eius quod est propria materia et potentia ad animam et vitam et eius opera. Et ideo male determinaverunt de anima nihil dicentes de susceptibili corpore, ac si quaelibet anima quodlibet corpus posset ingredi secundum pythagoricas fabulas. Nunc autem non est ita, immo secundum differentiam animarum, oportet esse et corporum susceptibilium differentiam, ita ut non differant membra leonis a membris cervi, nisi quia differt anima ab anima. Et hoc

tium De anima, written before the *De anima intellectiva*, in which Siger makes use of Aristotle's criticism of Pythagoras to defend the thesis of the unicity and separation of the intellect (since it does not enter just any body). The intellect is not multiplied substantially according to the multiplicity of human beings, although it appropriates the body of a human being.[54] Whether as the result of a misunderstanding of the *De anima* or simply because of bad faith, the critique of Pythagoras serves the cause of the separated intellect: a splendid example of radical Aristotelianism! In the *De anima intellectiva*, in which Siger distances himself from the doctrine of the unique intellect, he says that in any case, on the subject of the union of the intellective soul and the body, one must prefer the teachings of the Christian faith (*sententia sanctae fidei catholicae*); the truth is that intellective souls are multiplied in accordance with multiple human bodies.[55] This is what will be affirmed in the bull *Apostolici Regiminis* of 1513.

It is important here to compare the passage from Siger to the *Summa contra Gentiles* of Thomas Aquinas: "Si enim animae humanae multiplicantur secundum multiplicationem corporum, ut supra ostensum est, […]."[56] Thomas' doctrine was not yet the official doctrine of the Church at the time when Siger was writing his *De anima intellectiva*, which was written precisely in response to the attacks by Thomas.

Yet this was not the only time Siger referred to Pythagoras. In his *Commentary on the Metaphysics*, preserved in two *reportationes* from Cambridge and from Munich, he comes to speak of the use of fables in religion and in philosophy. In the Munich *reportatio*, Siger follows Aristotle's text word for word, and explains that it is common practice to invent fabulous stories or to affirm what

rationabiliter, quia forma non est nisi in propria materia; anima autem est corporis forma, ut visum est." See the text by Averroes cited above, note 51.

54 Cf. *Quaestiones in tertium De anima*, in Siger of Brabant 1972, 34: "[Q. 11, *Utrum anima separata pati possit ab igne*]. Et hoc quod Aristoteles dixit contra Pythagoram, quod non quaelibet anima ingreditur quodlibet corpus, debet intelligi per hoc quod ipse velit dicere quod intellectus, licet non sit nisi unus in substantia, non numeratus substantialiter secundum numerationem hominum, tamen ita appropriat corpus hominis quod non se inclinat ad corpus, id est, brutorum." This passage from the *Quaestiones* has been suggested as the source of article 32 condemned by Étienne Tempier in 1277 (see, on this subject at least, Hissette 1977, 191).

55 *De anima intellectiva*, in Siger of Brabant 1972, 88: "Hoc dicimus sensisse Philosophum de unione animae intellectivae ad corpus; sententiam tamen sanctae fidei catholicae, si contraria huic sit sententiae Philosophi, praeferre volentes, sicut in aliis quibuscumque"; ibid., 101: "Certum est enim secundum veritatem quae mentiri non potest, quod animae intellectivae multiplicantur multiplicatione corporum humanorum. Tamen aliqui philosophi contrarium senserunt, et per viam philosophiae contrarium videtur."

56 Thomas Aquinas 1918, II, 80, 504.

is false to guarantee and justify human laws; indeed, men give greater credence to the stories they are used to hearing, even if they are false. In this context, Siger mentions the use of the transmigration of the soul in the law (*lex*) of Pythagoras: the human soul enters the body of an animal, if it has behaved badly in a human body. According to Aristotle, Siger writes, it may happen that the legislator makes use of false and frivolous stories with a view to making men better.[57] The Cambridge *reportatio* includes a text that is very similar, although its arguments are arranged differently: Siger returns to the fact that by hearing the same false stories, one ends up believing them to be true, especially if they are told by a famous personage. In the constitution (*lex*) of Pythagoras, the transmigration of the soul has the function of frightening the citizens; it matters little that it does not correspond to the truth.[58] Thus, Siger of Brabant and Albert the Great independently attribute to the Pythagorean doctrine a function and usefulness in the political domain, although it is false. As we have seen, in his psychological works Siger does not focus on the philosophical content of transmigration, contenting himself with ascribing to it a just function in politics.

The theory of reminiscence that Plato sets forth in the *Meno*, the *Phaedo* and the *Phaedrus* in fact presupposes the immortality of the soul and its transmigration. Only the *Meno* and the *Phaedo* were translated into Latin by Henry Aristippus around 1154–1160, and their circulation seems to have been limited in the Middle Ages; however, this assertion, presented as self-evident and constantly re-transmitted by philosophical historiography, should probably be

57 Siger of Brabant 1981, 80: "[…] quia quando homines obediunt illis quae consueti sunt audire, etsi sint falsa, credunt tamen ea esse vera et eorum opposita falsa. Quod hoc sit verum probat Aristoteles in legibus humanis, dicens quod homo potest videre quod, quia homo magis assuetus est truffis quam aliis, ideo magis credit illis. Verum est quod lex facit multas considerationes malas secundum quod lex dicitur active, ut lex Pythagorae quae ponebat quod anima hominis ingrederetur corpus bestiae nisi bene esset operata in corpore humano; unde de poenis animarum nos multis illusere fabulosis antiqui poetae. Ratio huius est quam ponit Aristoteles XII° huius: legislator non ponit de primis principiis secundum quod opinatur, sed secundum quod magis conferens est hominibus, et secundum quod magis potest instruere bonis; aliquando autem per falsa et frivola possunt homines fieri boni."

58 Siger of Brabant 1983, 71–72: "Et ideo in lege Pythagorae tradebatur sub comminatione quod anima hominis boni post mortem intraret aliud corpus bonum, mali autem corpus alicuius bestiae; quod non fuit verum sed propter terrorem positum. Ratio autem quare consuetudo audiendi falsa facit credere ea haec est, quoniam audire aliquid, et maxime a famoso, est quaedam ratio probabilis: unde et locus quidam dialecticus est auctoritas." For an analysis of these passages from Siger on fables and laws, see Maurer 1990, 163–174 (on Pythagoras, see 165), and Bianchi 2018, 334–335. I thank Luca Bianchi for pointing out to me these quotations in Siger of Brabant.

qualified. In the *Summa contra Gentiles*, Thomas Aquinas refutes the theory of knowledge as reminiscence as he must have read it in the *Meno*; he criticizes Plato on the subject of the pre-existence and transmigration of the soul. If all souls existed prior to the bodies to which they unite, Thomas explains, it follows that the same soul unites with several bodies in the succession of time. This possibility was already envisaged by those who posit the eternity of world, Thomas adds. Indeed, in an eternalist perspective, since humanity is eternal, an infinite number of human bodies are generated and corrupted. If we exclude from the outset that an actually infinite number of souls exist, then one must accept that a finite number of souls animate an infinite number of bodies. Thomas combats the eternity of the world; however, this is not his main target in this chapter of the *Summa contra Gentiles*. The thesis that the soul exists prior to the body which it animates cannot hold, even if humanity is not eternal, for the generation of men can last infinitely by nature. We are thus back where we started, and it must be admitted that since the number of souls is limited, one and the same soul enters several bodies. Both pre-existence and the transmigration are rejected on the basis of this argument: human souls differ from each other in number, not in species; otherwise human beings would also differ in species. Numerical difference is dictated by material principles. It follows that the material principles that enable the differentiation of human souls are precisely the bodies with which they unite. Consequently, Thomas concludes, several bodies necessarily imply several different souls.[59]

Let us digress for a moment. Bonaventure also mentions the doctrine of transmigration, when he sets forth and criticizes the doctrine of the eternity of the world in the *Collationes in Hexaëmeron*, pronounced at Paris in 1273. He

59 Thomas Aquinas 1918, II, 83, 523: "Item. Si omnes animae praeextiterunt corporibus quibus uniuntur, consequens videtur quod eadem anima secundum vicissitudinem temporum diversis corporibus uniatur. Quod quidem aperte consequitur ponentes aeternitatem mundi. Si enim generatio hominum est sempiterna, oportet infinita corpora humana generari et corrumpi secundum totum temporis decursum. Aut ergo oportebit dicere animas praeextitisse actu infinitas, si singulae animae singulis corporibus uniuntur: aut oportebit dicere, si animae sunt finitae, quod eaedem uniantur nunc his, nunc illis corporibus. […] Quod autem sit impossibile unam animam diversis corporibus uniri, sic patet. Animae enim humanae non differunt specie ab invicem, sed numero solo: alioquin et homines specie differrent. Differentia autem secundum numerum est secundum principia materialia. Oportet igitur diversitatem animarum secundum aliquid materiale sumi. Non autem ita quod ipsius animae sit materia pars: ostensum est enim supra quod est substantia intellectualis, et quod nulla talis substantia materiam habet. Relinquitur ergo quod secundum ordinem ad diversas materias quibus animae uniuntur, diversitas et pluralitas animarum sumatur, eo modo quo supra dictum est. Si igitur sunt diversa corpora, necesse est quod habeant diversas animas sibi unitas. Non igitur una pluribus unitur."

affirms that if the world were eternal, humanity would be just as eternal, with, as a consequence, an infinite number of souls. To avoid this, only three solutions would remain: to posit the mortality of the human species, the transmigration of the soul from one body into another, or else the unicity of the intellect. All this follows from the eternity of the world, according to Bonaventure, not to mention the fact that there would be no reward or punishment after death.[60] The discussion is extremely concise, and Pythagoras' name is not mentioned.

Thomas Aquinas addresses the question of transmigration on several occasions. In Book II of the *Commentary on the Sentences*, a youthful work from the Parisian period, written before the *Summa contra Gentiles*, although he acknowledges that Plato and Pythagoras had the merit of understanding that the soul is incorruptible, he rejects both Plato's theory of pre-existence and Pythagoras' theory of transmigration, because both presuppose a union of body and soul that would be almost accidental, like a pilot and his ship or a man and his clothes. In contrast, Aristotle has explained that the soul bestows substantial, specific being upon a single given body, that it therefore has an "essential habit" toward the body, and that the soul is both the form and the motor of the body.[61]

60 Bonaventura 1891, Visio VI, collatio 4, 361: "Ex isto sequitur alia caecitas de unitate intellectus, quia, si ponitur mundus aeternus, necessario aliquod istorum sequitur: vel quod animae sunt infinitae, cum homines fuerint infiniti; vel quod anima est corruptibilis; vel quod est transitio de corpore in corpus; vel quod intellectus sit unus in omnibus qui error attribuitur Aristoteli secundum Commentatorem. Ex his duobus sequitur, quod post hanc vitam non est felicitas nec poena." See also Bonaventura 1934, Visio I, collatio 3, 92: "Et hoc modo veritate occultata incidit error de aeternitate mundi, ut etiam videtur ipse Aristoteles sentire, secundum doctores qui sibi hoc imponunt, scilicet Gregorius Nazianzenus, Gregorius Nyssenus. Ex quo sequitur unitas intellectus vel transcorporatio in aliud corpus vel quod sit corporalis, et quia non convenit intellectus infinitos ponere, cogitavit unum omnium ponere. Haec omnia sequuntur, si ponitur mundus aeternus. Et ulterius sequitur quod post hanc vitam non sit poena vel gloria." See Bianchi 1984, 144–148, concerning Thomas and Bonaventure on the eternity of the world. On the *Collationes*, see also Imbach 2013 (Imbach discusses the passage taken from the *Visio* VI, concerning the unicity of the intellect, at 381).

61 Thomas Aquinas 1929, lib. 2 d. 17 q. 2 a. 2 co.: "Respondeo dicendum, quod quorumdam antiquorum opinio fuit animas esse extra corpora a principio creatas: cujus erroris occasio fuit quod animam corpori uniri posuerunt quasi accidentaliter, sicut nautam navi, vel sicut hominem indumento, ut de Platone Gregorius Nyssenus narrat: unde dicebat, hominem esse animam corpore indutam: et ideo etiam, secundum Pythagoricos, de corpore in corpus transibat. Hoc autem Aristoteles reprobat, ostendens animam, cum det esse substantiale et specificum in tali corpore, habere essentialem habitudinem ad corpus, in tantum quod anima quae dat esse uni corpori, aliud perficere non possit; ibid., lib. 2 d. 19 q. 1 a. 1 co.: "[…] Ex quibus omnibus patet quod anima intellectiva habet esse absolutum, non dependens ad corpus; unde corrupto corpore non corrumpitur. Secunda

In Book IV, *distinctio* 44, Thomas distinctly affirms that besides the form of the mixture, the rational soul is the only substantial form of the human body.[62] A second argument is then advanced against transmigration. If one accepts that after death, the human soul can pass into the body of a dog or a lion, the result would be that the intellect differs from the senses only accidentally. Indeed, the intellect can only function in a human body, since it possesses the best complexion of all the animals, but remains inactive in the bodies of brute animals.[63] In other words, the intellect is of no use in a brute body, and the relation between intellect and body can by no means be accidental, which leads to the rejection of transmigration of the human soul into brute animals. Thomas attributes all these errors both to the ancient philosophers and to modern heretics, without being more specific. It is in the *Summa contra Gentiles* that Thomas simultaneously censures, on the one hand the Averroist doctrine of the unique possible intellect for all human beings, and, on the other, transmigration, on the grounds of arguments that we have already seen: namely, that the union of the soul and the body cannot be accidental, that the intellect is the form of the human body, and that each soul has its own body, precisely because, according to Aristotle's *De anima*, the soul cannot assume just any body in a contingent way. Thomas Aquinas thus associates Averroes and Pythagoras in the same critique,[64] an association we find again in Baldelli

fuit Pythagorae et Platonis, qui videntes incorruptionem animae, erraverunt in hoc quod posuerunt animas de corpore in corpus transire. Et hanc positionem improbat philosophus in 1° de anima, ostendens quod anima est forma corporis et motor ejus." The same arguments can be read in the *Quaestio disputata De anima*, a. 1. See George 1996.

62 Thomas Aquinas 1858, lib. 4 d. 44 q. 1 a. 1 qc. 1 ad 4: "[…] Corpus autem humanum praeter hanc formam mixtionis non habet aliam formam substantialem nisi animam rationalem."

63 Ibid., lib. 4 d. 44 q. 1 a. 1 qc. 1 co.: "[…] Sed haec opinio ex duabus falsis radicibus venit: quarum prima est, quia anima non conjungitur corpori essentialiter sicut forma materiae, sed solum accidentaliter, sicut motor mobili, aut homo vestimento; […]. Secunda est, quia ponebant intellectum non differre a sensu nisi accidentaliter, ut scilicet homo diceretur intellectum habere prae aliis animalibus, quia in eo propter optimam corporis complexionem vis sensitiva amplius viget; unde poterant ponere quod anima hominis in corpus animalis bruti transiret, praecipue facta immutatione animae humanae ad effectus brutales. Sed praedictae duae radices destruuntur a philosopho, in *Lib. de anima*; quibus destructis, patet falsitas praedictae positionis."

64 Thomas Aquinas 1918, II, 73, 459: "Ex praemissis autem evidenter ostenditur non esse unum intellectum possibilem omnium hominum qui sunt et qui erunt et qui fuerunt: ut Averroes, in III *De anima*, fingit. Ostensum est enim quod substantia intellectus unitur corpori humano ut forma. Impossibile est autem unam formam esse nisi unius materiae: quia proprius actus in propria potentia fit; sunt enim ad invicem proportionata. Non est igitur intellectus unus omnium hominum. Adhuc. Unicuique motori debentur propria instrumenta: alia enim sunt instrumenta tibicinis, alia architectoris. Intellectus autem comparatur ad corpus ut motor ipsius: sicut Aristoteles determinat in III *De anima*. Sicut

at the beginning of the 17th century.[65] Apart from objections of a philosophical nature, it must be added that in the *Commentary on the Sentences*, Thomas also advances an argument from faith: the doctrine of transmigration can be refuted, because it goes against the truth of resurrection taught by Holy Scripture. Indeed, for there to be resurrection, the soul must be able to return to the same body.[66] The *Sentencia libri De anima* is later than the *Commentary on the Sentences* and then the *Summa contra Gentiles*: it was written during the Roman teaching (1265–68). Thomas is still just as critical with regard to Plato and "other philosophers": they focused only on the nature of the soul, but not on the body capable of being moved by the soul. Thomas repeats that the soul cannot enter just any body, as is said in the Pythagorean fables. What is more, he adds an example taken from the *Paraphrase of the De anima* by Themistius, freshly translated by William of Moerbeke (the translation was completed on November 22, 1267), i.e., that the soul of a fly cannot enter the body of an elephant, that the body of a worm differs enormously from that of a dog, and that it is therefore inconceivable that one and the same soul could move them. Bodies, and especially those of animals, are not interchangeable. Thomas refines his doctrine as he goes: the soul cannot be understood without the body, and *vice versa*.[67]

igitur impossibile est quod architector utatur instrumentis tibicinis, ita impossibile est quod intellectus unius hominis sit intellectus alterius. Praeterea. Aristoteles, in I *De anima*, reprehendit antiquos de hoc quod, dicentes de anima, nihil de proprio susceptibili dicebant: *quasi esset contingens, secundum Pythagoricas fabulas, quamlibet animam quodlibet corpus indui*. Non est igitur possibile quod anima canis ingrediatur corpus lupi, vel anima hominis aliud corpus quam hominis."

65 On Baldelli, see *supra*, 322–323.

66 Thomas Aquinas 1858, lib. 4 d. 44 q. 1 a. 1 qc. 1 co. : "Et praedicti errores haereticorum destrui possunt ex hoc quod veritati resurrectionis praejudicant, quam sacra Scriptura profitetur. Non enim resurrectio dici potest, nisi anima ad idem corpus redeat: quia resurrectio est iterata surrectio; ejusdem autem est surgere et cadere; unde resurrectio magis respicit corpus quod post mortem cadit, quam animam quae post mortem vivit; et ita, si non est idem corpus quod anima resumit, non dicetur resurrectio, sed magis novi corporis assumptio." Cf. Thomas Aquinas 1918, II, 44, 369: "Amplius. [...] Si igitur anima rationalis hoc corpus consecuta est propter praecedens meritum vel demeritum, sequetur quod possit iterum coniungi alteri corpori: et non solum quod anima humana assumat aliud corpus humanum, sed etiam quod assumat quandoque corpus sidereum; quod est *secundum Pythagoricas fabulas, quamlibet animam quodlibet corpus ingredi* (*De anima* I, iii, 23). Hoc autem et secundum philosophiam apparet esse erroneum, secundum quam determinatis formis et motoribus assignantur determinatae materiae et determinata mobilia: et secundum fidem haereticum, quae animam in resurrectione idem corpus resumere praedicat quod deponit."

67 Thomas Aquinas 1984, lib. I, 8, 42. See, on the subject of the relation between body and soul, the article by Bazán 2013, who quotes this passage from the *Sentencia* (p. 249)

3 Pythagoras on the Fringe

In his *Quaestiones in Aristotelis De anima*, Book II, Nicolas Oresme combines several doctrines supported, according to him, by Pythagoras, Plato, and the Platonists. They maintained that the world is eternal, that the intellect is co-eternal with the world, that intellective souls are of a finite number equal to that of the stars, to which they return after the death of the body, and from which they re-descend into the body after several centuries. As for Pythagoras, he is said to have maintained that the soul of human beings enters the bodies of brute animals, in which they are incarcerated, since he had not established a hierarchical distinction between souls. In any case, according to Oresme, Plato's explanation concerning souls and their wanderings is preferable to that of Averroes, according to whom "the soul that is in me is the same as that which is in the person who is at Rome," without making explicit the reason why this unique soul would enter and leave this or that body. Oresme concludes by appealing to truth according to faith: souls are multiplied according to the multitude of human beings; they are eternal *a parte post*, not *a parte ante*; that is, souls are created and non-mortal.[68] Like Siger of Brabant in the *De anima intellectiva*, Oresme appeals to truth according to faith concerning the multitude of souls, a truth that was to be repeated and endorsed down to the *Apostolici Regiminis*. Nicolas Oresme is one of very few authors to mention the doctrine of transmigration in the 14th century. The latter does not appear any more among the subjects dealt with by the commentators on the *De anima*, no doubt because the genre of the continuous commentary, analyzing the text almost word for word, gave way to the commentary by questions, which dealt

without commenting on the reference to Pythagoras and to the Pythagorean fables. The expression "that the soul cannot be understood without the body, or the body without the soul" is used by Bazán 2013, 250.

68 Nicolas Oresme 1995, III, q. 7, 364: "Dicebant primo quod mundus est aeternus; secundo quod quilibet intellectus est ipsi mundo coaeternus; et ultra quod animae intellectivae sunt in numero finitae et correspondent stellis et post mortem revertuntur ad stellas, et quaedam puniuntur et etiam praemiantur secundum merita, et post multa saecula iterum revertuntur ad corpora, et sic infinities. Et credo quod ista fuit etiam opinio Aristotelis. Ideo dicit Virgilius rursum: quia incipiunt in corpora velle reverti. Tamen Pythagoras in hoc differebat, quia ponebat animas hominum intrare in corpora brutorum et ibi incarcerari; ideo dicebat omnes animas esse eiusdem rationis. Et si quaeratur qualiter post mortem animae moverentur ad caelum et postea descenderent, multo melius posset ipse Plato respondere quam Averroes, qui diceret quod illa anima, quae est in me, est ea quae est in illo qui est Romae. Nec posset causam dicere quo iret nec quibus a corpore recederet. Quarto est veritas et secundum fidem quod animae multiplicantur secundum multitudinem hominum et sunt aeternae a parte post, non tamen a parte ante." The editor Benoît Patar does not provide a source concerning the multitude of souls.

with key passages selected by the Masters. In short, neither John Buridan,[69] nor Blasius of Parma, nor John of Jandun, nor Thomas Wylton, nor Duns Scot, nor Taddeo of Parma, to mention only a few, discuss the doctrine of transmigration.[70] In contrast, it is in a non-university intellectual that we encounter Pythagoras once again. In the *De sui ipsius et multorum ignorantia* (II, 110–112), Petrarch sketches in a few lines the life and philosophy of Pythagoras, mentioning and rejecting the doctrine of "metempsychosis," which he sets forth following the *Divinae Institutiones* (III, 18, 15–17) of Lactantius, which are here explicitly cited. Petrarch makes no mention of the discussions of the university Masters on the philosophical content of transmigration, and in his style he seems to come close to another humanist, John of Salisbury.

We have seen that Thomas Aquinas criticizes the metaphor according to which the soul is like a pilot steering a ship, on the grounds that it refers to an accidental relation between soul and body. It should be noted that this example, which Aristotle uses in the *De anima* II, 1 (413a8–9), was subject to the most diverse interpretations in the Middle Ages.[71] In fact, the 7th article (in the order of the *Chartularium*) condemned in the *Syllabus* by bishop Étienne Tempier in 1277 seems to depend directly on Thomas' teaching: "the intellect is not the act of the body, except as the pilot of a ship, and it is not the essential perfection of man."[72] In contrast, we have seen that the Master of Arts of the *Lectura in Librum de anima* edited by Gauthier takes up the example of the pilot and his ship. Was he directly targeted by Tempier? This cannot be affirmed. With regard to article 193, it could be compared to the doctrine of transmigration: "the intellect can pass from one body to another, and thus be successively the motor of different bodies."[73] According to Roland Hissette, the article condemns the Averroist theory of the unique intellect[74] – other articles, moreover, openly mention the unique intellect. Alain de Libera has written that article 193 can "be understood in two ways: 1) in the Platonic sense of *metensomatosis* (the obvious meaning), 2) in the Averroist sense of a unique

69 John Buridan 2019.

70 The medieval commentaries on the *De anima* are very numerous, and for the most part unedited. The present study is, therefore, provisional and liable to be completed. In order to have some idea of the matter that remains to be explored, see the inventory of the commentaries on the *De anima* published in Mora-Márquez 2014.

71 For a very complete overview of the late ancient interpretations of this Aristotelian example, see Giardina 2009.

72 Piché 1999, 82–83: "Quod intellectus non est actus corporis, nisi sicut nauta nauis, nec est perfectio essentialis hominis."

73 Ibid., 138–139: "Quod intellectus potest transire de corpore in corpus, ita quod successiue sit motor corporum diuersorum."

74 Hissette 1977, 215.

intellect" united to the body like a motor, which, as it passes from one individual to another, activates his or her cerebral functions.[75] A Platonic interpretation of article 193 is advanced by an anonymous commentator of the 15th century (the commentary known as "Quod Deus"), which rejects the theory of the pre-existence of human souls, their localization in the stars, their descent into the body, and their return to the stars after the death of the body. It is God who creates the intellective human soul and immerses it within the human body.[76] As for Raymond Lull, author of a commentary on the *Syllabus* in 1298, he scarcely comments on article 193, and refers the reader to articles 78 and 32. Concerning the latter, Lull criticizes the doctrine of the unique intellect for several pages. Conrad of Megenberg comments on the articles condemned by Tempier in the context of his treatise *De regimine domus scolastice* (*ca* 1348–1352). He considers that article 193 deals with the transmigration of the soul, a doctrine that he refutes by using the argument that individual souls must submit themselves to the Last Judgment. But this would imply that one and the same soul could simultaneously be saved and damned according to the various bodies it had inhabited in its various lives. This would be absurd in his opinion.[77]

4 Conclusions

Nicola Baldelli is only the epigone of a long debate on the transmigration of the soul, which really begins with Manegold of Lautenbach at the end of the 11th century. This doctrine was almost unanimously rejected, on the grounds of

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75 De Libera 2004, 292, following the interpretation by Hissette, which he cites expressly.

76 Wels 2004, 106: "*Quod intellectus potest transire de corpore in corpus, ita quod sit successive motor diversorum corporum.* Error. Ratio falsitatis illius articuli est, quia, licet Plato dixerat, quod intellectus humanus esset in quadam stella et, quando vellet, indueret corpus humanum, et post separationem a corpore e converso daret se ad stellam, sed tamen hoc non potest esse, quia, si intellectus meus vel tuus fuisset in aliqua stella, utique sciret intellectus, quando fuisset ibi, quod tamen non est, ergo etc. Etiam sicut superius patuit, tunc anima intellectiva hominis a solo deo glorioso creatur et in creando corpori humano infunditur, quare patet, quod non est in aliqua stella." The commentary was written between 1415 and 1454–56, according to Wels 2004, lxxi. Bianchi 2008, 46 suggests advancing the *terminus ante quem* to the year 1439.

77 Conrad of Megenberg 1984, III, 1, 14, 120: "*Quod intellectus hominis potest transire de corpore in corpus, ita quod successive sit motor diversorum corporum.* Error est, quoniam unum illorum corporum poterit dampnari, aliud salvari, et in novissimo iudicio redibit anima ad corpus, et sic idem intellectus seu eadem anima simul dampnaretur et salvaretur cum corporibus diversis, quod est absurdum."

philosophical arguments taken mainly from Aristotle's *De anima*, beginning in the 13th century, especially the argument according to which just any soul cannot enter into just any body, as well as on the basis of theological arguments such as the resurrection of the body and the Last Judgment. Thomas Aquinas rejects transmigration, for it implies an accidental relation between soul and body; on the contrary, the soul is the substantial form of the body, and of only one body. The Pythagorean doctrine and the Averroist theory of the unicity of the intellect are associated and rejected by Thomas on the basis of the argument that the relation between soul and body cannot be accidental. Reservations against transmigration derived from common sense and logic are also often advanced, as is the case with William of Auvergne. The only shift is conceded by those authors, in fact not very numerous, who accept an allegorical reading of transmigration for moral purposes, especially in the 12th century, and by those who emphasize the use of the doctrine for political ends, particularly Albert the Great and Siger of Brabant in the 13th century. The Master of Arts of the *Lectura in Librum de anima* is the only medieval author to consider that the transmigration of the intellective soul from one human body to another can be envisaged from a nature point of view. However, the commentators on the *De anima* do not systematically debate the transmigration of the soul in the 13th century: the Anonymous of Siena, Boethius of Dacia (?), Gilles of Rome, John Peckham, to cite only a few, do not talk about it at all. Philosophical adhesion was thus moderate, but it can nevertheless be said that this doctrine met with a direct success, and was the subject of censorship. To follow its destiny in the Renaissance and the Early Modern period is not possible in the present study. However, it is sufficient to consider that an entry on *metempsychosis* is included in the *Dictionnaire de théologie catholique*, because this doctrine was followed by the spiritualist movements that flourished in 19th century Europe. Yes, the history of the transmigration of the soul is a long one.

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CHAPTER 11

Pythagoras Latinus. Aquinas' Interpretation of Pythagoreanism in His Aristotelian Commentaries

Marta Borgo and Iacopo Costa

There is no evidence to establish whether and to what extent the major Pythagorean sources circulating during the Middle Ages before the "rediscovery" of Aristotle[1] are relevant to the formation of Aquinas' knowledge of the philosophical key theses of Pythagoras and the Pythagoreans. The role of the Aristotelian corpus is by contrast clearly crucial for Aquinas, who becomes acquainted with Pythagorean physics, cosmology, and ontology through the Latin versions of the *Metaphysics* and of the *libri naturales*, especially of *On the Heavens*, where Pythagorean doctrines are reconstructed and criticized by the Philosopher in considerable detail.[2] In fact, a survey of the explicit references to Pythagoras and the Pythagoreans in Aquinas' corpus shows that most of them come from his philosophical commentaries, and more precisely from the paraphrases of Aristotle's doxographic passages. Basically, Aquinas takes an interest in Pythagoreanism whenever Aristotle leads him to think about it. Moreover, the relatively few mentions in the theological works appear to derive mainly from Aristotle. The few exceptions concern biographical anecdotes, which Aquinas gets, typically for his time, from classical authors and the Church Fathers, perhaps through collections of *exempla* or *auctoritates*.

Therefore, in order to sketch Aquinas' interpretation of Pythagoreanism, we will focus mainly on his Aristotelian commentaries. However, with the aim of better clarifying who Pythagoras and the Pythagoreans are in Aquinas' view, and what their role is as *auctoritates*, some general preliminary points will be made, taking into account other texts as well.

1 Brams 2003, 37–130.

2 Aquinas reads Aristotle's *Metaphysics* in parallel with the Latin versions of Averroes' commentary and of Avicenna's *Philosophia prima*. His knowledge of Pythagorean doctrines seems to be based mainly on the first book of the *Metaphysics*, which he was able to read in four different complete Latin versions plus a fifth partial one. More details in Borgo 2014, 30. Notice that Aquinas also knows the last two books of the *Metaphysics*, but does not comment on them, likely by choice. As far as the *On the Heavens* is concerned, from 1271 Aquinas reads it with the support of Simplicius' commentary, translated into Latin by Moerbeke.

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1 A Sketchy Portrait of Pythagoras

The terms "Pythagoras" and "Pythagorean(s)" occur almost 230 times in Aquinas' works. In one third of the cases it is specifically to Pythagoras, either his person or his thought, that he refers. In the remaining two thirds of the cases, it is to the Pythagoreans that he refers, considered as a fairly homogeneous group of ancient Italiote philosophers; and in this latter case there is no reason to suppose that Aquinas did intend to exclude Pythagoras, their *princeps*.[3] In fact, Aquinas appears to use the adjective "Pythagorean" broadly, so that the term "Pythagoreanism" designates the teacher as well as his followers, that is, anyone adhering to some of Pythagoras' major theses,[4] about which more will be said in what follows. Notice that not all of Aquinas' mentions of Pythagoras/ Pythagoreans are equally relevant for isolating a doctrinal core; and this not only when they figure in transitional contexts, such as a *divisio textus*. In fact, especially when the figure of the pioneer is at stake, Aquinas dwells on traditional anecdotic details, rather than over his effective philosophical impact.

According to Aquinas, it was after a career as a boxer that Pythagoras bumped into someone debating about the soul's immortality, and fascinated by the discussion, decided to dedicate himself to philosophy.[5] Active in Southern Italy,[6] Pythagoras was not only the foremost thinker among the Italiote philosophers, but more generally also the first ever to declare himself a philosopher.[7] Expert in music and talented, his hearing was told to be so refined that he could perceive even celestial harmony.[8] Pythagoras was an excellent teacher, capable of using words so effectively that he did not need to compose written works. In fact, as was the case with Socrates and Jesus Christ, he was able to impress his doctrine directly on the hearts of his disciples.[9]

In this sketchy portrait of Pythagoras, one specific aspect seems noteworthy: his interest in the immortality of soul, which in Aquinas' view represents at once the origin and one of the major limitations of Pythagorean philosophy. In fact, Aquinas rejects Pythagoras' thesis of the transmigration of immortal souls, not only as heretical, but also as philosophically erroneous. What is more, on a few occasions he labels this sort of psychological doctrine "Pythagorean myths" (*fabulae pythagoricae*), making use of a pejorative expression drawn

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3 Thomas Aquinas 1964, *In Metaph.*, §119.
4 Different scenario for "Epicurus/Epicurean," see Robert 2013, 42–43.
5 Thomas Aquinas 2014, *Serm.* XIV, 219, 210–215, with note.
6 Id. 1964, *In Metaph.*, §119.
7 Ibid., §56; Id. 1899, *IIa–IIae*, 186, 2, ad1. See further below, 352.
8 Id. 1886, *In De caelo*, II, 14, 6. About this passage, see more extensively below, 367–368.
9 Id. 1903, *IIIa Pars*, 42, 4, sol.

from Aristotle (*De anima*, 407b22),[10] diminishing its philosophical value. Pythagoras' philosophical relevance seems equally questioned by Aquinas in one of his sermons, when Pythagorean psychological enquiries are compared (indeed downgraded with respect) to the beliefs of common uneducated people: "We read that Pythagoras first was a boxer. He heard a *magister* disputing on the immortality of the soul […]. And he was enticed so much that he left everything and gave himself over to the study of philosophy. But which old woman nowadays would not know that the soul is immortal?"[11] These potentially disqualifying passages, however, call for some contextualization. First, this reference to Pythagoras in preaching has strictly no historico-philosophical weight; it comes rather under the genre of *exempla*: Pythagoras is the personification of pagan philosophers. Second, even taking this reference as a personal anecdote, the accuracy that Aquinas shows elsewhere in situating Pythagoras in the history of philosophy compels us to put things into perspective. Let us focus for example on the first mention of Pythagoras in Aquinas' commentary on the *Metaphysics*. Elaborating on Aristotle's well-known passage concerning wonder as the source of philosophy (982b11–21), precisely when paraphrasing Aristotle's description of "the lover of myth" as "in a sense a philosopher" (982b18–19), Aquinas rather compares philosophy to poetry, characterizing the philosopher as "aliqualiter philomythes, idest amator fabulae, quod proprium est poetarum." Moreover, he expands Aristotle's argument by adding an anecdote concerning Pythagoras, namely his foundational role with respect to philosophical practice: asked about his occupation, to distinguish himself from the experts and sages, Pythagoras refuses to pretentiously describe himself as an actual, perfected, wise man (*sapiens*) and opts for the humbler title of philosopher (*amator sapientiae*), a lover (in search) of wisdom for its own sake.[12] Given the juxtaposition of these two descriptions of the philosopher, as "aliqualiter philomythes" and "amator sapientiae," Aquinas' aforementioned critique of "Pythagorean myths" must be nuanced as targeting some particular aspects of Pythagoras' psychology, perhaps his articulation of such a theory; especially since Pythagoras' fundamental role in the history of philosophy is never questioned by Aquinas, who emphasizes more than once Pythagoras' pioneering interest in immaterial causes of material things. Therefore, when

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10 Cornelli 2016b, 5–10.

11 Thomas Aquinas 2010, 202–203. Cf. Id. 2014, *Serm.* XIV, 219, 208–216: "[…] Set que uetula est hodie que non sciat quod anima sit inmortalis? […]." On the ambivalent example of the old woman, see Agrimi and Crisciani 1993, 1289–1296.

12 Thomas Aquinas 1964, *In Metaph.*, §55–56. Cf. Cicero 1967, *Tusculanae disputationes*, V, 3, 7–4, 10; Augustine 1955, *De civitate Dei*, VIII, 2; Isidore of Seville 1971, *Etymologiae*, VIII, VI, 1, and XIV, VI, 31. About this exemplum see Ricklin 2006, 7–11.

juxtaposing Pythagoras' philosophical knowledge of the soul as immaterial[13] and immortal to the religious belief of an uneducated old woman, Aquinas is doubtless emphasizing the breaking role of Revelation in the development of human rational knowledge. Nonetheless he is ultimately neither trivializing Pythagoras' doctrines nor simply reducing causal philosophical knowledge to the *uetula*'s assent. Indeed, Pythagoras' philosophical experience – and pagan philosophy in general – is thereby situated in the past for determining its serviceability, in its complete autonomy, for theologians of Aquinas' age,[14] who are supposed to be something more than simply new instances of Pythagoras. Interestingly enough, in his sermon Aquinas refers to the *uetula* and Pythagoras just after having implicitly extended to theological knowledge what Aristotle says more specifically of philosophical investigation (*Metaph.* α 1, 993a30–b4): that studying the Truth is in some sense difficult can be shown by the fact that no thinker, as an individual, can either obtain an adequate grasp of it, or completely fail in the attempt; and that is why the modest results attained by individual thinkers – and even their mistakes – must be taken collectively for making significant advances in the study of truth. It is exactly from such a perspective that Aquinas looks at Pythagoras and the Pythagoreans in his commentary on Aristotle's doxographical sections, where he discreetly carries out the wise man's twofold task: showing the truth, and this by discussing previous errors.[15]

2 Pythagoras and the Pythagoreans in Context: A Turning Point in Ancient Philosophy

Unlike Albert the Great, who divides philosophers into three broad groups – Epicureans, Stoics and Peripatetics –, and considers Pythagoras and his disciples as exponents of the second,[16] Aquinas never explicitly connects them with Stoicism (which he considers a specific philosophical option, not a general category), and situates them (doctrinally, and not without anachronisms) in some

13 This seems to be the standard Pythagorean doctrine on the soul according to Aquinas. Notice, however, that Aquinas hints once at the agreement of some Pythagoreans with Democritus on the corporeality of the soul, which would be made out of an infinite number of invisible bodies (Thomas Aquinas 1984, *In De an.*, I, 3, 124–137).

14 Porro 2016, 47–48.

15 Cf. Thomas Aquinas 1964, *In Metaph.*, §180; cf. Id. 1886, *In De Caelo*, I, 22, 1–5.

16 Albert the Great 1960, *Metaph.*, I, IV, 1 (47, 5–13; 48, 49–50). Cf. De Libera 1997, 357–360; Santinello 1990, 64–67.

detail with respect to other early Greek thinkers.[17] Ultimately like Albert, however, while putting the Pythagoreans in some continuity with other so-called Presocratics – characterized as (*primi, antiqui*) *naturales*,[18] namely physicists, who inquired into nature (*de naturalibus*), giving strictly physical explanations for phenomena (*naturaliter loqui*) –, Aquinas considers them opponents of "material monism," adhering to some form of pluralism. Moreover, he puts them in closer relation to Plato than to any of the Presocratics. By adopting Aristotle's doxographical method, Aquinas demarcates original aspects from traditional ones in Pythagorean theses, providing helpful elements to substantiate also doctrinally a demarcation which he draws first geographically: Italiote versus Ionian philosophers.

In his view, the Pythagoreans are more properly concerned with metaphysics than any other thinkers before Plato, in so far as they endeavour to provide an account for beings of all kinds, not only corruptible ones.[19] For, as Aquinas remarks, while keeping their focus on natural things, unlike "physiologues" (and anticipating Plato) the Pythagoreans assign non-sensible, incorporeal, causes to the sensible world. They have, therefore, also access to some sort of intelligible beings.[20] More precisely, the Pythagorean approach to reality is distinct from that of the physical thinkers insofar it is theoretical, and mainly mathematical[21] (as will be Plato's too): being capable mathematicians, they adapt the principles with which they are most familiar, the mathematical ones (which are also better known by reason),[22] to sensible beings, which are thereby reduced to numbers, and ultimately to units.[23] Not only substances, but also accidents and habits of concrete things are assimilated by them to the properties of numbers.[24]

Notice that in Aquinas' view, in the history of philosophy thinkers succeed in coming nearer and nearer to the truth, almost forced by its necessity. Each one relates dialectically to his predecessor and goes beyond him. This presupposition results in teleologically, more than diachronically oriented

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17 On Aquinas' attitude towards *endoxa* in philosophical commentaries, see Porro 2016, 260–261.

18 Notice that strictly speaking Aquinas' expression "antiqui/primi naturales" is synonymous neither with the Aristotelian term *physiologoi*, often used also in philosophical contemporary literature, nor with the current label "Presocratics." Cf. Thomas Aquinas 1964, *In Metaph.*, §145–146.

19 Ibid., §201.

20 Ibid., §202–203.

21 Ibid., §119–123, 181.

22 Id. 1884, *In Phys.*, I, 10, 6.

23 Id. 1964, *In Metaph.*, §125–126, 506.

24 Ibid., §120, 123–124.

doxographies.[25] According to Aquinas, whose reconstruction is strictly Aristotelian (Aristotelian-centric indeed), any new philosophical doctrine is a sort of reaction to the previous, less perfect one. While its good intuitions are maintained, its mistakes are amended. Therefore, partially true philosophical doctrines succeed one another, and predecessors' errors serve as a catalyst for further investigations. Pythagoras is no exception. Nonetheless, Aquinas seems to assign him a pivotal role. In fact, precisely because of his mathematical turn, Pythagoras deserves, and he seems actually to be the first to deserve, the title of *philosophus* that he coins for himself, as mentioned above. Indeed, Aquinas appears almost to take the term in the Aristotelian sense of *philosophus primus*, since he intends to distinguish Pythagorean philosophers not only from the *sapientes*, but also from the *primi naturales* and the *philosophi naturales*.

Three major divergences between the Pythagoreans and other thinkers emerge neatly from Aquinas' paraphrase of *Metaphysics* III, where the aporias (which will guide Aristotle's and Aquinas' metaphysical reflections and structure their works)[26] are formulated.[27] First, like Plato and against material monists, the Pythagoreans admit extra-material, non-manifest, principles. However, while Plato maintains that they are separable, such is not the case for the Pythagoreans. Secondly, like Plato and unlike previous "physiologues," the Pythagoreans admit numbers, lengths, figures, and points as kinds of substances. However, while Plato considers such substances separate from sensible things, the Pythagoreans hold them to be inseparable from concrete substances. Thirdly, like Plato and unlike Presocratic materialists, monists, as well as pluralists, the Pythagoreans claim that unity and being signify the substance of things rather than something added to it.

Aquinas' commentary on the first book of *Metaphysics* furnishes helpful elements for detailing further these distinctive theses. If the Pythagoreans still consider contrariety as fundamental, for example, they conceive of it differently with respect to other "physiologues." In fact, while material monists postulate contraries as formally determining one corporeal principle, and while Empedocles assigns them an efficient role, Pythagorean contraries rather fulfil

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25 See for example ibid. §194–200. In this perspective, Anaxagoras and Empedocles are taken as "predecessors" of Pythagoras. Despite their breakthrough with respect to other Presocratics (especially as far as the recognition of causal agency is concerned), that Aquinas does not hesitate to acknowledge, both of them are considered by him as more distant from Plato than Pythagoras, in particular because of their philosophical language, which is still primitive.

26 Galluzzo 2005, 420–423.

27 Thomas Aquinas 1964, *In Metaph.*, §358, 366–367.

the role of material principles, insofar as they do not inhere in something else, and are somehow constituents of sensible substances.[28] Indeed, Aquinas remarks that this Pythagorean intuition lies at the root of Plato's Dyad.[29] By splitting the Pythagorean *infinitum* into a duality of great and small and in turn making them material principles, Plato unambiguously overturns the "physiological" perspective. Concerned strictly with mutable beings, the "physiologues" view matter as responsible for unity, and form as responsible for multiplicity. Therefore, in their perspective a unique material substrate going through change comes to be successively characterized by contrary determinations. For Plato, by contrast, matter causes diversity and plurality, since it is in virtue of its division that many individuals participating in one and the same separate form come to existence. According to Aquinas, the Pythagoreans can be situated between the two. Even if their numbers are not universals, neither separate in being nor immutable objects of knowledge – in Aquinas' view Pythagorean philosophy consists in an ontology, with no epistemology –, the Pythagoreans succeed in moving closer than "physiologues" to the essential dimension of concrete things.[30] Not only do they sketch some sort of very essential definitions of sensible things, but they also try to explain, albeit rudimentarily, how sensible things relate to, and are caused by, their incorporeal principles, conceiving of what Aquinas describes as the ancestor of the notion of participation.[31] It is worth mentioning that, in the Peripatetic perspective from which Aquinas looks at the history of ideas, the corrections proposed by Plato to Pythagorean deficiencies are not always so effective: as do numbers, ideas ultimately fail to account for physical movement. Moreover, they entail a useless multiplication of ontological levels, and show themselves powerless with respect to sensible things.[32] Furthermore, as it was the case with Pythagoras' technical language, also Plato's philosophical jargon, especially his choice of words to describe ideas as exemplars of sensible things participating in them, is branded by Aquinas as metaphorical, more apt for poets than philosophers.[33]

While as a philosophical exegete Aquinas does juxtapose, but does not situate precisely Pythagoras and his followers with respect to the Eleatics, in his *Sentences* commentary (II.17.1.1), he considers their contrasting philosophical views in a wide, and somewhat nuanced, perspective. Aquinas' target is

28 Ibid., §132–133, 147.
29 Ibid., §162.
30 Ibid., §148–149, 164.
31 Ibid., §156, 168.
32 Ibid., §201–208, 228, 241–242.
33 Ibid., §231–232.

the thesis that God and human souls would share the same (divine) essence. In his view, Parmenides' and Melissus' doctrine of unique being, Pythagoras' hypothesis of numbers as principles of sensible things, as well as Plato's theory of ideas, descend from the same fundamental mistake:[34] drawing fallacious ontological conclusions from strictly logical premises, namely moving illegitimately from commonness in predication to real uni(ci)ty in being (*ex intentionibus intellectis iudicium rerum naturalium sumere*).[35] As typical of him in such reviews of opinions in theological contexts, Aquinas tries to reconstruct a sort of long-term history of philosophical ideas, in order to show the ancient roots of current debates. He thereby establishes a further connection between the previous thinkers, including Pythagoras, and Avicebron, who infers the universality of a unique kind of matter on the basis of predicamental considerations.

Other interesting reviews intended to situate the Pythagoreans beyond the strict context of pre-Aristotelian philosophy concern the thesis according to which all good and evil things would descend from two contrary principles, respectively *bonum* and *malum*. While commenting on the *Metaphysics*, Aquinas confines himself to underlining its affinity with the Empedoclean postulation of love and strife as the ultimate efficient principles of sensible worlds,[36] elsewhere he emphasizes the theological posterity of such a doctrine, indeed its dualist drifts. This link is proposed by Aquinas in three passages[37] where the same fundamental question is asked: how many first principles must be postulated from a creationist perspective? To prove that there exists just one creating principle, Aquinas examines several fallacious opinions, among which is Pythagoras' option. In his view, unlike Plato – who runs the risk of making the world eternal by postulating one efficient first principle, as well as uncreated prime matter in addition to ideas –, Pythagoras opens the gateway to polytheism. And in fact, Aquinas assigns him a major role in Manicheism's prehistory. First the "physiologues" and Empedocles postulate two contrary principles, still overlooking what they have in common and the possibility of reducing them to something more fundamental. Pythagoras seems to go further: by considering the two contraries absolute principles and failing to realize that no absolutely bad things exist, he lays the groundwork for

34 Cf. Ibid., §158.

35 Id. 1929, *In II Sent.*, XVII, 1, 1, sol.

36 Id. 1964, *In Metaph.*, §147.

37 Id. 1929, *In II Sent.*, I, 1, 1, sol.; Id. 1918, *Contra Gent.*, II, 41, 362b; Id. 1953, *De potentia*, 3, 6, sol. and ad10. Notice the formula "verbum Philosophi est accipiendum secundum opinionem Pythagorae," used by Aquinas in this latter text and also occurring elsewhere, in slightly modified forms: the assumption is that Aristotle looks at things from his predecessors' viewpoint and expresses himself accordingly.

the hypostatization of good and evil. Finally come the Manicheans, who maintain that no imperfect things could be created by God and postulate another first creator, this one evil.

So far, Pythagoras and the Pythagoreans have been considered as a unity, for the general reasons explained above. There is some evidence, however, that Aquinas perceives some variety among them.[38] Two rare but highly significant situations are worth noting. On the one hand, Aquinas draws attention to Aristotle's remark that with respect to specific questions, especially those having to do with cosmological theses, there is no unanimity among the Pythagoreans. That the milky circle is a way, for example, is an idea shared just by some of them, with further nuances appearing within this subgroup.[39] However, on one occasion, by significantly expanding on Aristotle's text, Aquinas points out a variety of Pythagorean options concerning a central matter of ontology. At stake here is the number of pairs of contraries to be posited as ultimate principles:[40] while some Pythagoreans confine themselves to the most fundamental contrariety, even and odd, others list ten pairs of co-elements and arrange them in two columns. The latter seems to be the standard option according to Aquinas, as we will see below.

On the other hand, Aquinas refers sporadically to individuals who are described as either members of the Pythagoreans as a group, or somehow related to them. This is the case respectively with Paron and Alcmaeon of Croton.[41] The doctrine of random (so to speak) contraries formulated by the latter is mentioned by Aquinas as a third Pythagorean ontological option after the two others just mentioned. However, following Aristotle,[42] Aquinas points out that it is not clear whether Alcmaeon elaborates on Pythagoras' intuitions, or vice versa.

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38 The only case in which Pythagoras is supposedly distinguished from his followers (even if their doctrines are eventually joined up) is uncertain (Thomas Aquinas 1964, *In Metaph.*, §581): according to the manuscript tradition Aquinas mentions here only the Pythagoreans as a group.

39 Id. 1886, *In Meteor.*, I, 12, 2.

40 Id. 1964, *In Metaph.*, §126–127.

41 Also noteworthy is the peculiar case of Speusippus, whom Aquinas correctly describes as Plato's nephew, but explicitly considers doctrinally closer to Pythagoras. Interestingly, it is not uncommon to see Aquinas mentioning the Pythagoreans as supporters of doctrines introduced as anonymous by Aristotle, and which Ross suggests ascribing to Speusippus in his edition of the *Metaphysics*.

42 Thomas Aquinas 1964, *In Metaph.*, §131. Cf. Aristotle 1997, vol. 1, p. 152, note 29: in the Latin tradition of the *Metaphysics* the words considered by Ross as a later addition to Aristotle's text are taken for authentic.

3 Three Recurrent Pythagorean Doctrines in Aquinas' Corpus

Among others, there are three core doctrines recurrently ascribed by Aquinas to the Pythagoreans: the transmigration of souls; number (and ultimately unity) as an intrinsic substantial principle for sensible things; the table of opposites as an exhaustive account for remote and proximate principles of reality. Although Aquinas openly rejects them, he acknowledges their philosophical and theological interest. In what follows, we shall attempt to reconstruct these three Pythagorean doctrines from Aquinas' perspective.

Quaelibet Anima In Quolibet Corpore. The term "metempsychosis" is almost absent from Aquinas' corpus. Indeed, it figures in a couple of quotes from Jerome, characterized not only as a foolish philosophical doctrine, but also as a veritable heresy. Pythagoras and his school are not explicitly mentioned in such contexts. Once there occurs the expression "dogma of the transfusion of souls," and in this case Pythagoras is by contrast mentioned (with Plato) as the champion of this doctrine, according to which any soul can get out of one body and join, or go into, another.[43] Interesting enough, Aquinas' main concern in this context is not criticizing this psychological thesis' kernel, but rather denying (on chronological bases) its applicability to the specific case of John the Baptist's soul with respect to Jesus' body. More generally, in Aquinas' view the soul's migration as such is less troublesome than its philosophical presuppositions. Aquinas' foremost concern is in fact the accidental unity which is supposed to characterize Pythagorean hylomorphic living substances; whence the emphasis put on the randomness of soul's transitions, from any body to any other body.

Aquinas' criticism is rooted in Aristotle's *De anima* (407b21–24 with Themistius' paraphrase),[44] a text from which is equally taken the aforementioned expression "Pythagorean myths." From his perspective, the supposition of a lack of an essential *habitudo* of each single soul to just one single body entails some crucial errors. First, the pluralism of substantial forms: any body is supposed to be just a (inanimate) body, capable of subsisting as such with no extra-determinations, and it would become just accidentally vegetative, animal, human or celestial. The general order of the universe would thereby be jeopardized, in so far as souls would be supposed to migrate not only from one individual to another of the same species, but also from an individual of one

43 Thomas Aquinas 1951, *In Matth.*, §1219. Aquinas' targets here are some interpreters fallaciously maintaining that king Herod believed in the transmigration of souls, rather than in resurrection.

44 Id. 1984, *In De an.*, I, 8, 316–359.

species to an individual of another species, even of another genus. Moreover, if no soul is substantially tied to one specific body (and vice versa), its immateriality raises a problem of identity. Either one unique separate soul must be postulated, or if many souls exist, they must be specifically different from one another.[45] While the first option involves ultimately the elimination of individual thinking, by the second the reciprocal order of intellectual entities is ruled out: no longer would any ontological distinction between souls and angels be possible. It is, however, striking that, although some crucial theses by Aquinas are invalidated by Pythagorean assumptions about souls, his criticism of the doctrine of transmigration ends up being less severe than it could be.[46] According to Aquinas, in fact, nobody before Aristotle, including Plato, takes the body adequately into account when trying to define soul. The Pythagorean aberration is therefore not isolated.[47]

All Is Number. Aquinas no doubt considers the thesis according to which all reality can be reduced to number as the core of Pythagorean philosophy. Still, it is quite difficult to understand how he interprets it concretely, given that his adherence to the Philosopher entails that Aristotle's fluctuations (perhaps inconsistencies) in the formulation of this central doctrine[48] remain unsettled even in Aquinas' paraphrasis. Part of the problem derives from the fact that Aquinas accounts for Pythagorean notions by way of a non-neutral philosophical jargon, indeed by reinterpreting them in Aristotelian terms. Thus, in his view, the Pythagoreans turn to numbers in order to provide an explanation for the metaphysical structure of concrete things, but they ultimately do not consider numbers strictly fundamental: even and odd, finite and infinite are more effective principles of reality. And more importantly, Aquinas' understanding of the relation numbers bear to concrete things is not entirely clear, given that numbers are characterized not only as principles and causes of beings, but also as substances of concrete beings, and even identified *tout court* with them.[49]

Aquinas ascribes to the Pythagoreans two notions of number, not equivalent with one another, at least intentionally. If on the one hand he describes it

45 Id. 1929, *In II Sent.*, XVII, 2, 2, sol.; Ibid., XIX, 1, 1. Id. 1918, *Contra Gent.*, II, 44, 369b; Ibid., 73, 459a.

46 Theological theses are involved, given that the doctrine of the transmigration of souls precluded an adequate conception of the creation of individual souls and, to some extent, a correct understanding of the resurrection of bodies.

47 Thomas Aquinas 1984, *In De an.*, I, 8, 359–363.

48 Cornelli 2016a, 52–56.

49 Thomas Aquinas 1929, *In II Sent.*, XVII, 1, 1; Id. 1964, *In Metaph.*, §124, 160, 163, 363, 489, 900. Cf. also Id. 1884, *In Phys.*, III, 6, 5–6.

as a mere collection of unities,[50] on the other he characterizes number more specifically as a compound of, something made out of, unities.[51] According to the Pythagoreans, unity is no number in act, but all numbers in potency, and, like any number, it is made out of even and odd, as out of matter and form.[52] There are clues suggesting that Aquinas understands in similar terms the ontological role of unity and numbers with respect to sensible things. In fact, he sometimes refers to numbers as constitutive principles of things, saying that magnitude, world, and nature are made out of numbers.[53] However, his point of view on the subject calls for qualification.

While Albert the Great alludes to Pythagoras' differentiating instrumental numbers (obtained by abstraction from concrete things and useful for counting them), respectively from material numbers (things as countable) and formal numbers (which account for the existence of different concrete things as things of some kind rather than another),[54] Aquinas proposes a reductionist reconstruction, as it were, of Pythagoras' doctrine: there are numbers in any thing and just there.[55] In such a perspective, number turns out to be a sort of common nature,[56] shared by all existing things, making them countable and at the same time making any of them the very existing and countable thing it is, that is, one being of some specific kind. Accordingly, where encapsulating Pythagorean metaphysics in phrases like "any sensible being is substantially number," "all beings are numbers" or "without numbers there would remain nothing in things," Aquinas means to emphasize two complementary aspects. On the one hand, he intends to point to the essential variety whose numbers assure the Pythagorean cosmos. On the other hand, Aquinas draws attention to the necessity of a thing's being one, an undivided and countable substance, in order to just be. In fact, in Aquinas's view the Pythagorean doctrine of principles presupposes no real distinction, indeed no difference, between the notion of one as a transcendental and the numerical notion of one: no matter how the notion of one is taken,[57] it will be convertible with the notion of being, so that by asserting the unity of something, we will affirm nothing more than its existence. In this sense the notion of one means the substance of things: unity (as multiplicity) is not an accident supervening to another subsisting nature,

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50 Id. 1964, *In Metaph.*, §161.
51 Id. 1953, *De potentia*, 9, 7, sol.
52 Id. 1884, *In Phys.*, I, 10, 6.
53 Id. 1964, *In Metaph.*, §205–206; Id. 1886, *In De caelo*, III, 3, 1; Ibid., 4, 7.
54 Albert the Great 1960, *Metaph.*, I, IV, 2 (49, 49–68).
55 Thomas Aquinas 1964, *In Metaph.*, §163, 206.
56 Ibid., §900.
57 Id. 1953, *De potentia*, 9, 7, sol.

but is that very nature.[58] Therefore, there is no longer any difference between substance and quantity, and their categorial distinction is compromised. Notice that, as we will see more clearly below, within a Pythagorean perspective the notion of good is not comparable to those of one and being. One and good are not considered convertible notions; and in fact, they are mentioned as separate principles in the table of opposites. Moreover, there is not just one ratio of good, while there is a univocal notion of unity.[59] Good is an attribute of one and being, an accident with respect to the substance of things. For something to be good an already subsisting nature is required, while this is not the case with oneness. Therefore, goodness can be considered a constitutive cause of the substance of things only derivatively, in virtue of the fact that its subject of inherence is properly a constitutive cause of them.[60]

The Pythagorean thesis according to which "all is number" is openly criticized by Aquinas for several reasons. The first arises from an interior perspective. In fact, from the premise that number is something common to sensible beings, it does not necessarily follow that number is the *substance* of such beings. For it could also be a necessary property of such substances.[61] Secondly, Aquinas' target is the explanatory insufficiency of the thesis "all is number." On his view, the Pythagoreans are unable to account for the enormous variety of species observable in nature. In fact, once the substantiality of number is acknowledged, sensible things must be gathered according to quantitative criteria, such that beings naturally belonging to disparate classes would be unnaturally mixed up – e.g., triangles, syllogisms and three-dimensional bodies, in so far as each shares the number three as a constitutive principle.[62] Thirdly, Aquinas finds here a kind of circularity. If numbers and things are ultimately identical

58 Aquinas criticizes this doctrine, which he relates to Avicenna. According to Avicenna, however, the reason why one as a transcendental and the numerical notion of one are identical is that both mean something more than the simple being of things. For it is not the substance of a thing that makes it one. Aquinas maintains rather that one as a transcendental and as a number are not identical, the former adding nothing to the notion of being, the latter signifying by contrast something more, namely an accident with respect to being. Cf. Id. 1888, *Ia Pars*, 11, 1.

59 Id. 1964, *In Metaph.*, §178–179; Id. 1969, *In Ethic.*, I, 7, 68–84.

60 This assumption entails the impossibility of postulating the existence of a principle that is good *per se*. Notice that, following Aristotle, Aquinas provides a sort of empirical explanation for such a Pythagorean thesis: physical principles cause goodness and perfection in physical effects, without in turn being good and perfect. Cf. Id. 1953, *De potentia*, 7, 1, sol.; Id. 1964, *In Metaph.*, §2545.

61 Ibid., §901.

62 Ibid., §1512.

substantially, numbers (or things) end up as causes of themselves.[63] But nothing can be cause of something substantially identical. Fourthly, Aquinas raises a problem of adequacy. According to him, by postulating numbers as principles of sensible things, *in* sensible things, the Pythagoreans ultimately cannot separate physics from mathematics.[64] Their approach to sensible (properties of) bodies seems in fact to presuppose that there is nothing belonging to physical entities that cannot already be found in mathematical ones. Therefore, they reveal themselves not only unable to account for phenomena like natural change, but also unable to describe in specific terms beings undergoing it.

The Table of Opposites. Aquinas' reconstruction of the Pythagorean doctrine of principles – namely of what Aquinas seems to consider its standard version, according to which there are two series of ten principles, opposed to one another, from which all reality would be derived – is essentially grounded in four texts of Aristotle's corpus,[65] which Aquinas seems to read as complementary: (i) *Methaphysics* 986a22–26, wherein the contents of each series[66] are detailed; (ii) *Physics* 201b24–26, where it is made explicit that to the second series[67] belong indefinite principles, namely principles of privation; (iii) *Nicomachean Ethics* 1096b6, where Aristotle says that Unity is placed by the Pythagoreans under the series of goods;[68] and (iv) *Categories* 14a23–25, where there is actually no allusion to the Pythagorean series, but only to one of the pairs belonging to them: good and evil, which are here characterized as belonging to no genus, being themselves genera. The relevance of this latter text derives from Aquinas' reading of it as a passage where the Philosopher, permitted by its logical context, assumes a Pythagorean viewpoint rather than expressing himself from his own perspective.[69]

Aquinas' way of designating the Pythagorean series is not univocal. With very few exceptions, he exploits almost the whole lexical range emerging from

63 Ibid., §206.

64 Ibid., §205; Id. 1886, *In De caelo*, III, 4, 7.

65 Concerning *Metaphysics* 1093b11–14, which Aquinas neither comments on nor quotes elsewhere, there is no evidence for establishing its importance for him.

66 *Sustoichia* in Greek; variously rendered into latin: *ordo* in James of Venice's translation and the composite version (Aristotle 1970, 18, 15; 101, 16); *coelementatio* in William of Moerbeke's translation and in the anonymous version (Aristotle 1995, 25, 336; Aristotle 1976, 19, 1).

67 (*Co*)*ordinatio* in the Latin of James of Venice (Aristotle 1990, 103, 1) and Moerbeke, *pars* according to Michael Scot (in Aristotle 1562, 90va).

68 *Bonorum sistichia* according to Burgundio's rendition; *bonorum coordinatio* according to Grosseteste and the person revising his translation, possibly Moerbeke (Aristotle 1973, 729b).

69 Thomas Aquinas 1926, *Contra Gent.*, III, 8, 21b; Id. 1982, *De malo*, 1, 1, ad11.

the Latin translations at his disposal. Notice the absence of the transliteration *sistichia*, which Albert the Great, by contrast, uses quite often. Following Aristotle, Aquinas assigns the pair good-evil a twofold role. On the one hand, as we will see in a moment, he considers it a pair among the others. On the other hand, he assigns to good and evil a more fundamental role with respect to the rest of the series, which are thus reduced to good and evil as to their ultimate principles.[70] Openly inspired by the aforementioned passage (iii), this latter interpretation of the Pythagorean table of opposites stems also from the reading of passage (iv) in light of passage (ii).[71] In fact, according to Aquinas, given that one of the two terms involved in any opposition must be better, as it were nobler, more perfect, than the other, which by contrast necessarily involves some privation,[72] good and evil can be considered as the genera under which any other contrariety must fall. As Aquinas points out, *genus* is here to be taken in a broad, non-technical sense:[73] as something containing many things, indeed all things sharing some common character.[74]

Among the approximately twenty places wherein Aquinas refers to the Pythagorean doctrine of series, in four he explicitly lists the ten pairs of opposite principles concerned. In all these contexts, Aquinas substantially follows Aristotle's enumeration in the first book of his *Metaphysics* (i). In his view, to the two series belong respectively: finite, odd, one, right, male, rest, curve, light, good, square; infinite, even, many, left, female, movement, straight, darkness, evil, rectangle. Aquinas' access over time to different versions of this text generally accounts for the slight differences which can be observed. First, odd is once associated with infinitude rather to finitude (and consequently even with this latter). This reversed order is possibly inspired by the anonymous version (*Media*) of Aristotle's *Metaphyiscs*. Moreover, it is worth mentioning that when commenting specifically on *Metaphysics* A5, Albert the Great points out that if in general evenness of number is joined by Pythagoras to infinitude, sometimes it is joined rather to finitude.[75] Second, instead of "square," Aquinas once

70 Unlike Albert the Great, who applies this hierarchical reading also to passages in which Aristotle assigns no special role to good and evil, Aquinas accurately follows the Philosopher's various formulations and nuances in different contexts.

71 Also, the *Philosophia Prima* of Avicenna may have played some role, see Bertolacci 1999, 205–220; about Avicenna's confutation of Pythagoras and the Pythagoreans, see also Porro 2011, 128–135.

72 Thomas Aquinas 1953, *De potentia*, 3, 6, sol. and ad10.

73 Id. 1888, *Ia Pars*, 48, 1, ad1; Id. 1886, *In De caelo*, II, 2, 13.

74 Id. 1929, *In II Sent.*, XXXIV, 1, 2, ad1; Id. 1918, *Contra Gent.*, II, 41, 362b.

75 Albert the Great 1960, *Metaph.*, I, IV, 3 (51, 17–35). Cf. *Metaphysics* 986a17–19, with James of Venice's translation and the composite version. Aquinas' paraphrase of these lines seems consistent with the anonymous' and Moerbeke's rendition.

mentions "triangle," a reading attested by James' and the composite version of the *Metaphysics*.

As does his master,[76] but with a considerably different result, Aquinas chooses not to limit himself to enumerating the twenty principles included in the Pythagorean table of opposites. In fact, he also tries to pinpoint the criteria according to which such principles are established, namely closed off from the others, the accidental principles.[77] He proceeds thereby to deduce, as it were, the opposites belonging to the table, starting from finitude and infinitude, which can be attributed respectively to odd, because of its indivisibility by two, and to even, because of its divisibility by two. Odd and even are actually the first principles of all things, in so far as numbers derive from them. By consequence, the third opposition is obtained by looking at the most basic distinction that can be made among numbers, that is between one and many. By positioning numbers in space, extended magnitudes originate. In order to allow such a spatial distinction, right is opposed to left. By adding active and passive potencies to extended magnitudes, physical entities are obtained. From there we find the necessity to postulate male and female as the table's fifth opposition. But physical entities can perform merely natural or even animal actions. For assuring the former, there is added a sixth opposition, between rest and movement. As far as moving bodies are concerned, they can be furtherly divided by looking at the two possible kinds of local change ascribing to them. The seventh opposition, between curved and straight, makes this distinction possible. As far as animals are specifically concerned, principles for their cognitive and appetitive faculties are needed. Aquinas posits light and darkness as principles of knowledge, and good and evil as principles of voluntary actions. As far as the last, tenth, pair of contraries is concerned, Aquinas makes them principles of perfection and imperfection in extended magnitudes, namely of the figures supervening to them. Square is thereby opposed to rectangle. The former is originated by multiplying just one straight line by itself, while two separate lines are supposed to be multiplied one by the other in order to obtain the latter.[78]

It is worth remarking that Aquinas' criticism of the Pythagorean table of opposites is very moderate. In fact, with a single exception,[79] he tends to highlight the positive intuitions from which such a doctrine derives, rather than

76 Albert the Great 1960, *Metaph.*, I, IV, 4; Id. 1891, *Ethic.*, I, V (79); Id. 1968, *Super Ethic.*, I, VI, 32 (28, 44–50); Id. 1987, *Phys.*, III, I, 6 (161, 52–162, 33).

77 Thomas Aquinas 1964, *In Metaph.*, §125–130.

78 Cf. Id. 1984, *In De an., II*, 3, 55–72.

79 Id. 1953, *De potentia*, 3, 6, sol.: notice that Pythagoras is here joined to the *antiqui philosophi*, explicitly caracterized as materialists.

focusing on its weak points. In his view, the table as such is not the main source of philosophical concerns. His target is rather the radical interpretation of the subordination of the table's two columns to good and evil, at which we hinted above. In fact, while Aquinas refuses to assign evil any hypostatization or any ontological consistency, he is ready to consider it "a privation of being," to which any relative imperfection can legitimately be reduced.

4 Aquinas' Criticism of the Theory of Heavenly Harmonies

Before concluding our study, let us add a few considerations about Aquinas' interpretation of the Pythagorean theory of heavenly harmonies. Although he does not refer to this doctrine very often, Aquinas seems to be concerned with it for a philosophical reason, in so far as it strikes him as ultimately endangering the ordering of the cosmos. Notice that the person of Pythagoras is somehow directly involved in this debate, because of his presumed capacity to hear heavenly harmonies. The plausibility of this anecdote, which Aquinas seems willing to take in a non-literal way, does not matter much to him. His focus falls rather on the theory's psychological and cosmological implications.

In the second book of the *De caelo* (290b12–291a), after having treated the motion proper to the stars, Aristotle criticizes the Pythagorean position according to which this motion lies at the origin of harmonies. First, Aristotle presents the Pythagorean theory. This is grounded on the analogy existing between earthly bodies and the stars: since the former produce sounds when they move, the motions of the stars, which are much bigger and faster than earthly bodies, must produce even greater sounds. Moreover, because the mutual relations between celestial motions are analogous to the relations between musical notes, such sounds must be harmonious. Finally, these harmonies are rendered imperceptible to us by the faulty disposition of our sense of hearing.

Aristotle's refutation begins by this last point: even without taking hearing into account, these stars are so immense that their motions would produce other perceptible effects, as does thunder whose force can rip apart the toughest among the bodies; but since we cannot perceive such phenomena, we must reject the hypothesis of heavenly harmonies (290b30–291a6). Aristotle then explains that objects produce sounds only if they move in a body that does not itself; on the contrary, if they move within a body that is moved by the same kind of motion, they produce no sound, as we can see in the case of a moving ship, wherein things contained in the ship, and moved by its motion, produce no sound. Now the absence of sounds or any other perceptible effect produced

by the stars, according to Aristotle, provides an additional proof of his theory of celestial motions, according to which stars move in and are moved by the celestial bodies of their spheres (291a6–26).

Aquinas' main source for his commentary on this chapter is Simplicius' commentary, which he knew by way of the Latin translation produced in 1271 by William of Moerbeke. It is, however, important to note that Aquinas has tried to compliment Aristotle's account with other Pythagorean sources, specifically with respect to harmony. Indeed, Aristotle limits himself to saying that the voice of the stars is harmonious, since their speeds have the same ratios as the notes of a chord, as a consequence of the distances separating them. Simplicius' commentary is more explicit: the Pythagoreans, according to him, discovered that the distances separating the spheres from the earth are ordered in harmonious ratios, and that the same harmonious ratios must be found in the sizes of the spheres, as well as in their velocities (the exterior spheres being both faster and bigger than the inner spheres); moreover, the Pythagoreans extended the same ratios to the sounds caused by the motions of the stars, since the pitch of a sound depends on the speed of the body that produces it. In conclusion, the harmonious ratios of distances are analogous to the harmonious ratios of sizes, velocities and pitches.[80] Aquinas' explanation is quite close to that of the Greek commentator, but he adds a brief remark taken from Boethius' account on Pythagorean "symphonies": "determined proportion – writes Thomas – according to determined numbers of high and low, is the cause of harmony in sound; as the proportion of two to one produces the diapason (i.e., the octave), the proportion of three to two, called sesquialterate, produces the diapente (i.e., the fifth), and so on."[81] The last note seems to come from Boethius' *De institutione musica* (I, 10), exposing Pythagoras' analysis of the musical intervals.[82]

Although he largely depends on Simplicius' exegesis, Aquinas occasionally assumes a critical approach to it. Such is the case with the Greek commentator's defense of the Pythagorean position concerning celestial sounds.[83] In fact, Aquinas ultimately rejects it. His refutation focuses mainly on two of Simplicius' presuppositions, the first of which is that heavenly motions do not destroy life, but rather preserve it. The second is the admissibility of the possibility of sensory objects disproportionate to the sensory organs by which

80 Simplicius 1894, II, 9, 464; Id. 2004, 121.

81 Thomas Aquinas 1886, *In De caelo*, II, 14, 3 (the translation is ours).

82 Boethius 1867, pp. 196–198. The same Boethius' text is quoted in Aquinas 1984, *In De an.*, I, 7, 129–144. See also Calcidius 1962, *In Tim.*, XXXV.

83 Thomas Aquinas 1886, *In De caelo*, II, 14, 6.

they could be perceived. By consequence, as there are scents perceptible to other animals but imperceptible to humans, resulting from a disproportion between such scents and the human capacities for smelling, we could suppose that human hearing is commonly out of proportion to the sounds produced by stars. Still, there are people whose senses are sufficiently refined and pure as to be able to perceive what others cannot. Now, such is exactly the case with Pythagoras, recalls Aquinas following Simplicius: it seems he was capable of perceiving celestial harmonies, either because of the perfection of his hearing or because of his intellectual capacities. In the latter case, remarks Aquinas, it is in virtue of his capacity to grasp the proportions producing heavenly harmonies that Pythagoras would be said to be able to hear them. Now, in Aquinas' opinion both of Simplicius' presuppositions are false.[84] First, Aquinas draws attention to the destructive power of stars, taking the sun as an example. He points out that even if we can agree that it causes life, we must nevertheless acknowledge that its light can corrupt our vision, because of a disproportion between its power and our faculties for seeing. Second, Aquinas notes that if any sound is perceptible to no hearing faculty, either the sensible or the sense must be understood equivocally, contrary to Aristotle's theory of perception, according to which senses and sensible objects are understood univocally. In fact, every colour is visible to every (healthy) eye and, vice versa, there can be no such thing as an (healthy) eye incapable of perceiving colours.

Hence, Aquinas reaffirms Aristotle's position, according to which stars do not move by themselves, but by, in virtue of, the motion of the spheres in which they are fixed, as things contained in a moving ship. Here, Aquinas is most probably drawing on Alexander of Aphrodisias' criticisms of Pythagoras. Aquinas actually does not have access to his commentary on the *De caelo*, which is lost, but Simplicius quotes it extensively.[85] From one such excerpt it emerges that in Alexander's view, if stars move by themselves, they must move either within or on the surface of their sphere. If they move within their sphere, one out of three possible outcomes would occur: either (a) the stars would rip the heavenly body, or (b) they would move in a body existing within the body of the sphere, yet distinct from it, or (c) they would move in a void within the sphere. None of these solutions is plausible according to Alexander. Solution (a) is rendered impossible by the incorruptible nature of heavenly bodies. In fact, in order to be crossed by a star, any heavenly body would be expected to be soft, and as such, wet. But wetness characterizes corruptible matter. Therefore, no heavenly body can be soft. Likewise, solution (b) is

84 Ibid., 7.
85 Simplicius 1894, II, 9, 467–468; Id. 2004, 125.

impossible, since it would entail the existence of a corruptible body within the incorruptible body of the heaven, which, according to Alexander, would be unnatural. Alexander considers solution (c) similarly impossible, since it would require the existence of a void.

Moving to the second hypothesis, according to which the stars move on the surface of heavenly bodies, Alexander accounts for its impossibility by way of analogous arguments. In fact, neither a void, nor a destructible, nor any heterogeneous body can be supposed to exist between two heavenly spheres. Aquinas summarizes this position succinctly: "The stars – he writes – could not move by themselves without tearing apart either the spheres or an intermediate body." Thus, he concludes, their motions produce no sound.[86] It is worth noticing that, like Aristotle (291a24–26), Thomas recognizes a sign of providence in the fact that nature does not allow the stars to produce sounds, since this would subvert the earthly order, as the violence of such sounds would wreak havoc on the earthly phenomena.

This example from Aquinas' commentary on the *De caelo*, as well as the texts examined in our previous sections, allows us to conclude that he approaches doxographical materials in Aristotle's works as philosophically relevant to a more precise exegesis of the Aristotelian text and a deeper comprehension of its doctrinal core. Although he does not have precise historical grounds concerning Greek philosophical schools before Aristotle and his documentary knowledge is quite modest compared to our own, such doxographical passages offer him the occasion to acquaint himself with a few important Pythagorean theses which he takes seriously and which he tries to situate with respect to other physical and metaphysical positions. If Pythagoras and the Pythagoreans cannot be counted among the most inspiring and recurrent of Aquinas' (indirect) sources, they doubtless arouse his interest and contribute to his reflection on Aristotle as well as to the formation of his own physical and metaphysical thought.

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86 Thomas Aquinas 1886, *In De caelo*, II, 14, 10 (the translation is ours).

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CHAPTER 12

Latin Christian Neopythagorean Theology. A Speculative Summa

David Albertson

1 Reading the Pythagorean Past

In late 1454 the Flemish university master Heymeric de Campo (d. 1460) received some Pythagorean books from an old acquaintance that challenged his thinking. The ideas he found there did not sort easily into the customary schools and topics that he taught his students. Eventually, it seems, Heymeric reached the conclusion that he needed to reconsider where those books – and their strange mathematical ideas – stood in relation to the history of Christian theology, so that he could account for their eccentric position among other ancient sources.[1] Yet Heymeric's endeavor, an essay in what we would call reception history, proved to be a demanding task. Pythagorean ideas had (and have still) exerted only negligible influence on Christian teachings, at least compared to Platonic, Stoic, or Aristotelian currents. Moreover, even if he wished to memorialize those marginal Pythagorean contributions, it is not clear where he would begin. After all, there was no Pythagorean school to reference that was equal to the Thomists or the Scotists, although the Neoplatonizing Albertist school had always been fond of Boethius and his attempted Pythagorean retrievals. There were certainly no Sentence commentaries or *summae* in a Pythagorean vein for him to consult.[2]

Given this predicament, it seems Heymeric concocted a plan. If Pythagoreanism only appeared rarely and obscurely in the Christian past, then he could not afford to limit himself to the actual past. Instead, Heymeric could narrate other, hypothetical pasts that might well have determined present moments like his own in 1455, but for whatever reason had not. Fascinated by the prospect of a Christian Pythagoreanism, he simply suspended the criterion that his reception history needed to be disciplined by the contemporary moment from which he was narrating. And so without preamble or justification, Heymeric began to write down all of the diverse theological speculations that have

1 On Heymeric de Campo, see Hoenen 1994, 173–209; and Ziebart 2011, 473–476.
2 See Hoenen 1997, 81–103. In this volume, cf. the chapter by Marta Borgo and Iacopo Costa.

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filled, or might have filled, the Church's past, indeed the universal human past of various religions and philosophies. As his list multiplied, he began to win perspective on different roads taken or not taken, detours that might have ramified into new pathways but had been left untrod. Soon he had compiled one hundred chapters naming different species of *theologiae*. He named the treatise the *Centheologicon*, the Hundred-fold Theology, an unusual format that "escapes the literary genres of the philosophers and the theological schools."[3]

Today when we read Heymeric's experimental doxography, the work seems to lack systematic order, spanning angels and demons, Hebrew prophets and Greek philosophers, apostles and evangelists, scholastics and mystics. Like the biologist marveling at the riches of nature, Heymeric thought his exhaustive catalogue might inspire humility, if not reverence: the more scattered the spectrum of human opinions, the more singular the One who escapes their grasp. A few chapters in, Heymeric finally pauses to catch his breath and explain himself to his readers. Unlike the angels, our human efforts to know God grope about in tremendous darkness, *plus ignoranter quam scienter*. "For this kind of knowledge of God, *theologia humana*," he writes, "comes in diverse kinds in proportion to the different condition of human being: whether whole, fallen, or called back and restored from its original Fall [...]. They can be rationally divided by sect into different pursuits of knowing, explained with summaries of each sect, and compared each in turn with the orthodox scales of catholic truth."[4]

But while Heymeric's intentions are universal, he soon betrays his particular Pythagorean interests. Following his account of *theologia humana* (chap. 4), his first priority is *theologia pythagorica* and *theologia platonica* (chap. 5–6). For Heymeric, "Pythagorean" denotes the theologically resonant discourse of speculative number theory, outlined in Boethius' *Institutio arithmetica* and handed down by medieval commentators who developed its peculiar conceptual language.[5] To take one example: Heymeric explains that the tenfold, hundredfold, and thousandfold numbers mediate between God and creatures, much like the descent of lights from the Father in the Ps.-Dionysian *Celestial Hierarchy*. In this species of "Pythagorean theology," Heymeric writes, the principles of numbers are the true principles of all things, and the disciplines of the

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3 Bagnasco 2015, 512–513. Bagnasco shows that Heymeric knew the axiomatic and encyclopedic methods of Alan of Lille, Nicholas of Amiens, and Richard of St. Victor. See Heymeric de Campo, *Centheologicon*, MS Brussels, Bibliothèque Royale, Cod. 2177 (11571–75), fols. 1v–74v; edited by Bagnasco in Heymeric de Campo 2020. Earlier studies include Imbach 1983, 466–477; and Meliadò 2011, 385–412.

4 Heymeric de Campo 2020, 11–12 (*Centheologicon*, 4).

5 See Thierry of Chartres 2014, and the chapter by Cecilia Panti in this volume.

quadrivium teach mathematical but also metaphysical truths. In arithmetic, unity is the root of identity, equality, and concord, and the binary is the root of alterity, inequality, and difference; in geometry, the highest equality is thought as a square incommensurate with the circle.[6] Heymeric could have discovered such *theologia pythagorica* in twelfth-century Platonists like Thierry of Chartres (d. 1157) or Achard of St. Victor (d. 1171), drawing on their radical interpretations of Boethius. In fact, he almost certainly found it in the works of Nicholas of Cusa (d. 1464), the lawyer, cardinal, and Neoplatonist theologian.

After a brief discussion of Alan of Lille (d. 1202) and others (chap. 9–10), Heymeric devotes three chapters (chap. 14–16) to the most recent Pythagorean developments, the startling new books that he had just received from his old friend, Nicholas of Cusa. Decades before, back in 1425, the two young men had briefly overlapped at the University of Cologne, Heymeric as a freshly minted master and Nicholas enjoying a philosophical stint before commencing his career in canon law. In 1454, they traded letters again when Nicholas' episcopal duties brought him near Louvain where Heymeric taught. In the course of that exchange the cardinal's secretary sent the professor three works completed the year before: *De pace fidei* on interreligious dialogue, as well as a pair of volumes in speculative geometry, *De mathematicis complementis* and *De theologicis complementis* – a feat unlike anything Nicholas had ever attempted.[7] In the year after receiving these gifts Heymeric took up the project of *Centheologicon*. While his treatise covers a vast field of possibilities, his first priority is Nicholas of Cusa's mathematizing theology.

In Heymeric's first chapter on Nicholas, titled "ignorantly learned theology" (chap. 14), he references the kind of mathematical symbols that Nicholas had defended in his most famous work, *De docta ignorantia* (1440). In this species of theology, God's infinity is such that an absolute maximum coincides with an absolute minimum. Reason stabilizes the uncertain movements of the senses by reverting to dematerialized, geometrized images of pure intelligibility.[8] Heymeric's second chapter on Nicholas is named "conjecturally Triune theology" (chap. 15). Here Heymeric outlines the constructivist epistemology of Nicholas' *De coniecturis* (1441–1443), having already referenced its Pythagorean passages under the rubric of "Pythagorean theology" (chap. 5).[9]

6 Heymeric de Campo 2020, 13–15 (*Centheologicon*, 5).

7 See Hamann 2006, 56–59, 230–262; and Fiamma 2016, 217–257.

8 Heymeric de Campo 2020, 31–32 (*Centheologicon*, 14). Cf. Nicholas of Cusa 2002, 40–46 (*De docta ignorantia*, I.11–12 [§30–34]).

9 Heymeric de Campo 2020, 32–35 (*Centheologicon*, 15). Cf. Nicholas of Cusa 1972, 7–42 (*De coniecturis*, I.1–8 [§ 5–36]).

Heymeric's final Cusan chapter addresses what Heymeric names "geometrical theology" (chap. 16). Heymeric and Nicholas shared a passion for mystical geometrical figures. In *De docta ignorantia*, Nicholas proposed that infinite lines, collapsing triangles, centerless circles, and other unusual polygons could lead the mind upward to contemplate the invisible God. Even earlier in his treatise *Sigillum aeternitatis* (1433), Heymeric had designed complex triangular diagrams to combat Hussite dissidents in Bohemia, a kind of visual rebuttal against Jerome of Prague's *Scutum fidei* image. Heymeric's "seal" is an equilateral triangle trisected with lines running from the center to three apices, then circumscribed by colorful interlocking circles denoting ontological, ecclesiological, epistemological, and ethical levels of meaning.[10] Twenty years later in *Centheologicon*, Heymeric observes that this style of Pythagoreanizing "geometrical theology" has four aspects. First, geometry lifts the intellect above the flux of matter and grants the intellect greater certainty. Second, such shapes and figures can be freely constructed to orient the imagination toward the divine, especially the Christian Trinity. Third, where the circle reveals the identity and simplicity of the divine essence, the triangle reveals the divine "flow" among persons. Finally, even precise geometrical proofs can elicit a mystical "rapture" of reason.[11]

In short, Heymeric foregrounds Nicholas of Cusa's peculiar theology in *Centheologicon*, and categorizes it explicitly as "Pythagorean." Among the hundred-fold varieties, Heymeric features *theologia pythagorica* and *theologia geometrica* near the top of his list. Indeed he only set out to write this treatise after receiving the cardinal's first work of speculative geometry. We can reasonably surmise that Heymeric intended *Centheologicon*, in part, as a response to the conspicuous and notorious Pythagorean turn in Cusan theology. Not everyone shared Heymeric's enthusiasm. Gregor Heimburg (d. 1472), the prominent German humanist, charged that Cusanus had foolishly sought "to demonstrate with mathematical superstitions the sacred things of the true religion."[12] By contrast, although Heymeric politely leaves Nicholas unnamed, he refers with discreet admiration to a certain *mathematicus theologus*, a "mathematical theologian."[13]

Yet there is more going on here than simply a tribute to the Cusan project. By wedding Boethian number theory to the doctrines of Trinity and Incarnation,

10 Heymeric de Campo 2001, 93–128. See further Meier 2003, 755–781.

11 Heymeric de Campo 2020, 36–37 (*Centheologicon*, 16). Cf. Nicholas of Cusa 1994, 46–48 (*De theologicis complementis*, IX).

12 See Flasch 1972, 221–254.

13 See Nagel 2007.

Nicholas of Cusa had formulated a Christianized Pythagoreanism, or better, Neopythagoreanism, with very few precedents in the Latin West.[14] This achievement was so eccentric, so difficult to categorize by contemporary canons, that it frustrated its own reception. It almost demanded to be reconceived as its own school of thought, supplied with its own retroactive genealogy. Academic theologians like Heymeric in the fifteenth century, split as they were among competing schools, were preoccupied with constructing lineages of tradition from ancient foundations up to present masters. When Heymeric encountered a "school" as novel as Nicholas' Neopythagoreanism, his instinct was to buttress it with its own tradition. Unlike the established schools of Thomism, Scotism, nominalism, and Albertism, in this case Heymeric had to invent a heritage that never, in fact, had taken institutional form as *theologia pythagorica* or *theologia geometrica*.

Heymeric's odd treatise continues to intrigue us because his intentions are not easily named. Is he offering a historical survey of past theologies, yet one that encompasses sacred history as well? Or does it purport to be a principled division of all possible discourses? Does Heymeric intend his list doxographically, recounting ancient traditions from the perspective of contemporary Albertism, as Mario Meliadò has suggested?[15] Or does Heymeric speak in the subjunctive as well as the indicative, inventing counterfactual theological possibilities to fill out his hundred-fold complement? Viewed in this light, *Centheologicon* can help us reflect on the problem of the reception history of Pythagoreanism in medieval Christian thought. Initially Heymeric sought to understand the Cusan project within the canons of the ancient and medieval past. But ultimately, as Heymeric stretched his narrative to accommodate Christian Neopythagoreanism, historical synopsis shifted into imaginary reconstruction. In the end, he could only produce a hypothetical, speculative *Centheologicon*.

Likewise, when we ask today about the obscure fate of Pythagorean traditions in the Latin Christian west, we face the same unsettled mix of history and hypothesis, what was and what might have been. Augustine, Boethius, Thierry of Chartres, Alan of Lille, Achard of St. Victor, Robert Grosseteste (d. 1253), and Nicholas of Cusa all share Neopythagorean impulses to different degrees. But for a variety of reasons their voices never amounted to an integrated discourse. Latin Christian theologies were nourished by the influence of Peripatetic, Middle Platonic, Stoic, and Neoplatonist philosophies that interacted in different ways within competing medieval schools, as we know from decades of

14 See Albertson 2016a, 3–20.

15 See Meliadò 2018, 263–284.

historical scholarship. By contrast, we are only beginning to piece together the minor tradition of Latin Christian theologies influenced by Pythagoreanizing currents. Given their material absence from scholastic texts and institutions, we can only do so speculatively; they never developed actual *summae* because their ideas never took hold in the schools. Heymeric's predicament is also ours.

2 Major and Minor Doctrines

In what follows I survey some key moments in the reception of Pythagoreanism within medieval Christian theology in Latinate western Europe, from the fourth to fifteenth centuries, roughly Augustine to Cusanus. In short, we can call this tradition "mathematical theology," that is, "a species of Neopythagorean henology, oriented around the coeval mediation of Logos and Arithmos, which assumes a universal *mathesis* grounded in the quadrivium."[16] This tradition contends that alongside God's mediation by Word, there is a parallel mediation by Number, since the mathematical structure of the cosmos originates within the Trinity itself. In the language of Christian Platonism, God's primordial Wisdom is not only the Logos containing all forms, but also the Arithmos distinguishing all forms. Or as Augustine himself puts it, the *sapientia* of God and the *numerus* of God are both true mediators, and it is impossible to decide which has priority over the other.[17] God is the "Number without number" and the "Measure without measure."[18]

It is important at the outset to distinguish Latin Christian Neopythagoreanism, or mathematical theology, from neighboring aspects of Pythagorean traditions. First, it is not the same as so-called number mysticism or "arithmology," the view promoted by some Middle Platonists that each numeral correlates to a certain divine power or cosmic force.[19] This path was followed by Macrobius, later by philosophically curious Cistercians, and still later by Johannes Reuchlin.[20] By contrast, mathematical theologies contemplate number as such, the very notion of the mathematizability of the world, and consider the powers of discrete numbers only in their systematic operation as tetrad or decad.

Medieval Neopythagorean theologies also almost completely ignore "akousmatical" traditions about the sage Pythagoras, his ethical and ritual precepts,

16 Albertson 2014, 57.
17 See Augustine of Hippo 1970, 258 (*De libero arbitrio*, II.XI.30).
18 See Augustine of Hippo 1894, 99–100 (*De genesi ad litteram*, IV.3–4 [8]).
19 See Robbins 1921, 309–322, and the chapter by Jean-Pierre Brach in this volume.
20 See Lange 1972; Meyer 1975.

or physical doctrines about cosmic fire – quite unlike the direction taken by Renaissance Platonists like Giovanni Pico della Mirandola or Marsilio Ficino.[21] This is not to deny, of course, that there were also currents of akousmatical Pythagoreanism in the Middle Ages.[22] Authentic mathematical theologies also pass beyond mere henology. Other Christian theologies of the transcendent One by Ps.-Dionysius, Dominicus Gundissalinus, or Meister Eckhart – to name just a few – rarely consider numerical difference as a necessary mediation of the One and the Many, but most Christian Neopythagoreans do.

Finally, we can set aside medieval Latin theologians who were practicing mathematicians, such as Nicole Oresme or Thomas Bradwardine. The power of the quadrivium to unveil cosmic harmonies impressed many great minds of the past and remains a powerful heuristic today.[23] Nicholas of Cusa himself tried his hand at geometrical proofs in the dozen treatises he devoted to pure mathematics.[24] As Richard Oosterhoff has shown, by the turn of the sixteenth century Pythagoras was invoked "to raise the cultural status of mathematics" itself, even if in ways unrelated to Boethian arithmetic.[25] But the history of mathematics is distinct from Neopythagorean theology. Only rarely did medieval authors connect the fundamental structures of arithmetic and geometry immediately to Christian doctrines, as our mathematical theologians aimed to do.

I have told some of this story elsewhere in fragments.[26] Here I have collected, in abbreviated form, three foundational doctrines of medieval Christian Neopythagoreanism: on the nature of the Trinity, on the providence of the Creator, and on the path to Wisdom. Each implies the others, and each tends to germinate in Boethius, sprout in Thierry of Chartres, and flower in Cusanus, often in coordination with commentaries on quadrivial textbooks.[27] To be sure, there are several other doctrinal loci of greater or lesser prominence that we could also include. Three in particular appear consistently in medieval Christian sources: Creation, Incarnation, and Ethics. However, by contrast with the major loci of Trinity, Providence, and Wisdom, these three represent long-term, problematic irritants for the tradition. Each incurs the

21 On the akousmatical Pythagoreanism, see Burkert 1972, 192–208. On Florentine Pythagoreanism, see Celenza 2001; Id. 1999, 667–711; and Robichaud 2018.

22 See Hicks 2014, 416–434; and Joost-Gaugier 2006.

23 See recently Hicks 2017; and Hobart 2018.

24 See Nicolle 2001 and Böhlandt 2009.

25 Oosterhoff 2013, 531–552.

26 I ask readers to tolerate some immodest self-citation; this essay draws on other studies where I have provided more evidence than I could here.

27 See Albertson 2016a, 143–199.

conditions of tangible, physical reality – materiality, the body of Jesus, and practical moral action – that Neopythagorean mathematical models have struggled sometimes to accommodate. While we cannot explore these more complicated instances here, they merit a brief mention, if only to denote areas of creative ferment where the discourse of Christian Neopythagoreanism has not yet fully defined itself.

Thierry of Chartres starts with two rather anodyne terms in Boethius, *alteritas* and *explicatio*. But from them he generates highly original theories of material creation and created difference.[28] The graduated granularity of matter is structured through the plurality of numbers themselves, the gap between 1, 2, and 3.[29] The point enfolds the line, and the line is an unfolded point.[30] Nicholas of Cusa develops Chartrian alterity and folding from cosmological concepts into theological instruments that reveal the secret immanence of the Creator, as if God is hidden within between the infinitesimal degrees and folds of the fundamental structures of being.[31] In *De docta ignorantia*, Nicholas of Cusa expands the domain of folding to a whole series of temporal, arithmetical, and theological registers.[32] God enfolds the world, and the world is the unfolding of God. Then in his later work *De li non aliud* (1462), the alterity of matter does not distance us from God's Otherness, but deepens our access into God's more primordial non-Otherness.[33] Or in Thierry of Chartres' radical formula: alterity descends immediately from divine Unity.[34] To be sure, these examples only sketch some themes of a much longer tradition of Neopythagorean theories of matter, from the "binary" of twoness (the Platonic dyad) to the atomism of the geometrical point.[35]

If the fabric of reality is mathematical, and if the Trinity generates unity and equality, how is the Word made flesh? Ancient Christian theologians did not shy away from mathematical language about Christ, despite concerns that Neopythagoreanism was a crypto-Gnosticism.[36] Clement of Alexandria

28 On *alteritas*, see Boethius 2005, 167–168 (*De sancta trinitate*, I); and *Boethius* 2002, 119 (*Institutio arithmetica*, II.27.2). On *explicatio*, see Boethius 2002, 90 (*Institutio arithmetica*, II.4.6); and Id. 2005, 121–129 (*Consolatio philosophiae*, IV.6 [prosa] *passim*).

29 Thierry of Chartres 2014, 180–182 (*Super Arithmeticam*, II.27–28). Cf. Id. 1971, 77–82 (*Commentum*, II.28–43).

30 Thierry of Chartres 2014, 163–165 (*Super Arithmeticam*, II.4). Cf. Id. 1971, 155–156 (*Lectiones*, II.4–6).

31 See McTighe 1990, 55–71.

32 See, e.g., Nicholas of Cusa 2002, 22–30 (*De docta ignorantia*, II.3 [§ 105–111]).

33 Nicholas of Cusa 1944.

34 Thierry of Chartres 2014, 150 (*Super Arithmeticam*, I.32).

35 See Robert 2017, 181–206.

36 See Kalvesmaki 2013.

understands the incarnate Logos as the "Measurer" of God and humanity, and enjoins philosophers to ascend to the "quantity of Christ" (Eph. 3:18).[37] John of Damascus paints the kenosis of Incarnation as a "reduction to quantity and magnitude."[38] Even Augustine, in a nostalgic moment in *De trinitate*, identifies Christ as the "harmony" of God and humanity best understood as the number 6, the perfect integer.[39] Despite this exception, Augustine's most influential works rejected his own youthful interest in the Neopythagorean wing of Neoplatonism. The later Augustine wished to confront the pride of Neopythagorean number theory with the humility of the Incarnation. This proved to be a durable, almost ineradicable paradigm for western Christian theology. To embrace the flesh of the incarnate Word, it might seem, requires one to reject inadequate mathematical renderings of the materiality of that flesh.[40]

Thierry of Chartres does not solve this problem for Latin Christian Neopythagoreanism; in fact, he fails to record any theology of Incarnation. But in spite of this oversight, his theology of the eternal Word as *aequalitas* opens a new way into Christology. Thierry insists that the Word is both "equality of unity" (in the Trinity) and "equality of being" (in creation). But Thierry also suggests that the Word is a divine *Figura*, the "figure of the substance of the Father" (Hebrews 1:3).[41] More specifically, the divine Son is the figure of the "primal Square," who defines God's unity and equality, indeed even God's *divinitas* itself, through the "power of the square."[42]

Such hints towards a geometrized Christology were taken up energetically by both Achard of St. Victor and Nicholas of Cusa. Achard preached a sermon lauding Christ as God's Square: "This form is a square because it is stable and firm. […] Christ, our form […], became a spiritual square [*spiritualis quadratura*] for us. […] See there a vital, heavenly square! Approach and receive it, you stones […]. You have been hewn into this square form, and thus you have been transformed from dead to living!"[43] For his part, Cusanus takes on the problem of deficient Neopythagorean Christology as a positive agenda. In *De docta ignorantia*, he strikes upon a new cosmological model for construing the Incarnation in mathematical terms. The "concept of Jesus" is the singular point

37 Clement of Alexandria 1981, 143, li. 2–3 (*Stromateis*, v.11.71).
38 John of Damascus 1975, 82 (*Oratio*, III.8).
39 Augustine of Hippo 1968, 163–178 (*De trinitate*, IV.II–IX [4–12], *passim*). See Scully 2013, 93–116.
40 See Albertson 2014, 68–80.
41 Thierry of Chartres 2014, 147 (*Super Arithmeticam*, I.32); Id. 1971, 572 (*Tractatus*, 41).
42 Thierry of Chartres 2014, 186–187 (*Super Arithmeticam*, II.31); Id. 1971, 78–79 (*Commentum*, II.31–35).
43 Achard of St. Victor 1970, 150–151 (*Sermo* 13, §17); id. 2001, 229 (transl. Feiss modified).

of intersection between Word and flesh: between the transcendent Equality prior to numbers (*aequalitas essendi*) and the arithmetical grain of creaturely difference (*contractio universalis*).[44] In *De theologicis complementis* (1453), the divine Word is the "infinite Angle" and "infinite Number," a measure that measures itself, so that God is the supreme Geometer.[45] Finally, in the late work *Dialogus de ludo globi* (1463), Christ is the cosmic centerpoint whose Incarnation binds together the quadrivial contraries of rest and motion, center and circumference, unity and plurality.[46]

Although less common than the doctrines of Creation and Incarnation, a kind of Neopythagorean ethics can be found in some Latin Christian sources. Long before Plato, the "akousmatical" sect of ancient Pythagoreans propagated ethical mandates, while the opposing "mathematical" faction issued cosmological doctrines.[47] Yet even the latter retained the intuition that mathematical order engenders ethical order. Like Plato gazing at astronomical patterns to quiet the soul, like Aristotle calculating virtue through arithmetical mean, Boethius too discovered ethical lessons hidden within the arithmetic of Nicomachus of Gerasa (d. *ca* 150 CE). Evil, disorder, and imperfection stem from inequalities, either excesses or deficiencies of the Good. By the same token, "every kind of inequality proceeds from prior equalities."[48] This Boethian axiom has a mathematical meaning, but also an ethical one. Every evil reveals its derivation from the Good, and thus the possibility of its undoing. Reasoning with precise proportions in Neopythagorean arithmetic carries real moral force: it is the science of reducing inequalities back to their originary perfection, and thus restoring good to the world.

As he studied Boethian arithmetic, Thierry of Chartres encountered the same Nicomachean postulate of the priority of equality over inequality. In his gloss he applied it to natural philosophy and mathematics, but also to ethics. Evil and vice represent a declination (*discessio*) from the perfectly self-equal Good, a violation (*transgressio*) of the definite mean (*modus*) of the good, which functions as a "divine law" (*lex divina*).[49] Equality is the criterion (*iudex*) of evils, the standard from which they depart and return; when inequality is returned to equality, it does so through an "arithmetical proportion" (*arithmetica ratio*). If God is unknowable, the Good is finite and rationally known

44 See Nicholas of Cusa 2002, 2–32 (*De docta ignorantia*, III.1–4).
45 Nicholas of Cusa 1994, 64–67 (*De theologicis complementis*, XII).
46 Nicholas of Cusa 1998, 55–66 (*Dialogus de ludo globi*, I.50–60); and Ibid., 86–90 (II.73–75).
47 See Burkert 1972, 192–208.
48 See Boethius 2002, 66–67 (*Institutio arithmetica*, I.32.1–2). Cf. Nicomachus of Gerasa 1926, 226 (*Introduction to Arithmetic*, I.23).
49 Thierry of Chartres 2014, 144 (*Super Arithmeticam*, I.32); cf. ibid., 152 (I.32).

(*scientia*), but the range of possible evil is infinite in its possible transgressions, and therefore irrational.[50]

Cusanus rarely relates his ideas about the quadrivium to ethics, but he does in his last great Neopythagorean work, *Dialogus de ludo globi* (1463). As its name suggests, the dialogue concerns a geometrized game, in which the player rolls a spherical ball onto a gameboard of concentric circles. A roll that reaches the middle of the board wins the game, representing the soul passing through the cosmos to attain Christ, the governing center. Like human beings, however, the spherical ball is imperfect; it wobbles irregularly due to a concave section carved out of one side. The geometrical allegory allows Nicholas to draw several Augustinian conclusions about the weak human will, though in a Neopythagorean key. Given the freedom of our will, we never know in which circle or point a given ball will stop, after its wandering course comes to its final rest. By nature, "human motion" declines off course, but when greater momentum is applied, whether through habit or grace, one can manage a straighter roll toward the supernatural goal. Christ was the only player who rolled the ball of his human nature without a wobble to the centerpoint, where his ball still remains as model and as end.[51]

Having surveyed the lesser Neopythagorean doctrines of Creation, Incarnation, and Ethics, however briefly, we can now turn to the greater doctrines of Trinity, Providence, and Wisdom. As we begin to perceive the contours of Latin Christian Neopythagoreanism delineated as an interconnected whole, we can better appreciate its coherence as a discourse, even if one that always strayed, in Nicholas' own words, "beyond the customary schools of philosophy" (*supra philosophorum communem viam*).[52] In this way we can begin to reconstruct, if not a historical summa, then a speculative one, the summa that it never received – and better perceive the limits of the effective past that still constrain us.

3 On the Trinity: Numeration as Divine Name

Since antiquity Christian Platonists have taught that the order and harmony of mathematics provide a trace of divine presence, and hence a path of return to God. But Christian Neopythagoreans take the further step of claiming that the

50 Ibid., 147–148 (*Super Arithmeticam*, I.32).

51 Nicholas of Cusa 1998, 6–8 (*Dialogus de ludo globi*, I.5–7), and 66 (I.59–60).

52 See Nicholas of Cusa 2002, 100 (*Epistola auctoris ad dominum Iulianum cardinalem*, in *De docta ignorantia*, III [§ 263–264]).

inner mystery of God is itself a kind of eternal numeration that nevertheless preserves its unity. On this view, the divine Trinity is uncountable by number specifically because God is the very fount of number – not extrinsically, as its Creator, but intrinsically in Godself. To be God is to be the primordial One (*unitas*), the One eternally the same as Oneself (*aequalitas*), and to be so to such a maximal degree that the One's unity and equality themselves unite into a secondary, higher-order equality (*conexio*) that resounds, so to speak, from the power of the first. Before God is named Father, Son, and Spirit in the lexicon of reproduction or respiration, God is unity, the equality of unity, and the equality of unity and equality. This is what it means to state that God is the primordial fount of number, the Number beyond number.[53]

This entails that for Christian Neopythagoreans, number is not a symbol of God, but touches something true beyond other kinds of divine names. Mathematics equips the human mind with the ultimate conceptual instrument, not by accident, but because God is transcendent numeration itself. If numbers are the infrastructure of being and thought, they are even more unsurpassable when used theologically. Hence Nicholas of Cusa's maxim: "If something is true in mathematics, it will be even more true in theology."[54] As the founding axiom of every mathematical theology, this doctrine of the mathematical Trinity – God as unity, equality, and connection – merits special attention.

We can spy the origins of this triad in the first stirrings of Greek Neopythagoreanism, well before the Neoplatonism of Plotinus.[55] Struggling to bridge the *Republic*, *Timaeus*, *Philebus*, and *Parmenides* dialogues, the Middle Platonist Eudorus of Alexandria (fl. *ca* 50 BCE) proposed a bold new synthesis. The supreme divine One reigns over two lesser principles, the limiting monad and the unlimited dyad.[56] A century later, Moderatus of Gades (fl. *ca* 50 CE) suggested a slightly different threefold henology: the One beyond Being, the One of Intellect (the Logos) containing the forms, and the One of Soul engendering quantitative numbers.[57] According to Porphyry, Moderatus also taught that the Pythagorean "One" signified the principles of "oneness" (*henótēs*), "equality" (*isótēs*), and "concord" (*sumpnoia*).[58] Although the dating remains controversial, some scholars think Marius Victorinus (d. 364) might have been drawing on Pythagorean ideas to combat the Arians, if it was he who wrote:

53 See McGinn 2002, 237–264.
54 Nicholas of Cusa 1994, 14 (*De theologicis complementis*, III).
55 See Dillon 1996, 126–128, 344–351.
56 On Eudorus, see Dörrie 1976, 276–309; and Bonazzi 2007, 365–377.
57 On Moderatus, see Dodds 1928, 129–141; Rist 1962, 389–401; Hubler 2010.
58 Porphyry 1963, 43–44 (*Vita Pythagorae*, 48–50). Cf. Guthrie 1987, 133.

"God is the One that generates the monad from himself and reflects love in his unity."[59] After all, Augustine, who famously received the *libri Platonici* from Victorinus, casually repeats a similar triad in *De doctrina christiana*: "In the Father there is unity (*unitas*), in the Son equality (*equalitas*), and in the Holy Spirit a harmony (*concordia*) of unity and equality."[60]

Augustine never discussed the triad again, but medieval readers would note similarities between his theological *unitas* and *equalitas* and the parallel terms they found in Boethian mathematics. In his influential *Institutio arithmetica*, Boethius paraphrased, and sometimes simply translated, works by a leading architect of ancient Neopythagoreanism, Nicomachus of Gerasa. Along with his technical analysis of numbers and proportions, Boethius also imported Nicomachean doctrines of the divine *unitas* and *equalitas*. For example, unity is the fecund "mother" from whom all difference streams through the procession of numbers. The self-equality of unity preserves the possibility of cosmic justice and an ultimate return to the One, as the evil of inequalities is restored to the good of equality.[61] Scholars have wondered if Augustine himself had direct access to Nicomachus' writings.[62]

During the twelfth-century renaissance in western Christian theology, this mathematical triad made a comeback amidst resurgent interest in the Boethian corpus and widespread efforts to systematize Trinitarian names.[63] An echo had already sounded at Cluny the century before, when Stephen of Liège penned a new antiphon for the feast of the Trinity drawing on Marius Victorinus via Alcuin: "Eternity dwells in the Father, equality in the Son, and in the Holy Spirit, the connection of eternity and equality."[64] But the most original voice in the *aetas Boetiana* was Thierry of Chartres, born in Brittany, who according to his students was the top humanist of his generation at Paris and Chartres. Thierry proposed a radical hermeneutical innovation. Boethius had named the same God as the Neopythagorean One in *Institutio arithmetica* and as the Christian Trinity in *De trinitate*. Thierry simply started reading Boethius' doctrinal works and mathematical works univocally, trusting that they would fit together like two sides of a puzzle. This subtle maneuver effectively relaunched Christian Neopythagoreanism as a discursive option, however short-lived, within medieval Latin theology.

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59 Hudry 2009, 150. Hudry contends that the *Liber XXIV philosophorum* was not a medieval text, as widely believed, but in fact was written by Marius Victorinus.

60 Augustine 1995, 16–17 (*De doctrina christiana*, 1.12 [5]).

61 Boethius 2002, 31–32 (*Institutio arithmetica*, 1.14.2–4).

62 See Solignac 1958, 114–148.

63 See Hödl 1965; and Chenu 1957, 142–158.

64 See Feiss 1983, 39–66.

In his early commentary on *Institutio arithmetica*, Thierry of Chartres links the metaphysics of "equality" in Boethian number theory to the "equality" of Father and Son in Augustine's triad. In the Word, the Creator forms all things, shaping each creature into itself, neither greater nor lesser, but the precise equality of its own being (*aequalitas entitatis*).[65] For the Spirit, Thierry replaces Augustine's musical "harmony" (*concordia*) with the arithmetical function of linking or bonding numbers (*connexio*), so that the Trinity can be named through number alone: "There is between unity and the equality of unity a certain harmony and connection. This connection we also name as unity itself: namely, there is the first unity, then the equality of unity, which is the connection."[66] Thierry's mathematical Trinity became a signature theme in his later lectures and never ceased to fascinate him.

For instance, Thierry concludes his own Genesis commentary, *De sex dierum operibus*, with a coda in praise of the quadrivium.[67] The systematic rationality of the fourfold mathematical arts returns the mind to the Creator by manifesting God's presence within every unity, equality, and connection. The triad allows Thierry to uncover deep sympathies between Augustine's theology of creation and Boethius' aesthetics of measurement. The power of unity unlocks the secret workings of creation and even allows us to glimpse the Creator. "Unity is all-powerful in the creation of number," he writes, "but the creation of number is the creation of things. […] Unity is therefore necessarily deity."[68] Unity generates other numbers by generating itself: the number 5, for example, is also always 5 x 1. Yet the very inequality of 5 and 1 directs us back to the root 1, the ever-prior self-equality of unity (1=1) that makes departure from itself even possible. Since there cannot be two eternal equalities, divine Unity must be a Tri-unity of self-equal ones.[69]

Equality not only orders the arithmetical domain, but also the geometrical. "Just as unity creates all numbers from itself," Thierry explains, "so too the same equality of unity produces all the proportions and inequalities of all things from itself, and in itself resolves them all again."[70] Equality is the "mean" (*modus*) and "measure" (*mensura*) of everything, by which each singular creature is equal to itself and no another. As the governing force of created difference, Equality is divine Wisdom, the "mind of God" that contains

65 See Thierry of Chartres 2014, 146–148 (*Super Arithmeticam*, I.32); ibid., 149 (I.32); and ibid., 152 (I.32).

66 Thierry of Chartres 2014, 146 (*Super Arithmeticam*, I.32).

67 Thierry of Chartres 1971, 568–575 (*De sex dierum operibus*, 30–47).

68 Ibid., 570 (*Tractatus*, 36).

69 Ibid., 570–572 (*Tractatus*, 37–40).

70 Ibid., 573 (*Tractatus*, 43).

the primordial divine ideas within itself, and the form (*forma essendi*) that delivers individual beings into their truth.[71]

Thierry's Genesis commentary breaks off before he can address *connexio*, the third moment of the triad, but leaves some hints that are fulfilled in later commentaries.[72] If unity grounds the arithmetical order (numbers), and equality the geometrical order of measured proportions (ratios of numbers), then *connexio* grounds the harmonics of the complex, second-order "proportionalities" discovered in the remaining quadrivial arts, *musica* and *astronomia* (ratios of ratios of numbers).[73] In subsequent commentaries, Thierry adds that the eternal, divine triad of unity, equality, and connection is reflected in another "perpetual," cosmological triad of alterity, integrity, and motion.[74]

Due to Augustine's endorsement, the arithmetical account of the Trinity enjoyed broad attention in the middle decades of the twelfth century. Thierry's Neopythagorean reading of the triad in terms of the Boethian quadrivium was unusual, however. Most other early scholastics read Augustine's triad in a weaker, non-mathematical sense. For them, unity, equality, and connection were not the proper, mathematical names for the three persons of the Trinity, but rather denoted attributes of the one God that could be provisionally attributed to various divine persons, much like power, wisdom, and goodness. The Son can be called Equality, but Equality itself does not define Sonship, as Thierry had proposed. Following the controversies of Peter Abelard and Gilbert of Poitiers in the 1140s, the 1150s and 1160s were decades of consolidation. Simon of Tournai, Peter of Poitiers, and Alan of Lille all affirmed the more modest reading of Augustine's triad in their *summae*, although Alan would experiment with Thierry's Neopythagoreanism in his later *Regulae theologiae*. Above all, Peter Lombard's *Sententiae* established the non-mathematical interpretation as the dominant one, still legible in later commentaries by Albert the Great, Thomas Aquinas, and Duns Scotus.[75]

Until Nicholas of Cusa in the fifteenth century, only Thierry's minor students repeated his Neopythagorean reading, such as Clarembald of Arras or the author of the Hermetic treatise *De septem septenis*.[76] The one conspicuous exception is Achard of St. Victor, the abbot of St. Victor and later bishop of

71 Ibid., 572–574 (*Tractatus*, 41–46). Cf. Id. 2014, 146–148 (*Super Arithmeticam*, I.32).

72 Thierry of Chartres 1971, 575 (*Tractatus*, 47).

73 Ibid., 225 (*Lectiones*, VII.7). Cf. Id. 2014, 186 (*Super Arithmeticam*, II.31); ibid., 199 (II.42).

74 Thierry of Chartres 1971, 80–82 (*Commentum*, II.39–43); cf. Id. 2014, 150 (*Super Arithmeticam*, I.32).

75 See Albertson 2012a, 101–144.

76 See Häring 1965; and (Ps.-) John of Salisbury 1855, 945–964 (*De septem septenis*).

Avranches.[77] Amidst the decline of the mathematical Trinity, Achard sought to develop the triad into a theory of divine aesthetics in his remarkable *De unitate dei et pluralitate creaturarum*. Achard posits that the Triune God is not simply oneness, but rather a supreme plurality, the only perfect plurality. Creation is manifold, but finite creatures lack true plurality because each falls short of divine unity. But in God's perfect plurality, each unity is equal to every other unity, gathering into a harmony (*congruentia*) of unity and plurality that radiates maximal beauty (*summa pulchritudo*). "The beauty of each would of itself coalesce into the complete unity of the other, and somehow fuse with its beauty," writes Achard. "Nothing can be or can be thought which is more beautiful [...]. It is therefore necessary [...] that it be God."[78] For Achard, this is what it means to hold that God is – in his version of the triad – unity, the equality of unity, and equality itself.[79]

Like Thierry, Achard draws on Boethian arithmetic to demonstrate the Neopythagorean Trinity. It is even possible that he had access to Thierry's breakthrough *Arithmetica* commentary.[80] For Achard, God is the primal odd (1), the primal even (2), and the original prime (3); likewise God is the triangular root of all other geometrical space.[81] Since equality precedes every inequality, as Boethius teaches, there must be an eternal equality that is also the equal of each of its equivalent terms.[82] Achard correctly directs his readers to Augustine's theology of number in *De musica* and his Genesis commentary.[83] If Thierry's attention to self-multiplying numbers grounds the mathematical Trinity in arithmetical science, Achard's emphasis on harmony and equality effectively substitutes music as the governing quadrivial basis.

Achard's contributions are significant, but there is really no comparison to Nicholas of Cusa when it comes to the mathematical Trinity. One can hardly read any of his works without encountering the triad of unity, equality, and connection in some way or another.[84] From his late-medieval vantage point, Nicholas would have known the triad from Augustine, and certainly heard the Boethian resonances. But Nicholas' primary sources came from Thierry of Chartres and his early readers. Nicholas puts the triad front and center in

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77 See Albertson 2018, 353–386.

78 Achard of St. Victor 1987, 74 (*De unitate dei et pluralitate creaturarum*, I.5); id. 2001, 382.

79 Id. 1987, 86 (*De unitate dei*, I.14); id. 2001, 390. Cf. Id. 1987, 90–96 (*De unitate dei*, I.17–23).

80 See Albertson 2019.

81 Achard of St. Victor 1987, 90–92 (*De unitate dei*, I.18–19); id. 2001, 394–395.

82 Id. 1987, 92 (*De unitate dei*, I.20); id. 2001, 395. Cf. Boethius 2002, 66–67 (*Institutio arithmetica*, I.31.1–2); and Thierry of Chartres 2014, 143–146 (*Super Arithmeticam*, I.32).

83 Achard of St. Victor 1987, 150 (*De unitate dei*, II.5); id. 2001, 443–446.

84 See Haubst 1952; McGinn 2003, 90–117; and Resch 2014.

his first major theological work, *De docta ignorantia*. It is even possible that the architecture of the three books of *De docta ignorantia* was inspired by the mathematical triad. The first book is on God as "absolute maximum," and the second is on the world as "contracted maximum." But the third book explains their common intersection in the incarnation of Jesus, whose humanity is the *conexio* of all created difference.[85]

Nicholas begins Book I with the mathematical Trinity. God as absolute maximum escapes the continuum of numbers altogether as absolute unity, and therefore coincides with the absolute minimum.[86] After all, unity is both minimum, as the beginning of numbers, and maximum, as their end.[87] But we in our measurements can only deal in relative degrees of precision; we never attain perfect equality, since one thing can always be more equal to another. The intellect can only approximate transcendent equality, like the polygon that can only infinitely approximate the circle.[88] Cusanus then adduces arguments reminiscent of Boethius' *Institutio arithmetica*, and especially Thierry's *Arithmetica* commentary, to argue that all inequality derives from eternal equality. Since God's unity is eternal, and God's equality is eternal, they must be united in a co-eternal connection: *unitas, aequalitas, conexio*. Interestingly, however, Cusanus does not attribute the triad to Augustine or a Chartrian source; instead he conspicuously traces it back to Pythagoras himself.[89]

In subsequent works, Nicholas continues to experiment with Thierry's triad, either modifying individual terms or else draping a different vocabulary upon the same arithmetical frame. In his middle works, Cusanus tests different strategies for combining the Boethian quadrivium with the mathematical Trinity. In *De coniecturis* Cusanus superimposes the two conceptual bases of the quadrivium, multitude and magnitude, upon the mathematical Trinity. Inscribed into the very structure of the four mathematical sciences, God's triune nature preserves the difference of number and the difference of quantity as *multitudo*, *magnitudo*, and *compositio*. As the triune image of God, the human mind mathematizes naturally, itself composed of quadrivial proportions and measures.[90] By *De theologicis complementis*, Nicholas simply fills out the triad with geometrical terms. God is Center, Line, and Circumference,[91] or again,

85 Nicholas of Cusa 2002, 24 (*De docta ignorantia*, III.3 [§ 201]); and ibid., 6 (III.1 [§ 185]).
86 Ibid., 16–18 (I.4 [§ 11–12]).
87 Ibid., 22 (I.5 [§ 14]).
88 Ibid., 12–14 (I.3 [§ 9–10]).
89 Ibid., 18–21 (I.7 [§ 18–21]).
90 Nicholas of Cusa 1972, 9–10 (*De coniecturis*, I.1 [§ 6]).
91 Nicholas of Cusa 1994, 17–18 (*De theologicis complementis*, 3).

the geometer's inscribing, being inscribed, and circumscription.[92] Later in *De aequalitate* (1459), the harmonic proportions of *aequalitas* take over the whole triad: God is absolute equality, the equality of equality, and their connection.[93]

In his later works, Nicholas became increasingly interested in physics and the power of pure possibility. In *Trialogus de possest* (1460), he attempts to mathematize Aristotelian hylemorphism in terms of the triad. Creatures reflect the Trinity in their immanent structure as possible (*posse*), actual (*actus*), and their composition (*nexus*). But we can also see the Father as absolute possibility (*posse absolutum*) and the Son as the existence (*esse*) of all that the Father can possibly be. Upon the scaffolding of the mathematical triad, Cusanus constructs a new name for God as the divine "Can-Be" (*poss-est*), a Trinitarian neologism that verbally unifies *posse* and *esse* in a literal *nexus*.[94]

4 On Providence: Numbers as Divine Ideas

For Christian Neopythagoreans, God is the very fount of number, as the harmony of unity and equality. Beyond this foundational Trinitarian theology, the tradition teaches that every created being is ordered by essential numbers, and that God's Wisdom or Providence is an infinite repository containing and preserving those numbers of things. Only the Mind of God (*mens dei* or *mens divina*) knows all of the numbers; indeed these infinite numbers are the ideas that fill God's Mind. If each creature has a numerical signature, then the Creator's blueprint for the cosmos must be mathematical in structure, the hidden code of the created order.

Already in the first generations after Plato in the Old Academy, Speusippus and Xenocrates contended that the forms were actually a set of primordial numbers or geometrical objects. Some Middle Platonists, especially monotheists like Philo of Alexandria, asked if the mind of God contained transcendent numbers as divine ideas.[95] For Nicomachus of Gerasa, the demiurgic mind is not simply filled with numbers or shapes, but with the very principles of the quadrivium. In his widely read *Introduction to Arithmetic*, Nicomachus offered this gloss on Plato's *Timaeus*: "arithmetic existed before all the others in the mind of the creating god like some universal and exemplary plan, relying

92 Ibid., 30 (*De theologicis complementis*, 6).
93 Nicholas of Cusa 2001, 46–48 (*De aequalitate*, 35–36).
94 Nicholas of Cusa 1973, 54–63 (*Trialogus de possest*, 44–51).
95 See Dillon 2010; Dillon and Toland 2021, 34–52; but also classic studies by Rich 1954, 123–133; and Wolfson 1961, 3–32.

upon which, as a design and archetypal example, the creator of the universe sets in order his material creations." Number, he explains, is the eternal pattern and paradigm of the world structure, precisely because numbers abide in the divine mind.[96] In later centuries Iamblichus envisioned tiers of greater and lesser numbers cascading like a champagne tower from the One to the Many. The mundane numbers that we use in mathematical calculations are but lower-order refractions of the divine numbers radiating from the divine Intellect.[97]

Although this Platonist doctrine predates Christianity, medieval Christian Neopythagoreans would become some of its greatest exponents. They could not help but notice it recurring throughout the most venerable sources. "God has these models of all things within himself," they read in Seneca, "and has embraced the numbers and measure of all things which are to be accomplished in his mind. He is filled with those shapes which Plato calls Ideas: immortal, immutable, indefatigable."[98] We find the same image of the *mens Dei* brimming full of numbers in the Latin Neoplatonism of Calcidius and Macrobius.[99] Augustine is more complicated on this point. He accepts the forms as the ideas of God's mind in his *Quaestio de ideis*.[100] In *De libero arbitrio* and other works he suggests that God's Wisdom can be construed as God's Number, such that lower and higher numbers eventuate in divine numbers.[101] But Augustine's concessions to Neopythagoreanism never add up to the doctrine that God's ideas as such are numbers.

Instead, within Latin Christian traditions, we must wait for Boethius to pass along Nicomachean doctrines in the mathematical handbook *Institutio arithmetica*:

> From the beginning, all things whatsoever which have been created may be seen by the nature of things to be formed by reason of numbers. Number was the principal exemplar in the mind of the creator. [...] Arithmetic is prior to all not only because God the creator of the massive structure of the world considered this first discipline as the exemplar of his own thought and established all things in accord with it; or that

96 See Nicomachus of Gerasa 1866, 9 (*Introductio arithmetica*, 1.4, 2); ibid., 12 (1.6, 1). Cf. Helmig 2007.

97 See O'Meara 1989.

98 Seneca the Younger 1917, 448 (*Ad Lucilium*, 65 [7]); and Caiazzo 2005–2006, 91–116.

99 See Theiler 1930, 1–38; and Krämer 1964, 275–279.

100 See Gersh 1986, 403–413; and De Rijk 1975, 204–213.

101 See Horn 1994, 389–415; and Ladner 1959, 214–223, 454–459.

> through numbers of an assigned order all things exhibiting the logic of their maker found concord.[102]

God's Mind is arithmetical, since God thinks numbers as the ideal exemplars of every created being – an image not far from Galileo's famous suggestion that the universe is a great book written in the language of mathematics, and hence only legible in terms of geometrical exemplars.[103] Yet, even while he hands down Nicomachus' words, Boethius subtly shifted their meaning for the next millenium, altering the theological function of number in Christian Neopythagoreanism. On the one hand, Boethius consolidated the four mathematical arts in a way that Nicomachus had not, and grounded them in the primacy of arithmetic; on the other, Boethius chose to leave Nicomachus' arithmological treatise on the sacred decad untranslated.[104] Thereafter, the numbers in God's Mind were not a supreme Decad, but rather arithmetical science itself, the basis of the entire quadrivium. God is not the supreme number; God is the supreme mathematician.

Boethius' vision of numbers residing within the mind of God captivated medieval Platonists theorizing the causal order of nature in the twelfth century.[105] If the mathematical order of creation subsided deep within the Creator, how could they fail to discover the same structures and rhythms? At the turn of the twelfth century, Bernard of Chartres (d. 1130) posited that a network of secondary causes (*formae nativae*) dwelled within matter to mediate the organizing force of the World Soul. Grasping such native forms, human reason can understand with necessary certainty the autonomic regularity of nature without reference to God or divine illumination. Bernard further proposed that this new physical science, distinct from theological or poetic adumbrations of nature, was governed by the four disciplines of the quadrivium that Boethius had theorized in such detail. Number becomes the immanent principle of natural order.[106] After Bernard's intervention, in the words of one historian, nature becomes "a play of numbers only accessible to thought, numbers which ground its outward appearance and are expressed through it [...] as something that owes its determinacy to corresponding mathematical structures."[107] Descartes' mathematization of nature in the seventeenth

102 Boethius 2002, 11 (*Institutio arithmetica*, 1.2.1); Id. 1983, 75–76 (transl. Masi modified); and Boethius 2002, 8–9 (*Institutio arithmetica*, 1.1.8); Id. 1983, 74.

103 See Galileo Galilei 1957, 237–238.

104 See Guillaumin 1990, 139–148; Id. 2003, 341–355.

105 See Speer 1995, 105–126.

106 Bernard of Chartres 1991, 76–78.

107 Schrimpf 1995, 208.

century would be wider and deeper, but it does find precedent in these fellow French philosophers from 500 years before.[108]

Thierry of Chartres' retrieval of Boethian number theory was sure to include his doctrine of numbers as divine ideas. In his commentary on *Institutio arithmetica*, Thierry glosses the key passage from Boethius (cited above) as follows:

> God is said to have created according to the exemplar of number, since all things have being from differentiation, which comes from numbers. And note that God is said to possess arithmetic (that is, number) as the exemplar of his reasoning (that is, his arranging), since it is according to number (that is, according to the differentiation of things foreseen in God's mind) that God arranged ALL THINGS. I say "all things" which are harmonized through number, that is, by means of their designated proportion, namely, the order they have been assigned in the divine Mind. For the order of all things proceeds just from such order and proportion.[109]

All things are foreseen in the reasoning of God, who arranges all creatures harmoniously, despite their differences, within the divine Mind. This providential order is, in essence, arithmetical activity, because God is dealing with the numerical essences of creatures eternally resident in God's Wisdom. Later in the commentary, Thierry identifies the "divine mind" or "providence" with the eternal "equality" or "mean" (*modus*) of the Good. In both cases, God operates as *praefinitio aeterna*, a biblical phrase for God's providential purpose (Eph. 3:11), which Thierry uses to connote the primordial measure of creaturely difference through numbers.[110] As Thierry writes in his later hexaemeral work, the divine Word as Equality "circumscribes and delimits" each created thing. That Word is the Creator's *praefinitio aeterna* measuring the total categorial location of every creature – what, how much, what kind, in what space and time – demarcating its "equality of existence" as neither greater nor lesser than itself.[111]

Thierry had developed an account of God's Equality as mean in his *Arithmetica* commentary.[112] But now he applies that insight to the theory of divine ideas from his *De trinitate* commentaries. First, Thierry shows that the

108 Speer 1995, 293–294.
109 Thierry of Chartres 2014, 114 (*Super Arithmeticam*, 1.8).
110 Ibid., 144 (*Super Arithmeticam*, 1.32); cf. id. 1971, 572 (*Tractatus*, 42).
111 Id. 1971, 574–575 (*Tractatus*, 45–46).
112 Id. 2014, 143–149 (*Super Arithmeticam*, 1.32).

divine ideas, now called "notions" (*notiones*), operate not only as forms but as measures of being.

> The equality of unity is a certain mean beyond or below which something cannot exist. And this mean cannot be anything other than primal, eternal Wisdom. Wisdom alone is that according to which the being of any given thing is determined, beyond or below which it cannot rightly be itself. Hence the forms of all things possess existence also by measure. There [in Wisdom] are contained the notions of things, for the knowledge of a thing is always contained in its equality.[113]

Next, Thierry points out that such "notions" are mathematical objects:

> Therefore, just as the Equality of unity contains within itself, and generates from itself, the notions of things, so too Equality contains and produces the forms of all things. And just as Unity engenders numbers from itself, so Equality of unity produces the proportions and inequalities of all things.[114]

As Thierry demonstrates, the Trinitarian theology and sapiential theology of creation that dominate Latin Christianity after Augustine, in effect, remain open to fusion with Boethian Neopythagoreanism. This is the thrust of Thierry's radical judgment that "the creation of number *is* the creation of things" – or, as it could also be translated: "creation means: the creation of the numbers of things."[115]

Thierry's closest readers, Achard of St. Victor and Nicholas of Cusa, both endorsed the exemplar doctrine enthusiastically, likely influenced on this point by his *Arithmetica* commentary. For Achard, God's Wisdom is not only One, but Many, "an impenetrable and infinite multitude."[116] God is the "there" (*ibi*) where the eternal reasons of all the things we know "here" (*hic*) dwell.[117] Here there is only one moon; there they are infinite. Here every creature has one shape and its own peculiar beauty; but there it inhabits every dimensional possibility and infinite degrees of beauty. The beauty of a single eternal reason

113 Id. 1971, 573 (*Tractatus*, 42).
114 Ibid. (*Tractatus*, 43).
115 Ibid., 570 (*Tractatus*, 36).
116 Achard of St. Victor 1987, 186–188 (*De unitate dei*, II.18).
117 Ibid., 110 (I.40).

exceeds the beauty of the entire material cosmos, making God's mind a kind of virtual multiverse rippling with unmeasurable depths.[118]

Achard divides eternal reasons in God's mind into formal causes, final causes, and explicative causes.[119] His major argument in *De unitate* is that the eternal forms in God's mind cannot be a simple unity in God, but abide as an infinite multitude, even if they are collected into a diverse whole by God's Word or Wisdom. There are finite forms here on earth (*in actu*) and infinite forms there in heaven (*in intellectu Dei*), and yet – although reason can scarcely conceive it – Achard insists these two are "the same in number" (*numero idem*).[120] Created forms and eternal forms are linked by number rationally, since God, as Triune fount of number, remains amenable to number. The multitude of eternal forms in the mind of God are distinct in the forms themselves, and distinct in eternity not only time; and this compromises nothing of God's sovereignty, since after all, God is more plurality than unity.

For Achard the numerical identity of forms reveals God's intimacy with creation. Each eternal form expresses a virtual infinity of possibilities, a harmonious (and thus maximally beautiful) reconciliation of singularity and multiplicity. Whether looking at the infinite cosmos or a single grain, God sees in a single glance of the unique "divine eye," the "inestimable magnitude" of each creature and therefore its "inestimable beauty."[121] The infinitely beautiful parts flow together and converge into an inconceivable whole that preserves every spot and wrinkle of finite difference – not as a abstract composite, but as a living network (*connexio*) of infinite numerical possibilities, whose vast expanse only God's Mind can measure. Achard ends by furnishing his readers a florilegium of Seneca, Boethius, and the early Augustine on the exemplar doctrine.[122]

Another mathematical theologian who merits greater attention is Robert Grosseteste, bishop of Lincoln. While eminently comfortable with mathematical language for God, Grosseteste was informed more by the new wave of scholastic interest in Aristotle than by the Boethian-Nicomachean stream bridging Thierry, Achard, and Nicholas. For example, Grosseteste expounds a complex mathematical doctrine of the Trinity in his hexaemeral commentary (*ca* 1230), not unlike Thierry's *Tractatus* the century before. But where Thierry begins with the mathematical triad of unity, equality, and their connection, Grosseteste weaves together other alternatives from the early Augustine,

118 Ibid., 116–118 (I.45).
119 Ibid., 108 (I.39).
120 Ibid., 120 (I.46).
121 Ibid., 130 (I.48).
122 Ibid., 150–152 (II.5).

including the physics of matter, form, and composition; Peter Abelard's power, wisdom, and love; and the biblical triad measure, number, and weight (Wisdom 11:21). Finally he adds *magnitudo*, *figura*, and *ordo* – a geometrizing revision of Augustine's *modus*, *species*, and *ordo*. Taken as whole, Grosseteste writes, this latticework of Trinitarian patterns suggests that "number leads the understanding to wisdom, since according to Augustine number and wisdom are the same thing."[123]

Just as Grosseteste's mathematical Trinity offers an alternative to the Chartrian triad, so too his account of numbers in the mind of God is as original as it is Neopythagorean. Grosseteste invokes the Senecan doctrine of mathematical exemplars in the divine Mind, grounding necessary reasons in divine Providence.[124] In his commentary on Aristotle's *Physics*, he posits that all cosmic measures are rationally founded on an ultimate measure. Yet such knowledge is impossible in principle, since one cannot discern one basic unit for an infinite continuum of atomistic points. God's infinite mind, however, knows such infinities with ease, as if they were finite. Hence God's own mathematical knowledge is the foundation of human mathematics. God is, in Grosseteste's remarkable phrase, *Mensurator primus et certissimus*, the transcendent *Numerator*.[125] Likewise the Trinity: the *mensura* of the Son is equal to the *mensura* of the Father.[126] As if turning Achard's number theory on its head, Grosseteste's infinite numbers remain uncountable, until God, counting infinities *simpliciter* as finite, preserves our finite numbers in their objective and rational order.

Nicholas of Cusa also frequently invoked the Boethian exemplar doctrine. In fact, in later centuries, Cusanus' most devoted readers like Jacques Lefèvre d'Étaples would be most excited by this doctrine.[127]Already in *De docta ignorantia*, Nicholas writes that "our Augustine, and later Boethius, declared that of the things to be created number was undoubtedly the 'principal exemplar in the mind of God.'"[128] That is, the doctrine of numbers as divine ideas serves as a signature, shared with Augustine and Boethius alike, of the nascent tradition of Christian Neopythagoreanism that Nicholas wants to support. In Book II of *De docta ignorantia*, he returns to the same problem:

123 Robert Grosseteste 1982, 222–223 (*Hexaëmeron*, Pars VIII, Cap. IV, 3–6). Id. 1996, 226–227.
124 Id. 1912, 167–168 (*De libero arbitrio*, Cap. 5).
125 McEvoy 1982, 175–180. Cf. Robert Grosseteste 1963, 86ff.
126 Robert Grosseteste 1912, 148 (*De ordine emanandi causatorum a Deo*).
127 Oosterhoff 2018, 339–366.
128 Nicholas of Cusa 2002, 42 (*De docta ignorantia*, I.11 [§ 32]); id. 1997, 101.

> Therefore, just as number arises from our mind because we understand as individually many that which is commonly one, so the plurality of things arises from the divine mind, in which the many exist without plurality because they exist in enfolding unity. [...] Who, I ask, could understand how the plurality of things is from the divine mind, since God's understanding is God's being and God is infinite unity?[129]

Nicholas continued to puzzle over the ideas of number residing in the divine Mind. He notes that divine numbers must be the source of all plurality, but also that God is the source of unity who cannot be multiplied like creatures.[130] In short, he stumbles into the same problem of numbers "here" and "there" that Achard of St. Victor had already posed in *De unitate*.

The problem continued to stimulate Nicholas of Cusa's thought as he turned to his next works. In *De coniecturis* Nicholas posits numbers in human beings as mental exemplars that drive us to pursue *mathesis universalis*. "Inferring symbolically and surmisingly from the rational numbers of our mind to the real, ineffable numbers of the Divine Mind," Nicholas writes, "I say that in the Mind of the Creator number is the first exemplar of things, just as number that arises from our reason is the first exemplar of our corresponding world."[131] The human mind is the image of the divine Mind. If God is the mathematical Trinity of unity, equality, and connection, our minds reflect God's by being triune mathematical measures: measuring multitude (as diversity), measuring magnitude (as inequality), and measuring their composition (as division).[132]

Later in *Idiota de mente* (1450), Nicholas reaffirms the anthropological doctrines of *De coniecturis*, yet now in terms of enfolding and unfolding. Just as the divine Mind contains all numbers, so too the human mind has special affinity with mathematics.[133] The divine Mind enfolds all things in their truth, yet in a miraculously simple unity that has the power to encompass infinite cosmic difference. God's concepts produce things actively, but our concepts assimilate things passively. "All things are present in God, but in God they are exemplars of things," he writes. "All things are present in our mind, but in our mind they are likenesses of things."[134] Cusanus emphasizes that the human mind does

129 Id. 2002, 24–26 (II.3 [§ 108–109]); id. 1997, 135–136.
130 Id. 2002, 26 (II.3 [§ 109]).
131 Nicholas of Cusa 1972, 14 (*De coniecturis*, I.2 [§ 9]); id. 2000, 167 (transl. Hopkins).
132 Id. 1972, 9–10 (*De coniecturis*, I.1 [§ 6]).
133 Id. 1973, 132–133 (*De mente*, VI [§ 88]).
134 Ibid., 110–112 (III [§ 73]); id. 1996, 543 (transl. Hopkins); id. 1973, 148 (VII [§ 98]).

not stand in any relationship of folding with God, but is merely an image of God's eternal enfolding.[135]

Such anthropological developments only deepen the mystery of number in the divine Mind. We know plurality because of plural numbers in our mind. But how can there be plurality of number in the simplicity of God's mind? In answering this question, Nicholas sketches a vision of infinite divine plurality not unlike Achard of St. Victor's before him. Strictly speaking, he explains, the plurality of things comes from God's mind, not ours, when God understands one thing in more than one way: as a unity, as a replicated unity (equality), and as their mutual connection. Our mind generates plurality in a similar manner when, through the activity of our own enumeration, we imitate God as a lesser triune *imago Dei*.[136] We encounter God's plural numbers immediately in the beauty and order of the proportions surrounding us in the world, and indeed in our mind's capacity to discriminate and measure the sharp edges of distinct things. If plurality begins with God's numbers, Nicholas contends, there is no need to separate God and world with an intermediating tier of mathematicals, as Plato had suggested. Rather, with the Pythagoreans, Nicholas affirms that "between the Divine Mind and things there is no actually existing intervening number. Instead, the number of things *are* the things."[137]

5 On Wisdom: *Mathesis Universalis*

The third major doctrine of Christian Neopythagoreanism is a matter of both epistemology and ethics, or perhaps a principle of method. We might just as well have started with it as prologomenon as ended with it as conclusion. As Cusanus wrote, all theology proceeds in a circle (*theologia circularis*).[138] This is also perhaps the Neopythagorean doctrine both most commonly recognized and commonly misunderstood. It is the notion that all thinking should be disciplined by mathematics, that number and extension are the key to interpreting any phenomenon, and that the so-called mathematization of nature liberates the intellect, through the objective precision of measurement, from the vagaries of myth, illusion, and even dogmatic theology. In his *Rules for the Direction of the Mind* (1628), Descartes named this enterprise "*mathesis universalis*," a

135 Ibid., 114 (IV [§ 74]); id. 1996, 544.
136 Id. 1973, 138–142 (I.6 [§ 93–95]).
137 Ibid., 143–145 (I.6 [§ 96]); id. 1996, 555.
138 Id. 2002, 86–88 (*De docta ignorantia*, I.21 [§ 65–66]).

universal mathematical method. He explicitly names the four sciences of the quadrivium, but wants to expand their purview into "a general science which explains all the points that can be raised concerning order and measure irrespective of the subject-matter."[139] In 1936, Edmund Husserl singled out the Cartesian "mathematization of nature" as the critical rupture with the human past that marked the advent of modernity, associating the authority and certainty of mathematical knowledge with individualism, freedom, progress, and secularism.[140] Martin Heidegger's influential historiography follows Husserl closely on this point.

Since then, historians have found more nuanced ways to compare ancient Neopythagoreanism and Cartesian *mathesis*, but the notion that modern secularism is advanced by the mathematical method of natural science remains pervasive.[141] The term itself was invented as a pejorative. In 1633, the Jesuit theologian Melchior Inchofer warned that whomever followed Galileo's new teachings was no longer Christian but "Neopythagorean."[142] Yet this disjunction between the quadrivium and theology would surprise the Christian Neopythagoreans described above. For them, universal mathematical method was profoundly theological, following directly from the other two foundational doctrines. If God is the fount of all number and measure, and if God contains and preserves the secret arithmetical blueprints of the world, the wise theologian would know all things through number, precisely in order to reach God through the created order. In this view, *mathesis universalis* is a spiritual practice. Far from rendering God obsolete, or evacuating the material cosmos of divine presence, it increases the proximity of God and world. In mathematics one discovers a trace of God's order and measure.

Well before Plato, stretching back to Philolaus and Archytas, there has always been a feedback loop between awe at the power of concrete mathematical sciences, on the one hand, and inductive theological conclusions about transcendent mathematical structures and beings, on the other, all way up to the divine One. Universal mathematical methods have implied Neopythagorean theologies, and naturally such beliefs have enjoined thinkers to adopt exclusively mathematical methods.[143] Starting from Archytas and Aristotle, but adding his own distinctions, Nicomachus theorized a fourfold

139 Descartes 1908, 377 (*Regula*, IV); id. 1985, 19 (transl. Cottingham *et al.*).
140 See Husserl 1970, 21–59; cf. Roux 2010, 319–337.
141 See Rabouin 2009; Bechtle 2007, 129–154; and Crapulli 1969.
142 Dörrie 1984, 756–758.
143 See Albertson 2014, 25–27 and 136–139.

system of universal mathematical knowledge: arithmetic (absolute number), geometry (quantity at rest), harmonics (relative number), and astronomy (quantity in motion).[144]

Boethius adopted the same fourfold system and renamed it the "quadrivium," and by doing so sustained the Neopythagorean doctrine within Latin Christian scholasticism for the next thousand years. For the most part, the quadrivium organized scientific pursuits in different physical domains.[145] But some philosophers took up the deeper truth that Boethius emphasized at the beginning of *Institutio arithmetica*:

> Among all the ancient men of authority who, following the lead of Pythagoras, have flourished in the purer reasoning of the mind, it is clearly obvious that hardly anyone has been able to reach the highest perfection of the disciplines of philosophy unless the nobility of such wisdom was investigated by him in a certain four-part study, the quadrivium, which will not be hidden to a just and penetrating mind. For this is the wisdom of things that are, and the perception of truth gives to these things their unchanging character.[146]

Wisdom comes from knowing the most enduring things of the world. But since these necessary reasons are known through the eternal categories of number and quantity, the only sure path to knowledge of any kind passes through the quadrivium:

> If a searcher is lacking knowledge of these four sciences, he is not able to find the true; without this kind of thought, nothing of truth is rightly known. This is the knowledge of those things which truly are; it is their full understanding and comprehension. He who spurns these, the paths of wisdom, does not rightly philosophize. Indeed, if philosophy is the love of wisdom, in spurning these, one has already shown contempt for philosophy.[147]

144 See the essays on Nicomachus by Robbins and Karpinski in D'Ooge's translation, in Nicomachus of Gerasa 1926, 3–145; and Napolitano Valditara 1988.

145 See Klinkenberg 1959, 1–32.

146 Boethius 2002, 6 (*Institutio arithmetica*, 1.1.1); id. 1983, 71 (transl. Masi modified).

147 Id. 2002, 7 (*Institutio arithmetica*, 1.1.5); id. 1983, 72–73.

After these solemn admonitions, Boethius revisits the theological status of mathematics in *De trinitate*. Paraphrasing Aristotle, he weighs the differences between physical, mathematical, and theological abstract forms.[148] The mediating level of mathematical knowledge is closest to God, but also conceptually unstable given its intimate proximity.[149]

Again it is Thierry of Chartres who moved the Neopythagorean discourse forward in his attempts to integrate the fragments of the received Boethian legacy. Thierry sought for years to formulate a universal theory of science that could bind together the three *rationes* that Boethius names in *De trinitate* (physics, mathematics, and theology), as well as the four levels of knowledge in Boethian anthropology (sense, imagination, reason, intellect). In seeking a master concept, Thierry also sought a unifying perspective on his own commentarial work, which spanned the hexaemeral physics of creation in Genesis, the quadrivial writings on arithmetic and music, and the theological opuscula on Trinity, Christ, and God's being. In Thierry's first attempt at a synthesis, he tries a fivefold modal theory (God, prime matter, actuality, created spirit, number) and a fourfold Pythagorean ontology (God as unity, matter as duality, with the forms and actuality as two means in between). He even experiments with materialist psychology of fluid *spiritus* coursing through the arteries of the brain, and with a pantheistic account of matter and form descending immediately from the Trinity.[150] But soon Thierry abandons these because he has finally struck upon a solution: the superior model of the "four modes of being."[151]

His mature modal theory is nothing less than a Neopythagorean *mathesis universalis*. To paraphrase Descartes, Thierry's system of four modes is a general science explaining all the ways of knowing in terms of "order and measure," irrespective of the particular disciplinary subject matter. The system rests on two oppositions: necessity (*necessitas*) and possibility (*possibilitas*); and enfolding (*complicatio*) and unfolding (*explicatio*). Together they generate four modes of being:

148 See Boethius 2005, 168–171 (*De sancta trinitate*, II).

149 Merlan 1953, 56–59, 71–75.

150 Thierry of Chartres 1971, 68–82 and 97–98 (*Commentum*, II.1–43 and IV.6–10). Cf. Albertson 2014, 121–139.

151 Thierry of Chartres 1971, 155–166 (*Lectiones*, II.4–34); and ibid., 271–276 (*Glosa*, II.12–36).

	Complicatio	Explicatio
Necessitas	*Necessitas absoluta* (1st mode) = *Deus, simplicitas*	
		Necessitas complexionis (2nd mode) = *ordo, fatum*
Possibilitas		*Possibilitas determinata* (3rd mode) = *actualia*
	Possibilitas absoluta (4th mode) = *materia primordialis, chaos*	

The first mode of being is absolute necessity, which enfolds the second mode, the necessity of composition or enfolding. Divine simplicity unfolds into the "order and measure" of mathematical difference. The third mode is ordinary actual beings, unfolded into the manifold of determinate possibilities. The fourth mode of being is pure, absolute possibility, a universal enfolding that is opposed to God as dyad is to monad. These four modes also index each speculative discipline and each faculty of human understanding.[152] The first mode corresponds to theology, known to intellect, while the second corresponds to mathematics, known by reason. Together the third and fourth modes circumscribe the domain of physics, known empirically through sense and imagination.

The brilliance of Thierry's system is that it harmonizes mathematical and theological inquiries, at least in principle. The second mode is properly subordinate to the first, and yet it is inextricably bound up with the first through reciprocal folding. Theology is the enfolding of mathematics, so to speak, and mathematics is simply unfolded theology. Moreover, both conceptual foundations of Thierry's modal theory share Neopythagorean origins. First, necessity and possibility stem from the "Pythagorean" opposition of the One (*unitas: Deus*) and the Two (*binarius: materia*), as Thierry understands it.[153] Second, the concepts of enfolding and unfolding are themselves first theorized as models of geometrical space. Points spread into lines, and lines express the full potential

152 See id. 1971, 158 and 164 (*Lectiones*, II.11 and II.30).
153 See ibid., 77 (*Commentum*, II.28).

of points, each the fold of the other.[154] Hence the folding that structures the modes of being is ineluctably mathematical; the whole can be collapsed into one unified Fold, a divine Measure.

Thierry of Chartres was not alone in his devotion to the wisdom of the quadrivium. Alan of Lille reimagined theological dogmatics as a universal deductive method, a series of axioms *more geometrico* unfolding stepwise into conclusions of increasing scope. Alan's chief model was Boethius' axiomatic ontology in *De hebdomadibus*, but his results resembled Euclid's *Elements* as well, and anticipated others like Spinoza's *Ethics* or Wittgenstein's *Tractatus*. Like a mathematical proof, each postulate of *Regulae theologiae* rigorously followed from the previous. His first rule is that God is not simply one (*unus*) but unity itself (*unitas*): "unity begets unity from itself; from itself it brings forth equality."[155] The second rule is that God's unity is utterly unique (*unitas singularitatis*).[156] By the fourth rule, Alan has demonstrated that "in the Father is unity, in the Son equality, in the Holy Spirit the connection of unity and equality."[157]

Robert Grosseteste applied geometrical form to physical science, anticipating in some ways the mathematization of the cosmos in the seventeenth century.[158] In one geometrical work he proposes that causes, effects, and all of natural philosophy about the universe must be exclusively understood as various "lines, angles, and figures."[159] Yet it is Grosseteste's brief but powerful *De luce* that envisions the most exhaustively mathematized vision of the physical reality, one that might have been influenced by Thierry of Chartres.[160] The first corporeal form is a point of light, which instantaneously multiplies and extends itself in every dimension into a spherical cosmos. Such material space operates as a substrate for all beings, empty in itself and endowing all with visibility.[161] Its omnidimensionality, however, is mathematically structured. Because light is simple per se, the primal *punctum* must be infinitely multiplied to produce a finite quantum of corporeal dimensionality. Since there are greater and lesser infinities, it follows that every spatial relation, every possible

154 Thierry of Chartres 2014, 163–165 (*Super Arithmeticam*, 11.4).

155 Alan of Lille 1981, 125 (*Regulae theologiae*, 1.5). See further id. 1995; Evans 1980, 36–52; and Dreyer 1996, 106–170.

156 Alan of Lille 1981, 126 (*Regulae theologiae*, 2.3).

157 Ibid., 128 (*Regulae theologiae*, 4).

158 Speer 1996, 73–90.

159 Robert Grosseteste 1912, 59–65 (*De lineis angulis et figuris seu de fractionibus et reflexionibus radiorum*).

160 McEvoy 1987, 91–110.

161 Robert Grosseteste 2013, 226–228 (*De luce*, 1–40). See further Lewis 2013, 239–247; and Hedwig 1980, 119–140.

corporeal observation, is subjected to the original geometry of the luminal space radiating from point to sphere. This space is mathematically cognizable through numerical ratios, but to do so one must activate every possible rational and irrational ratio, given the infinite distance between the first point and its explosion into boundless space. For this reason, Grosseteste concludes, all bodies are composed of points, lines, and surfaces, and the wise person searches for the mathematical structure of creation.[162]

In the first book of *De docta ignorantia*, Nicholas of Cusa joins the Boethian tradition of *mathesis universalis* when he elevates mathematical concepts above all others. Not unlike Descartes, Nicholas embraces mathematics for the order and measure it conveys in the face of uncertainty:

> Every inquiry consists in a comparative proportion that is either easy or difficult. Because the infinite escapes all proportion, the infinite as infinite is unknown. But since proportion expresses agreement in some one point and also expresses otherness, it cannot be understood apart from number. Number, therefore, includes all that is capable of proportion. […] Perhaps this is why Pythagoras insisted that all things are constituted and understood through the power of number. […] Since our approach to divine things is through symbols, we can appropriately use mathematical signs because of their incorruptible certitude.[163]

Accordingly, Cusanus is proud to call his own theology a "Pythagorean investigation" (*inquisitio Pythagorica*).[164] He praises Pythagoras by name repeatedly and attributes the mathematical Trinity to him.[165] He follows Albert the Great in applauding "Pythagoreans and Peripatetics" who defeated the godless Epicureans, led by "Pythagoras, the first philosopher in name and deed."[166]

Beyond his enthusiasm for Pythagoras, we should also observe how Cusanus is among the first self-aware genealogists of the tradition of Christian Neopythagoreanism. He places Augustine and Boethius in sequence and outlines their common faith in universal mathematics as a theological instrument. First, he affirms Boethius' commendation of the quadrivium: "Boethius, most learned of the Romans, maintained that without some training in mathematics

162 Robert Grosseteste 2013, 228–230 (*De luce*, 41–84).
163 Nicholas of Cusa 2002, 6–8 (*De docta ignorantia*, 1.1 [§ 3]); id. 1997, 88 (transl. Bond). Id. 2002, 44 (1.11 [§ 32]); id. 1997, 101.
164 Id. 2002, 36 (*De docta ignorantia*, 1.9 [§ 26]).
165 Ibid., 26–30 (1.7 [§ 18, 21]).
166 Ibid., 42–44 (1.11 [§ 32]).

no one could attain a knowledge of divine things."[167] Then he traces this idea from Pythagoras through "the Platonists and also our own major thinkers." "Our Augustine, and later Boethius" defended the doctrine of numbers as exemplars in the divine Mind, Nicholas explains. Augustine himself "turned to mathematics for assistance" to formulate the mathematical Trinity in *De doctrina christiana* and to prove the immortality of the soul in *De quantitate animae*. Finally: "our Boethius seemed to be so pleased with this method that he constantly asserted that all true doctrine is contained in the doctrines of plurality and magnitude."[168]

That singular phrase – "our Boethius" – encapsulates Nicholas of Cusa's awareness of an emerging Christian Neopythagorean tradition, one he wishes to sustain and augment. It includes a tacit reference to the writings of Thierry of Chartres and his students, whose (for him) anonymous texts Cusanus cites, glosses, and generally attempts to harmonize throughout large swaths of Books I and II of *De docta ignorantia*. From Nicholas' point of view, the Chartrian materials are a collection of like-minded treatises commenting on "their" shared Boethius, alluding to Pythagoras and combining mathematics with Christian contemplation. They are partners with whom he can cooperatively steward the Boethian legacy of Neopythagorean arithmetic and theology, the wisdom of the quadrivium in its full profundity. By 1449, Nicholas will celebrate all of the Chartrian ideas stemming from that author whom he could only characterize as a nameless "commentator on Boethius," calling Thierry "easily the most brilliant man of all those whom I have read." This is an even more striking statement in a work that praises Ps.-Dionysius and Meister Eckhart with fainter words.[169]

Even Heymeric de Campo, Nicholas' old friend, found the quadrivium to be a storehouse of universal wisdom. Heymeric's search for all-encompassing knowledge extended to Llullian logic, systems of the liberal arts and sciences, and most of all visual geometrical symbols of his own invention.[170] But the *mathesis universalis* that Heymeric discovered in the quadrivium also provided useful arguments in unexpected domains like ecclesiology and church politics. As younger men both Heymeric and Nicholas had attended the Council of Basel (1431–1449). Both eventually would side with the papalist party, but at first they defended a strong conciliarist position.

167 Ibid., 42 (I.11 [§ 31]); id. 1997, 101.

168 Id. 2002, 42–44 (I.11 [§ 32]); id. 1997, 101.

169 Nicholas of Cusa 1932, 24 (*Apologia doctae ignorantiae*, 35).

170 See Hamann 2006.

Heymeric even presents Neopythagorean arguments for the superiority of church councils over popes. In his *Disputatio de potestate ecclesiastica* (1433), Heymeric contends that geometry itself contends against the papacy. The universal council has a spiritual unity like a circle, of which Christ is the center-point; but the hierarchical church (crowned by the papacy) is a polygon that, while sharing the same center, can only infinitely approximate the perfect circular unity of the council. Heymeric adds further arguments from arithmetic, music, astronomy, and optics. He concludes that the "human wisdom of Christ" is the foundation of number theory (arithmetic), refracted through the quadrivial order into either harmonies of number (music, the greater science) or distributions of quantity (astronomy, the lesser).[171]

A few years after Heymeric's *Disputatio*, Nicholas discovered his own Neopythagorean voice in *De docta ignorantia* and the other writings he eventually sent to his friend Heymeric. By those years, unlike Heymeric, Nicholas found himself more committed than ever to the project of universal mathematical wisdom. By 1450, in *Idiota de mente*, Nicholas even records, as if for himself, a telling moment of self-reflection.

While Nicholas does not appear in the dialogue, his views clearly align with the amateur philosopher, the Layman, whose practical experience as a craftsman makes him wiser than the academic Philosopher. At one point the Philosopher flatly accuses the Layman of being a "Pythagorean." When the Layman responds, we can hear Cusanus answering the query for himself: "I don't know whether I am a Pythagorean or something else. But I do know that no one's authority guides me, even if it attempts to influence me. However, I deem the Pythagoreans – who, as you state, philosophize about all things by means of number – to be serious and keen philosophers."[172] Their sole mistake, Nicholas explains, was that they misrecognized the true object of their contemplation. In all of their doctrines, the ancient Pythagoreans thought they were speaking about number qua number, or about number proceeding from the human mind. In fact, all along the Pythagoreans were speaking about the divine Number and the divine Mind. In spite of themselves, they had always been theologians.

171 See Albertson 2012b, 149–169.

172 Nicholas of Cusa 1973b, 132–133 (*De mente*, VI [§ 88]); id. 1996, 551.

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PART 4

New Trends in Early Modern Pythagoreanism

CHAPTER 13

Pythagoras and Pythagoreanism in the Renaissance. Philosophical and Religious Itineraries from Pico to Brucker

Denis J.-J. Robichaud

> Who will not wonder at this chameleon of ours? Or rather, who will admire any other being more? Not without reason, Asclepius the Athenian said that man was represented in the secret rites by Proteus because of his changing and metamorphous nature. Hence the metamorphoses renowned among the Jews and the Pythagoreans. Indeed, even the most secret Hebrew theology at one time transforms holy Enoch into an angel of divinity, whom they call [מטטרון] *Metatron*, and at other times it reshapes other men into other spirits. According to Pythagoreans, wicked men are deformed into brutes and, if Empedocles is to be believed, into plants as well.[1]

1 Giovanni Pico della Miradola and Pythagoreanism's Vanishing Point

"There is no knowledge that is a greater guarantor of Christ's divinity than magic and Kabbalah."[2] This proposition from the 900 *Conclusiones* that Giovanni Pico della Mirandola (1463–94) proposed to debate publically in Rome in 1486, when he was only twenty-four years old, was one of thirteen propositions that helped him earn two excommunications: from the Catholic Church and from the history of philosophy. The first came from the commission set up by Pope Innocent VIII to examine the young count's works. They condemned this last proposition as wrong, erroneous, superstitious, and heretical. The second came at the hands of Johann Jakob Brucker (1696–1770) who dismissed Giovanni Pico's Kabbalistic magic as a syncretic Pythagorean disease, which "corrupted

1 Giovanni Pico della Mirandola 2012, 123–27. All translations are mine unless otherwise indicated. Thank you to Viveca Pattison Robichaud for helping to prepare figures 13.1–3, and Brian Copenhaver for offering a few helpful suggestions.

2 Giovanni Pico della Mirandola 1557, 105 (*Conclusiones magicae numero xxvi. secundum opinionem priopriam*, 9).

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Johann Reuchlin [(1455–1522) along with other northern humanists] with his new prejudice, and infected him with his new error."[3] Pope Alexander VI pardoned Giovanni Pico and lifted his first excommunication in 1492 but his reputation continued to suffer long after.

In March 1514, news reached Giovanni Pico's friend Girolamo Benivieni (1453–1542) that a priest had preached from the pulpit of the Florentine Cathedral Santa Maria del Fiore against Giovanni Pico and Marsilio Ficino (1433–99). Benivieni, who was then sixty-two years old and physically unable to travel to meet the priest in person, chose to write a concerned letter defending his deceased friend: "In these past days a few gentlemen, who heard you speak, have mentioned to me that on occasion in your sermons, which they praise highly, you used to say how the good memory of count Giovanni della Mirandola sought for a while with Marsilio Ficino in Careggi and elsewhere to unite with God, to perform miracles, and to prophesize, by means of natural magic and by virtue of Kabbalistic doctrines, along with their experiences, prayers, and incenses."[4] There is no mention of Pythagoreanism per se in Benivieni's letter, but to speak about Giovanni Pico's interests in Kabbalah and magic necessarily entails Pythagoreanism.

In the section on how Kabbalah confirms the truth of Christianity in his 900 *Conclusiones* Giovanni Pico writes obscurely that "What the Kabbalists say, that the light deposited sevenfold shines greater than the remaining light, agrees miraculously with Pythagorean arithmetic;" and in the proposition immediately thereafter, "Whoever knows how to unfold the quaternion into the decade will have the method, if he is skilled in Kabbalah, to deduce the name of seventy-two letters from the ineffable name."[5] Giovanni Pico is decidedly cryptic in these propositions, but presumably by the first he intends that Kabbalah explains that the seven days of creation were revealed to Moses after he ascended through 49 gates of understanding (or once he had received the sevenfold illumination of 7X7) to unite with God (the 50th gate of Jubilee), and that one can identify an analog to the sevenfold revelation of Genesis' creation in the traditional Pythagoreans' veneration of the number seven.

To make sense of creation, Giovanni Pico also compares Moses' Genesis with Plato's *Timaeus*, in particular with Plato's use of the triangular figure of the Greek letter lambda (Λ), which in Giovanni Pico's mind signifies Plato's

3 Johann Jakob Brucker 1743, IV.1, Period. III. Pars. I. Lib. II. Cap. IV, 373. On Brucker and Renaissance philosophy, see Catana 2005, 72–90; 2013a, 50–98 and 2013b, 166–200; Celenza 2013, 367–401; Matton 2016, 5–68.

4 The letter is transcribed by Marcel 1958, 541–42, n. 1.

5 Giovanni Pico della Mirandola 1557, 112 (*Conclusiones Cabalisticae numero LXXI. secundum opinionem propriam, ex ipsis Hebraeorum sapientum fundamentis Christianam religionem maxime confirmantes*, 55 et 56).

employment of Pythagorean triangular mathematics, to explain the generation of the cosmic soul from the Same, Other, and Being as intervals corresponding to the seven integers 1, 2, 3, 4, 8, 9, 27 (i.e., the monad, the first even and odd integers, followed by their squares and cubes).[6] Giovanni Pico thus conceives of the Kabbalistic sevenfold cosmic generation as harmonious with Timaean mathematics, according to which the emanation of multiple beings in the cosmos from a common, prior, and single source is understood as a mathematical sequence that can be represented by the triangular figure of lambda (figure 13.1: Λ) made up of two strips of multipliers of seven odd and even numbers emanating from the triangle's apex, the One:

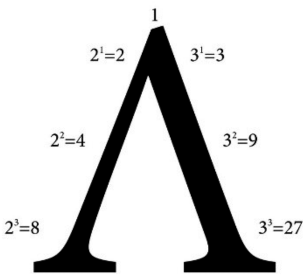

FIGURE 13.1 Lambda diagram, *Timaeus* 35b–c

This numerical sequence from the *Timaeus* was so important to Giovanni Pico that he placed it first among the set of propositions dedicated to his own conclusions based on Plato's doctrines from his *Conclusiones*.[7]

As for the second proposition, Giovanni Pico associates the numerical value of the Biblical Tetragrammaton (יהוה / *YHWH*), or the ineffable sacred name of God revealed to Moses, with the the Pythagorean Tetractys (1+2+3+4 = 10). According to him this explains how "the quaternion [unfolds] into the decade." Both the Tetractys and the Tetragrammaton result in 10, the perfect decade for the Pythagoreans, and the symbol of the Sefirot, or 10 emanations of the divine, for the Kabbalists. Once gain, one can represent this proposition as a triangular figure (figure 13.2):

6 Pl., *Ti.*, 35b–c.

7 Giovanni Pico della Mirandola 1557, 95 (*Conclusiones secundum propriam opinionem numero LXII. in doctrinam Platonis, de qua pauca hic adducuntur, quia prima paradoxa conclusio totam sibi assumit Platonis doctrinam discutiendum*, 1). The ancient interpreters of the *Timaeus* Crantor and Theon illustrated Plato's sequence as a Λ and later interpreters did so as well.

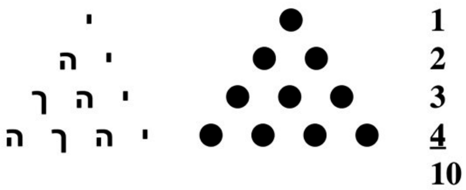

FIGURE 13.2 Giovanni Pico's numerical association of the Tetragrammaton with the Tetractys

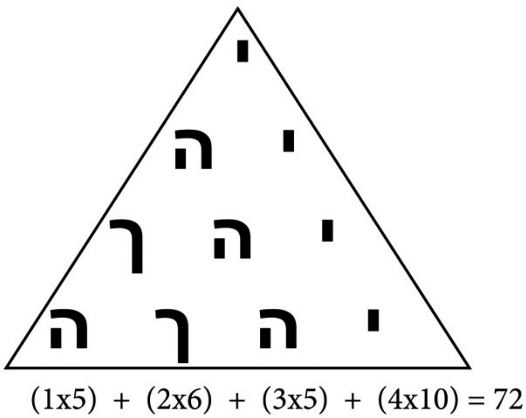

FIGURE 13.3 Giovanni Pico's use of *gematria* for the ineffable Hebrew name of God

Giovanni Pico's Pythagorean Kabbalah functions according to the basic method of *gematria*, whereby the numerical value of each Hebrew letter determines the power of words. In order to arrive at the number 72 for the ineffable Hebrew name of God from the Pythagorean Tetractys, Giovanni Pico follows the traditional calculation of the algebraic value of each letter of the Tetragrammaton, where the value of *yod* (י), *he* (ה), and *waw* (ו) are 10, 5, and 6 (figure 13.3).[8]

There is an imaginative genius in this Pythagoreanizing Platonic lambda (Λ) as a symbol of the metaphysical generation of multiplicity in the cosmic soul from a higher source, in that it integrates simple arithmetic – demonstrating the mathematical principles of one, odd, even, integers and their multiplication

8 On Giovanni Pico's mathematics and Kabbalah, see Bacchelli 2001; Copenhaver 2000, 25–76; id. 2002: 56–81; Zatelli, Lelli and Avenzinelli 1994, 159–191; Lelli 1994, 193–223; Toussaint 2014a, 79–110; id. 2014b, 111–20; Wirszubski 1989. On Giovanni Pico's understanding of Genesis in the Heptaplus, see Black 2006. On the Kabbalah in the Renaissance more broadly, see Secret 1964 and Schmidt-Biggeman 2016, 383–94. More generally on the Kabbalah, see Idel 1988 and Scholem 1960.

as squares and cubes – into a geometric form. Since, moreover, the lambda (Λ) figure itself represents the generation of multiplicity of beings it becomes a representation of the mathematization of nature and the cosmos. This mathematical derivation continues in the *Timaeus* by also integrating astronomy and music into the cosmic procession.

Giovanni Pico's geometric imagination Hellenizes Moses' account of genesis. He claims that Platonists and Hebrews signify the same thing when they speak of congregations of souls on Mount Ida and Mount Sinai.[9] The two-dimensional geometry of the shape of lambda (Λ) thus also symbolizes in his mind the three-dimensional conical shape of mystical mountains. His mathematics does not just operate on the plane of Plato's reason (λόγος) and Pythagoras' number (ἀριθμός), it also operates on the mystical planes of Christianity's creative logos and the numerical pattern of the Hebrew letters in Genesis that God revealed to Moses – and one is tempted to call this μῦθος. Giovanni Pico's method seeks a vanishing point where reason and revelation converge at the intellectual apex of our rational soul. He conceives the marriage of Kabbalah and Pythagoreanism as a way for our mind to ascend mystically to the truth of the Bible's divine mysteries and of the nature of the cosmos. However, according to his Renaissance critics, at best, his method obfuscates the Bible with ancient philosophy, and at worst, it aspires to an unholy alliance between pagan magic and nefarious Hebrew learning.

Benivieni wished to set the record straight about his friend Giovanni Pico's Pythagorean Kabbalah in his letter from 1514 to the accusatory priest. Since Benivieni claims to have spent *every day* with Giovanni Pico during his time in Florence – as he recounts in his letter to the unnamed priest, he went horseback riding with the Count *every evening* – he tries to convince this priest that he knows better than anyone else that although Giovanni Pico might have indulged his curiosity for Kabbalistic magic, he never gave it any credence. To make his case, Benivieni even lays the onus of the blame for these activities on Marsilio Ficino, adding that Giovanni Pico only went to visit Ficino at his Careggi villa less than three times. Nonetheless, Benivieni could not deny that Giovanni Pico had a number of books about Kabbalah translated to help him interpret and comment on the mysteries of the Bible. In the closing summary of Benivieni's defense one hears that Giovanni Pico might have employed Hebrew learning and Pythagorean philosophy in his *Heptaplus* to interpret the account of creation in Genesis, and he might even have been curious about

9 Giovanni Pico della Mirandola 1557, 95 (*Conclusiones secundum propriam opinionem numero* LXII. *in doctrinam Platonis, de qua pauca hic adducuntur, quia prima paradoxa conclusio totam sibi assumit Platonis doctrinam discutiendum*, 4).

Kabbalistic magic, but he never believed it was effective for reaching God, nor did he ever practice it.

Upon closer examination Benivieni is neither the best witness nor the best advocate for Giovanni Pico's defense. For one, the alibi that the two friends spent every day and every night together can easily be discredited. Case in point, it is known that Giovanni Pico had begun to learn Chaldean or Aramaic with Flavius Mithridates, a convert from Judaism to Christianity who was also known as Guglielmo Raimondo Moncada and who was one of Giovanni Pico's teachers of Arabic, Kabbalah, and Hebrew (Giovanni Pico's other instructor was Yohanan Alemanno). Mithridates apparently made his student swear to keep his Aramaic teachings secret, and Giovanni Pico himself reports in a letter from 10 November 1486 that on one occasion, Mithridates threw out Benivieni in anger after Benivieni interrupted his lesson with Giovanni Pico.[10] And for another, despite concluding his letter to the priest with the afterthought, "whatever I say about the count, I believe to be able to state the same thing without scruple about Marsilio Ficino," it is clear that Benivieni wishes to defend Giovanni Pico by passing the blame for experimenting with Kabbalah onto Ficino.[11] Here too Benivieni is not very credible.

In his youth Marsilio Ficino translated and later circulated in manuscripts the first four books – and the only books extant – of Iamblichus' *De secta Pythagorica*: (i) *On the Pythagorean Life*; (ii) the *Protrepticus*; (iii) *On General Mathematical Science*; and (iv) *On Nicomachus' Arithmetical Introduction*.[12] One of the readers of Ficino's translation of Iamblichus was unsurprisingly Giovanni Pico who mined it for his reconstruction of ancient Pythagoreanism. This large compendium of Pythagorean mathematics often quotes from older Pythagorean pseudepigrapha written in an archaizing Doric Greek that purport to be from Pythagoras' contemporaries and followers but that, it is now generally accepted, mostly date to a period from the first century BCE to the first century CE. One of the primary ways in which Ficino used this work was as an interpretive guide to what he considered to be the Pythagorean dimensions of the Platonic corpus. Inspired by Iamblichus and Proclus, Ficino also proposed his own elegant theory to explain the mathematical unfolding of the multiplicity of all beings according to a system of nine Pythagoreanizing Platonic principles (understood as the multiplication of two triads, 3X3)

10 Giovanni Pico della Mirandola 1557, 384–86.

11 Marcel 1958, 542, n. 1.

12 On Ficino and Iambl. *De sect. Pyth.*, see Allen 1994, 32–34; Celenza 2001, 15–34; Gentile 1990, 57–104; Robichaud 2016, 437–52; id. 2017, 44–87, 149–86; id. 2018, esp. 77–95. On Iambl. *De sect. Pyth.*, see especially O'Meara 1989.

derived progressively from the One.[13] Ficino's Pythagorean studies also helped him to theorize about the mathematization of prayer by means of ordered numbers and figures as a way to unite (ἕνωσις) and come into contact with the divine (συναφή), and even earned him the reputation of being something of a Pythagorean sage in Renaissance Florence.[14]

In short, Ficino might certainly have been one of the greatest Renaissance revivers of ancient Pythagoreanism, he might have believed that Pythagoras learned much foreign wisdom during his travels in Egypt and Persia, and he might also at times have associated Pythagoreanism with Hebrew learning, repeating, at least on one occasion, Ambrose's claim that Pythagoras had a Jewish family, but unlike Giovanni Pico, Ficino never exerted himself to become a scholar of Hebrew nor of the Kabbalah, nor again of wedding Pythagoreanism and Kabbalah into a detailed intellectual program.[15] These were Giovanni Pico's goals. Simply put, Benivieni's claim that Giovanni Pico only went to Careggi three times to see Ficino is nothing more than misdirection.[16]

As it turns out, this was not the only time Benivieni tried to manipulate the public memory of Giovanni Pico. For example, when he and his friend Biagio Buonaccorsi (1472–1521/22) published editions of Giovanni Pico's *Commento sopra una canzone d'amore di Girolamo Benivieni* in 1500 and 1519 they not only deleted any mentions of Ficino, they also suppressed Platonic references in the commentary.[17] Benivieni's motives are clear: the priest from Santa Maria del Fiore had attacked one of his dearest friends from his youth. But what about the unnamed priest's motives? Why would he deliver a sermon in 1514 in the religious heart of Florence, attacking the memory of Giovanni Pico and Marsilio Ficino, both of whom had long since passed away? A dispute that began north of the Alps is the likely reason for renewed theological anxieties over Ficino and especially Giovanni Pico's orthodoxy. It serves as evidence that

13 On Marsilio Ficino's derivation of nine Pythagoreanizing Platonic principles to explain how the multiplicity of beings emanate from the One, see Robichaud 2018, 163–72 and Id. 2020. More generally on Marsilio Ficino and Pythagoreanism, in addition to the works cited in n. 12 above, see Allen 1982, 171–92; id. 2014, 435–53; Celenza 1999, 667–711; Toussaint 2014a and 2014b. For a general survey on Pythagoras in the Middle Ages and the Renaissance, see also Joost-Gaugier 2006 and 2014.

14 On Ficino and Pythagorean soteriology, see Celenza 2001, 21; as well as on Ficino, soteriology, mathematical prayers, and Iamblichus, see Robichaud 2017 and 2018, 77–95, 149–86.

15 Marsilio Ficino 2000, 1: 30.

16 See also Gentile 1994, 127–47.

17 On the revisions to Giovanni Pico's *Commento*, see Giovanni Pico della Mirandola 1942 (*De hominis dignitate, Heptaplus, De ente et uno*); id. 1989.

Giovanni Pico's Pythagorean work continued to invite religious opposition into later stages of the Renaissance.

Between 1511 and 1516 the German humanist, lawyer, and scholar of Hebrew and the Kabbalah, Johann Reuchlin (1455–1522) was embroiled in a virulent controversy, in which he argued against the Catholic convert from Judaism Johannes Pfefferkorn's (1469–1523) scheme to destroy all Jewish books throughout the Holy Roman Empire. Since Reuchlin, who was selected by the Emperor Maximilian to adjudicate on the question, refused to go along with Pfefferkorn's plans, Pfefferkorn wrote an anti-Semitic tract, the *Handspiegel* (1511), directed against the humanist, who in turn replied with his own tract, the *Augenspiegel* (1511). By 1513 the Church's inquisition was investigating Reuchlin and had banned his *Augenspiegel*. The University of Paris denounced Reuchlin's work, and by 1516 Rome intervened to end the quarrel. What is of present concern about this controversy is that when he wished to defend the use of Kabbalistic books in the *Augenspiegel* Reuchlin appealed to the aforementioned propositions from Giovanni Pico's 900 *Conclusiones*.[18] News of the controversy circulated widely, and Reuchlin's alignment with Giovanni Pico's Pythagorean Kabbalah must have enflamed the Florentine priest to deliver sermons against Giovanni Pico and his acolytes at Santa Maria del Fiore, which in turn prompted Benivieni's reply.

Reuchlin was far from being a disinterested party in the controversy. He had been introduced to Florentine intellectuals during his travels to Italy and Florence in 1482 and 1490, and he remained in epistolary contact with Ficino and Giovanni Pico's nephew, Gianfrancesco Pico della Mirandola (1470–1533). He also acquired the works of Ficino, Pico, and Angelo Poliziano (1454–94) as soon as he could with the help of Johannes Sträler, who was in Florence with Reuchlin's younger brother, studying Greek with Demetrios Chalkokondyles (1423–1511) and introducing himself to Giovanni Pico and Ficino.[19] The preface of Reuchlin's *De arte cabalistica*, which is addressed to Pope Leo X, under whom Reuchlin's case was pending, makes it abundantly clear that he considers himself the heir to Giovanni Pico's particular brand of Pythagorean Kabbalah. He declares in its opening lines that Pythagoras is the true father of Italian philosophy, and that Pythagoreanism lay dormant until it began to resurface with Ficino and Giovanni Pico at the Florentine court of Cosimo and Lorenzo de' Medici, that is, Leo X's father and great-grandfather. Insinuating that this prior Florentine revival of Pythagoreanism only foreshadowed its true

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18 See Reuchlin's *Augenspiegel* in O'Callaghan 2012, 130, 168–70. On Reuchlin and his controversy with Pfefferkorn, see also Price 2011; Rummel 2002; and Schmidt-Biggeman 2016.

19 Johann Reuchlin 1558, 5–8.

rebirth, Reuchlin states confidently that he will do for Pythagoras what Ficino did for Plato, and Jacques Lefèvre d'Étaples (*ca* 1455–1536) for Aristotle: bring him back to life.[20] What this proclaimed rebirth of lost Pythagorean philosophy actually looks like in Reuchlin's *De arte cabalistica* is religious arithmology. He reconstructs and examines it through a dialogue between three interlocutors, a Jew, Simon, a Muslim, Marranus, and a Pythagorean, Philolaus. Reuchlin was certainly inspired by a number of contemporaries, particularly by Nicholas of Cusa's (1401–64) mathematical theology in his *De possest*, which had recently been published by Lefèvre d'Étaples, but it is Giovanni Pico's notion of a religious Pythagoreanism that influenced him the most.[21]

Giovanni Pico's fusion of Pythagoreanism and Kabbalah also had a strong pull on other northern humanists. For instance, in his *De magia naturali*, written *ca* 1492–1495, Lefèvre d'Étaples, who travelled to Florence in 1491–92, continues to try to excavate Pythagoreanism by means of a Kabbalastic tool kit.[22] Lefèvre only circulated his *De magia naturali* in manuscript, but he nonetheless dedicated it to an important patron, Germain de Ganay (†1520), a councilor of the *Parlement* and the King of France, canon of the Cathedral Notre-Dame de Paris and eventually the Bishop of Cahors and Orleans, and a promoter of Italian humanism in Paris who had also previously corresponded with Ficino.[23] The second book of this work, entitled *De Pithagorica philosophia quae ad magiam*, combines theology, Pythagorean mathematics, and Kabbalah in order to explain the magical power of characters, numbers, and names. Lefèvre apparently grew tired of this approach towards ancient Pythagorean mathematics, but traces of it are still very evident in his later work. For example, in his study of the Psalter from 1509, Lefèvre attests to the fact that he learned from Reuchlin, Giovanni Pico, and Nicholas of Cusa that the names of Jesus and the Tetragrammaton (חדח′ / *YHWH*) hold the same numerical value. Lefèvre's sources – Reuchlin's *De verbo mirifico*, Giovanni Pico's *Conclusiones*, and Cusanus' *Sermons* – all investigate whether the numerical value of the divine names of the Tretragrammaton and Jesus are identical, and whether their vocalizations hold any effective power. In one of his *Sermons*, Cusanus scrutinizes the mathematical value of the names of God but also concludes: "It must not be believed that there is any power in these words."[24] In his *De*

20 Johann Reuchlin 1517, Aiv.

21 Schmidt-Biggeman 2016, 386. On Cusanus' mathematical theology, see Albertson 2014.

22 On this work, see Copenhaver 1977, 189–211; Kieckhefer 2007, 63–78; Mandosio 2013, 37–79; and Rice 1976, 19–29. For Lefèvre d'Étaples' and mathematics, see more generally Oosterhoff 2013, 532–52.

23 Gentile 1986, 51–76.

24 Quoted from Copenhaver 1977, 202.

arte cabalistica and *De verbo mirifico*, Reuchlin explains that the incarnation of Christ reveals the pronounceable name of Jesus in order to break the code of Pythagorean silence over the ineffable name of God, the Tetragrammaton that was revealed to Moses, and like Pico before him, he explains that the Tetragrammaton is an analog to the Pythagorean Tetractys (1 + 2 + 3 + 4 = 10).[25] For his part, Giovanni Pico resolves in his *Conclusiones* that the Pythagorean numerical sequence of three Hebrew four-letter names of God, *Ehyeh* (אח'ח), *YHWH* ('חדח), and *Adonai* (אדכ'), correspond to the three persons of the Trinity.[26] To reiterate the proposition that helped him earn his excommunications, "There is no knowledge that is a greater guarantor of Christ's divinity than magic and Kabbalah."[27]

2 Non Sunt Multiplicandi Pythagorici Sine Necessitate

While many humanists like Reuchlin and Lefèvre, as well as Symphorien Champier (1471–1539), Guillaume Postel (1510–81), and Cornelius Agrippa (1486–1535) followed Giovanni Pico's approach to studying Pythagoreanism by way of the Kabbalah, others began to change the course of Pythagorean research. In 1496, shortly after he wrote his *De magia naturali*, Lefèvre printed a mathematical textbook containing Boethius' *Arithmetic*, Jordanus de Nemore's *Elementa arithmetica*, and a rulebook for the game *Rithmomachia*, which he dedicated to another powerful patron Jean de Ganay (1455–1512), the brother of Germain (to whom he had dedicated the *De magia naturali*) and *chancelier de France* under Louis XII. In the dedication to this volume, Lefèvre still argues that Pythagorean mathematics can lead the mind to God, but he also begins to emphasize the practical applications of studying mathematics, like land surveying. Lefèvre became an influential teacher in Paris and imprinted on his students, including Josse Clichtove (*ca* 1472–1543) and Charles de Bovelles (*ca* 1475–*ca* 1566), this reformed notion of Pythagorean mathematics.[28]

Although these French humanists still thought of Pythagorean mathematics in theological terms, they nonetheless explored its useful applications in university curricula. For example, Peter Ramus (1515–72), the humanist professor of eloquence and dean at the College Royal in Paris, seized Lefèvre's baton and

25 Ibid., 203; Schmidt-Biggeman 2016, 387–88.

26 Giovanni Pico della Mirandola 1557, 108 (*Conclusiones Cabalisticae numero LXXI. secundum opinionem propriam, ex ipsis Hebraeorum sapientum fundamentis Christianam religionem maxime confirmantes*, 6).

27 Ibid., 105 (*Conclusiones magicae numero xxvi. secundum opinionem propriam*, 9).

28 Oosterhoff 2013 and 2018.

ran farther towards this new goal. Like Lefèvre, he was interested in reforming the arts curriculum at the university by making mathematics integral to it. Ramus, who had been a student of the mathematician Oronce Fine (1494–1555), was confronted with an important obstacle before he could oversee the mathematical curriculum. He had to dislodge ancient Pythagorean mathematics from its associations with the Kabbalah, and integrate it into his conception of logic and dialectics.[29] His solution was simple and clear, though complicated in its execution. It involved a *pars destruens*, analyzing the sources for the history of Pythagoras and Pythagoreanism, and a *pars construens*, rewriting the history of ancient Pythagoreanism to suit his needs. Humanists had often drawn on ancient biographical accounts of Pythagoras' travels to Egypt and Babylon (as Diogenes Laertius, Porphyry, and Iamblichus record) or of his Jewish ancestry (as Ambrose reported) to connect Pythagoreanism to Hebrew learning. To disconnect these two traditions, Ramus wrote an alternative historical account of Pythagoras and Pythagoreanism that rivaled the Kabbalistic ones.

The immediate impetus that prompted Ramus to re-evaluate Pythagoras and Pythagoreanism was not so much Christian Kabbalists but a rancorous controversy with a fellow professor in Paris, Jacques Charpentier (1524–74), over the tenure of a vacant chair in mathematics at the University of Paris. Ramus' student Jean Pena (1528–58) formerly held the chair before his hasty death. When Charpentier, a scholastic philosopher with no specific interests in Greek philosophy nor any training in mathematics, was appointed as his replacement, Ramus objected. The matter was brought to the Parlement in 1566, which adjudicated in favor of Charpentier, offering him the chair as long as he included some mathematics in his otherwise Aristotelian curriculum. Ramus' response was to weaponize Pythagoras by writing a history of mathematics in his *Prooemium mathematicum* (1567). Like Reuchlin, Ramus too pays homage to Medicean Florence in the preface of this work, which he dedicates to Catherine de' Medici, but unlike Reuchlin, not to Giovanni Pico's Kabbalistic Pythagoreanism. Instead, he wishes to demonstrate the truth and utility of mathematics in order to justify its place in university curricula. With this goal in mind, Ramus collects ancient fragments and testimonies about Pythagoras and Pythagoreanism and recasts them into the mould of his educational program, reshaping the historical figure of Pythagoras into a mathematical reformer and school teacher.[30] Culling evidence from a number of ancient texts, Ramus argues that mathematics should oversee all the arts, since on the

29 Margolin 1976, 109–155.

30 Goulding 2009, 54. On Ramus and the history of mathematics, see also Goulding 2010.

one hand mathematics was revealed in ante- and post-diluvian sacred history, and on the other, secular history documents how the modern arts curriculum originated in the schools established by Pythagoras.

To construct his history Ramus salvages information about sacred history he found in Flavius Josephus' *Jewish Antiquities*. He relates that Adam's son Seth invented astronomy and Adam preserved antediluvian mathematical knowledge on two columns of brick and stone; that Abraham the Chaldean taught arithmetic and astronomy to the Egyptians; and that studying the movement of the heavenly bodies led Abraham to become a monotheist.[31] Quoting Wisdom 11: 21, Ramus reasons that mathematics is therefore useful for theology, alongside the knowledge of Latin, Greek, and Hebrew: "The theology of the pagans is completely enveloped with mathematical coverings. Instead, I consider and examine Christian theology, in which God creates all according to the number, measure, and weight of seven, that is according to an arithmetic, geometric, and isorropic rule."[32] Just as Pythagoras, Plato, and Aristotle arranged their school curricula so that mathematics was prioritized and studied in the correct order, so too should Christian professors. Although Ramus claims that mathematics originated in sacred history, crossed the sea from Egypt to Greece, and is useful for present-day Christian learning, he does not explicitly connect Pythagoras' travels nor Pythagorean mathematics with Hebrew learning and the Kabbalah. Instead of receiving mathematics as a revelation, Pythagoreans, according to Ramus, train themselves in mathematics in their schools by way of an aphaeretic method.[33] Ramus even claims that Pythagoras was the first to collect mathematical theorems and write textbooks, a practice similar to Ramus' own activities in Paris. Robert Goulding succinctly phrases the circular reasoning in Ramus' Pythagoreanism: "A great Ramist systematizer of mathematics would, in fact, be a second Pythagoras (since, as Ramus had shown, Pythagoras was in some sense a *first Ramus*)."[34] Ramus' opponent, Charpentier, did not hesitate to characterize himself as a Catholic savior protecting Christian learning from Ramus' reformed university curriculum, which proposed to place mathematics at the foundation of humanist trilingual Christian learning.[35]

Despite reorienting the history of ancient Pythagoreanism away from Giovanni Pico's Kabbalistic approach, Ramus too, like his Italian counterpart,

31 Joseph. *AJ*., 1.2.3; 1.8.2; Ramus 1567 (*Prooemium mathematicum ad Catharinam Mediceam*), 3–7.

32 Ibid., 220–21. Cf. Augustine, *De doctr. christ.*, 2.16.25–26; *De Gen. ad litt. XII*, 4.3–6.

33 Ramus 1567 (*Prooemium mathematicum*), 16–22; 45.

34 Goulding 2009, 76.

35 Ibid., 61–62; Girot 1998, esp. 73–74.

was confronted with the similar problem of finding a theological value in the ancient sources for Pythagoreanism. On the one hand, the humanist rediscovery of sources for Pythagoras and ancient Pythagoreanism seemed to make Pythagoreanism a nearly ubiquitous tradition in the ancient world. On the other hand, there seemed to be no clear consensus on how to interpret these sources. The esoteric nature of many of these texts often invited interpretive leaps or the use of traditional religious hermeneutics. Giovanni Pico's approach to the problem of sources is comparative and structural. By comparing the two traditions of Pythagoreanism and Kabbalah, he wishes to find their common origins. In other words, he is attempting to swim upstream through two schools that are, in his mind, two distributaries of oral traditions flowing out of the same esoteric source, containing the same message: God ordered all things – words, numbers, minds, the cosmos and the heavens – according to a measure. Ramus' solution is as historical as it is presentist. Like Josephus before him, he enters into an old scholarly contest, comparing chronologies to prove that certain Greek intellectual traditions or inventions, in this case mathematics, have their origins among the so-called "barbarians."[36] Nevertheless, according to Ramus, Pythagoras and his school did not arrive at mathematical truths by revelation but by training and method. His source criticism, therefore, discards aspects of the tradition that diverge from schoolroom mathematics in order to uncover an ancient Pythagoreanism that justifies his immediate needs.

The mathematical theologies and practical mathematics of each of these Renaissance Pythagoras*es* are clearly different; but the multiplication of Pythagoreanisms did not just happen in the Renaissance. The proliferation of different Pythagorean sources is also an ancient phenomenon, especially in the form of the large production of pseudepigraphic Pythagorean literature. Diogenes Laertius once claimed that four men named Pythagoras lived near each other in the same period of history.[37] By the time Pythagoreanism reaches the Early Modern period, the number of Pythagoras*es* (this time understood as versions of the same well-known philosopher) multiplied exponentially. Only a quick survey is necessary to see how there were Jewish, Catholic, Protestant, humanist, scientific, philosophical, theological, mathematical, Italian, French, German, Polish, and English Pythagoras*es* that followed in Giovanni Pico's wake. In the biographical entry on Pythagoras in his *Commentarii Urbani* of 1506, the humanist and Servite monk, Raffaello Maffei (1451–1552) describes Pythagoras in his own image, as a proto-Christian monk who established a philosophical school that resembled something like a monastic order. Michael Neander

36 On comparative and historical chronologies, see especially Grafton 1975, 156–185.

37 Diog. Laert., 8.46–47.

(1525–95) later adapted Maffei's biography of Pythagoras for his own purposes in his dedicatory letter to his translation of the pseudo-Pythagorean *Golden Verses* (1559). His Pythagorean history, Ada Palmer has recently explained, "systematically repositions the arguments made by fifteenth-century humanists to defend classical learning in a Christian era, adapting them to strip out medieval authorities unpalatable in anti-Catholic lands, and to create an alternative history of learning, which jumps from Jerome to Erasmus."[38] The list goes on to include a who's who of scientists, theologians, and philosophers, each in turn claiming some form of Pythagorean ancestry for their preferred school of thought. Based on a fragment from Philolaus stating that Pythagoreans conceived of a central and unlimited fire in the cosmos, which was called hearth or Hestia, Nicolaus Copernicus (1473–1543) and Copernicans argued that Pythagoras and Pythagoreans demonstrated heliocentric science like them.[39] Gottfried Wilhelm Leibniz (1646–1716) too reimagined Pythagoras' famed travels to learn from Egyptian and Brahmin wise men as evidence that Pythagoras was at the center of an ancient republic of letters.[40] Leibniz, who studied the Pythagorean writings of Erhard Weigel (1625–99) and Johannes Scheffer (1621–79) and who also wished to disconnect Pythagoreanism from "vulgar Cabbala," found in Pythagoreanism an ancient precursor for his own theories on a *mathesis universalis*, and seemed to have encouraged the identification between himself and the ancient philosopher.[41] His contemporaries willingly obliged. Andreas Clavius, for instance, called Leibniz a new German Pythagoras on account of his monadology. To cap it all off, the Scottish professor of mathematics, Colin Maclaurin (1698–1746) similarly claimed that his own preferred scientist, Isaac Newton (1642–1727), revived the work of mathematizing nature of his forerunners Pythagoras and his school.[42] In the ancient world Pythagorean pseudepigraphy facilitated the construction of authority. In the Renaissance and Early Modern Europe, when one wished to increase the authority of one's position in religion, science, and philosophy, one still had the option of praying to the polycephalic demigod Pythagoras.

38 Palmer 2016, 211–26.

39 Neumann 2016, 458. Ficino had also speculated on the nature of the central Pythagorean fire, known as Hestia, or the hearth of the cosmos: Robichaud 2017, 57; id. 2018, 158.

40 Neumann 2016, 456. Galson 2016, 406.

41 Ibid., 399.

42 Neuman 2016, 460–461.

3 Source Criticism, Chronology, and Apologetics; or What Goes into a History?

The Renaissance recovery of ancient sources for Pythagoras and Pythagoreanism eventually met head-on with philological and historical critical methods. Writing at the end of the nineteenth century, Eduard Zeller (1814–1908) could look back and distill the research results of a few centuries of scholarship into a general thesis about the expansion of the Pythagorean tradition, namely that the sources, on the one hand, say more and more about Pythagoreanism and its founder the further removed in time they are from the relative historical facts (filling silences with dogmatic prejudices, doubtful legends, and spurious texts); and that the sources, on the other hand, say conversely less and less in the same magnitude the closer they are to the tradition's origins. If Zeller's thesis holds true, then surely by the seventeenth century Pythagoreanism must have snowballed into a mass of confusion.[43]

The three principal accounts of Pythagoras' life – Diogenes Laertius' *Life of Pythagoras*, Porphyry's *Life of Pythagoras*, and Iamblichus' *On the Pythagorean Life* (the first book of Iamblichus' larger summary of Pythagoreanism, the *De secta Pythagorica*) – were rediscovered in the Latin West during the Renaissance. Material from Diogenes Laertius had already begun to circulate in the late Middle Ages after Henricus Aristippus (*ca* 1110–1162) partially translated the *Lives* in Sicily in the twelfth century. In the fourteenth century an anonymous – but likely Italian – scholar used Aristippus' partial translation to compose another work entitled *De vita et moribus philosophorum*, which circulated in over 270 extant manuscripts and was long attributed incorrectly to Walter Burley.[44] These two very important traditions of Diogenes Laertius, however, pale in comparison to the popularity and wide circulation of the 1433 translation by the Camaldolese monk and scholar from Santa Maria degli Angeli in Florence, Ambrogio Traversari (1386–1439). His Latin translation circulated in manuscript before it underwent at least twenty-two different printings between 1472 and 1600. Johannes Boulierius, Johannes Sambucus (1531–84), and Henri Estienne (1528/31–98) revised Traversari's translation, and another translator, Tommaso Aldobrandini (†1572), tried to replace it by retranslating Diogenes Laertius in the late 1500s. The first complete Greek edition of Diogenes Laertius was printed in 1533, followed by Estienne's edition in 1570, which was printed again with Isaac Casaubon's (1559–1614) annotations

43 Zeller 1856, 1: 206–365, esp. 206–216. On Zeller's thesis of the expansion of Pythagorean tradition, see Burkert 1972, 1–14. Cornelli 2013, 3–45.

44 Grignaschi 1990, 131–190.

in 1593. Bartolomeo, Lodovico, and Pietro Rositini published their Italian translation in 1545, Giosefo Salviati his Italian version in 1598, and François de Fougerolles (*ca* 1560–1626) his French translation in 1602.[45] Hence, by the beginning of the seventeenth century Diogenes Laertius' *Life of Pythagoras* was widely available in Greek as well as in Latin and vernacular translations.

It took a little longer for Iamblichus and Porphyry's Pythagorean lives to be printed by Dutch and German scholars. Johannes Arcerius (1538–1604) first printed his edition and Latin translation of Iamblichus' *On the Pythagorean Life*, along with the *Protrepticus*, in 1598. Konrad Rittershausen (1560–1613) printed his edition of Porphyry's *Life of Pythagoras* with notes by Daniel Heinsius (1580–1655) in 1610. Two other editions of Porphyry's text followed shortly thereafter, the first by Johannes Donatus Ferrarius in 1629 and the second by Lucas Holstenius (1596–1661) in 1630. In 1668, Samuel Tennulius further produced an edition of Iamblichus' book on Pythagorean arithmetic *In Nicomachi Geraseni arithmeticam introductio* (the fourth book of the *De secta Pythagorica*).

Long before these printed editions spread Pythagorean teachings to the four corners of Europe, manuscripts of the Greek texts and their Latin translations were already circulating in fifteenth-century Italy. When the humanist Giovanni Aurispa (1376–1459) travelled to Byzantium for Tommaso Parentucelli (later Pope Nicholas V) in 1441 he returned with a number of manuscripts of Iamblichus, likely including his *De secta Pythagorica*, and Hierocles' *Commentary* on the Pythagorean *Aurea verba*. The personal library of Cardinal Bessarion (1403–72), which he donated to the Republic of Venice in 1468, also contained copies of Hierocles, Diogenes Laertius, and Iamblichus' *De secta Pythagorica*. Bessarion argued in his *In calumniatorem Platonis* (completed by 1459 but printed in 1469) that Plato was above all a Pythagorean. To make his point in this work, he quoted and therefore was responsible for circulating a Latin translation, done largely by Niccolò Perotti (1429–80), of the pseudepigraphic letter of Lysis on revealing Pythagorean mysteries.[46] And, we have already seen how Ficino translated Iamblichus' *De secta Pythagorica* and Traversari translated Diogenes Laertius into Latin.

When it came time for Brucker to assess the ancient history of Pythagoras and Pythagoreanism, as well as its Renaissance rediscovery, in his *Historia Critica*

45 Hankins and Palmer 2008, 62–63.

46 There are two extant versions of this letter. According to Delatte's classification, variant A is from Iambl. *De sect. Pyth.* 75–78, and part of it is also found and attributed to Hippasus in Diogenes Laertius and Clement of Alexandria. Bessarion quotes variant B of the letter in his *In calumniatorem Platonis*. See Delatte 1915, 83–106; Burkert 1961, 17–28. On Bessarion's use of this pseudepigraphon, see Robichaud 2018, 72–74.

Philosophia, his judgment on this tradition was even more severe than Zeller's, writing "there is no part of philosophical history that is so plagued with many difficulties; everything bursts out to such an extent with incertitude, falsehoods, contradictions, and myths, everything relies on the same testimonies that merit very little faith, and once they are examined according to the Lydian touch stone of historical credibility, they become most doubtful."[47] Brucker is just as much interested in finding a way out of the labyrinth of ancient and Renaissance sources to identify an "authentic" Pythagoreanism uncorrupted by later traditions as he is in giving reasons to leave Pythagoreanism behind in the history of irrational and anti-Christian mistakes. He begins his assessment with a brief survey of ancient histories of Pythagoras, relating that although Pythagoras' wife Theano is said to have been the first to write about Pythagoras, it is really Aristoxenus who is supposedly the first to have had an influence on later writers, especially on Hermippus, Lyco of Iasos, and Moderatus. Brucker thus ostensibly begins at the beginning of the history of Pythagoreanism, but he is much more concerned to examine the three later authors who are principally responsible for shaping the narrative about Pythagoras' life: Diogenes Laertius, Porphyry, and Iamblichus. That Diogenes Laertius, and especially Porphyry and Iamblichus controlled the reception of the history of Pythagoras and Pythagoreanism is a cause for concern to Brucker: "the few rags that are extant for us, are torn out from those same men who conduct themselves suspiciously in their work, I mean Porphyry and Iamblichus, writers worthy of such little credence for the history of Pythagoras."[48] Brucker might be examining these biographies in search of older authentic sources inadvertently stitched into their texts, but he is also criticizing them because he considers them pagan adversaries to Christianity and because he is troubled that their works had gone through a revival in the Renaissance.

Christian apologetics clearly dominate Brucker's comparison of the parallel lives of Christ and Pythagoras as they also did among late ancient church fathers.[49] According to him, Porphyry and Iamblichus are not simply poor historians, they are also imposters who deliberately forged their histories of Pythagoras by including superstitious tales about his birth, life, travels, learning, and miracles in order to fabricate Pythagoras into an anti-Christ thaumaturge to rival Jesus.[50] Brucker, who is also inspired by the anti-Platonic writings of the Lutheran church historian Johann Lorenz Mosheim (1693–1755),

47 Johann Jacob Brucker 1767, Period. III. Pars. II. Lib. II. Cap. X: 989.

48 Ibid., 991–992.

49 On late ancient parallel lives, see, for example, Smith 1990.

50 Ibid., 992–993.

dispenses with a number of episodes from Iamblichus and Porphyry's biography of Pythagoras. They are absurd, he believes, because they seem to emulate episodes from the life of Christ: "For why would Pythagoras be born of Apollonian parentage and declared by his oracle to come to aid mankind, than to rival, it seems, the miracle birth of the Savior? Why did sailors admire him as though he were God in their ship? Is this not what sacred history testifies happened to Christ? Why do they boast that he acts as a mediator between God and men, than because this is the first glory of Jesus of Nazareth?"[51] Brucker's inquisition proceeds through a litany of questions. He bemoans that Medicean Florence corrupted Christianity by fueling a Pythagorean revival and he wishes to understand what he considers to be a syncretistic hybrid of Kabbalistic-Pythagorean-Platonic-Aristotelian-Hebrew-Christian learning in order to contain this dangerous and anti-Christian virus. To help him purge philosophy of what he considers to be a problematic history, Brucker turns to the technical chronologies of the life of Pythagoras by three English scholars: Richard Bentley (1662–1742), Bishop William Lloyd (1627–1717), and Henry Dodwell (1641–1711). Their historical critical methods, Brucker believes, will finally cleanse the waters.

Their breakthrough in Pythagorean chronology and source criticism is a result of Richard Bentley's entry into a now-famous controversy with Charles Boyle (1674–1731) over the authenticity of the so-called *Epistles of Phalaris*.[52] In 1695, Boyle had published an edition of the letters, which he contended were authentic. A few years later, Bentley published his *Dissertation on the Epistles of Phalaris* (1699) in which he argued against Boyle that the letters were spurious. Since the letters mention Pythagoras three times, Bentley also undertook an examination of the chronology of Pythagoras' life in order to advance his case that the *Epistles* were forgeries. Despite making a number of reasoned arguments proving the incoherence of a number of ancient sources on Pythagoras' life, Bentley admits that he is far from arriving at a definitive chronology for Pythagoras' biography. In fact, he provides a chronological table with no less than nine dates for Pythagoras' death, along with a list of their corroborative sources. He thus requested the aid of Bishop William Lloyd to settle the specific portion of the controversy dealing with the life of Pythagoras. Lloyd agreed, publishing *A Chronological Account of the Life of Pythagoras* in the same year. Since Boyle had drawn on Henry Dodwell's book on chronology, the *De cyclis*, to date the *Epistles of Phalaris*, he too became the target of Bentley and Lloyd's

51 Ibid., 993.
52 On this controversy: Levine 1991, 47–120; Levitin 2015, 225–229.

criticism. This provoked Dodwell to write a response, *Exercitationes Duae; prima de aetate Phalaridis, secunda de aetate Pythagorae Philosophi*, in 1704.

One could easily get lost in the weeds of this controversy, but for Brucker the consequences of what he thinks to be the first critical examination of Pythagoras' biography are clear: contradictions in the received chronology of Pythagoras' life, especially in Iamblichus, justify doubts that Pythagoras ever learned from Egyptian, Hebrew, Babylonian, Chaldean, and Persian wise men, and was ever initiated into their religious mysteries; this in turn, he believes, allows one to purge any 'oriental' religious elements from ancient and modern Pythagoreanism. As a case in point, Brucker examines a passage in Porphyry that recounts how Pythagoras studied with a certain Chaldean magus with a dubious name in order to demonstrate how the sources disagree as to whether Pythagoras sought out Chaldean wisdom:

> Chief among these mages whom Pythagoras is said to have frequented at that time is a certain magus called Zabratus, by whose power his life was purged of its former sordidness, and he learned from him what a sage ought to abstain from, and then heard him teaching about the nature of things and the principles of the universe. He is also called 'Nazaratus' by Alexander in his *De symbolis Pythagoricis*, as it is reported by Clement of Alexandria, 'Zares' by the *Suda*, 'Zaran' by Cyril, and 'Zaratas' by others, which have a certain resemblance with the name Zoroaster, the most famous magus among the Persians, in order to establish that Pythagoras listened to Zoroaster himself, which in our age the greatly missed [Isaac] de Beausobre adorned with eloquent erudition. Others think that the age of Zoroaster came before Pythagoras' time, and that Pythagoras listened not to Zoroaster, but to someone from the Israelite prophets living in captivity in Babylone at that time, and they believe that by this name 'Zaratae' one ought to understand either Ezechiel, Daniel, or some other man from that holy people. Certainly the mighty tribe of erudition is in agreement, following a great many doctors of the ancient church, in the fact that Pythagoras learned from the Jews during the time he spent in this great Persian and Babylonian sojourn. To strengthen their case, some appeal to the testimony of Hermippus in Josephus and Origen, and of Aristobulus in Clement and Eusebius, others invoke the harmony of Pythagorean doctrine with Jewish scripture, nay even they wish to believe that Pythagoras learned there the mystery of the venerable divine name חדח′ and imitated it in his Tetractys.[53]

53 Johann Jacob Brucker 1767, Period. III. Pars. II. Lib. II. Cap. X: 1003–05. On the pseudonym Zaratas and variants of Zoroaster or Zarathustra, and the Chaldean magi, see Bidez and

Brucker still has Giovanni Pico's association of the Tetragrammaton and the Tetractys in his sights, as he also takes aim at later Renaissance transformations of Ficino's notion of a *prisca theologia*, which in its most famous formulation articulated a transmission of wisdom through six exemplary sages: Zoroaster, Hermes, Orpheus, Aglaophamus, Pythagoras, and Plato.[54]

With the view of demolishing any connections between Pythagoreanism and 'oriental' wisdom, whether Chaldean or Hebrew, Brucker develops a philological strategy of avoiding ingenious conjectures, proposing instead to judge whether there is any textual evidence for Pythagoras' wide travels to the east. He proposes five arguments to his readers. He first treats Iamblichus as a fool or *nugator* by demonstrating that his chronology for Pythagoras' travels, and specifically his capture by Cambyses in Egypt and his captivity in Babylon, does not add up. Second, he argues that the evidence from Porphyry, Diogenes Laertius, Themistius, and the *Souda* is not much better. Third, he contends that whatever sources exist for Pythagoras' history they are too prejudiced to be trusted since they are written by Neoplatonists and Church Fathers (*recentiores Platonici vel doctores ecclesiae*). Fourth, that there is no reliable evidence at all that Pythagoras encountered any Jewish teachers. Aristobulus and other writers who make this claim should not be trusted since they sought to establish the origins of all Greek philosophy in earlier Jewish learning. Brucker further claims that the expression "κατ᾽Ἑβραίους" in the key passage from Porphyry's *Life of Pythagoras* that states that Pythagoras learned from the Hebrews is a later interpolation that must have originated as a marginal gloss. Finally, he argues that there is neither a textual basis nor a chronological argument to support Pythagoras' encounter with a Chaldean magus by the name of Zoroaster, or with any other magus whose name resembles Zoroaster's.[55]

Brucker, therefore, utilizes the results of Bentley, Lloyd, and Dodwell's innovative scholarship to find contradictions in the ancient biographical traditions of Pythagoras. But it turns out that these English scholars were not the first to do so. Joseph Scaliger (1540–1609) beat them to the punch. When he studied Arcerius' 1598 edition of Iamblichus' *On the Pythagorean Life*, he took the time to write two long critical notes on his book's flyleaf (figures 13.4–5). The first is an excerpt from Jerome's (347–420) *Apologia adversus libros Rufini* that

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Cumont 1938, 30–50, esp. 36–38. Bidez and Cumont also collect the relevant fragments of the different testimonies in volume two of this work.

54 Allen 1998, 1–49; Brisson 2016, 45–59; Campanelli's introduction to *Mercurii Trismegisti. Pimander sive de Potestate et Sapientia Dei* (Campanelli 2011); Gentile 1990, 57–104; Hankins 1990, vol. 2, 460–64; Robichaud 2016 and 2017; id. 2018, 153–154, 185–186, 234; Walker 1972. On the 'oriental' thesis, see Jeck 2004; on this thesis' relationship to Renaissance Platonism, see Soranzo and Robichaud 2017, and Robichaud 2018, 236–237, 243.

55 Johann Jacob Brucker 1767, Period. III. Pars. II. Lib. II. Cap. X, 1004–1008.

contains a brief account of thirteen Pythagorean symbols. Since Jerome had acknowledged in a letter to Rufinus (344–411) that in his youth he confused the teachings of the Apostles with what he had read in Pythagoras, Plato, and Empedocles, Rufinus apparently asked him to produce a list of titles of Pythagoras' books. Their ensuing disagreement hinged on Jerome's assertion that Origen incorporated Pythagorean doctrines into his Περὶ Ἀρχῶν, while also on the fact that Jerome doubted Rufinus' claim that six thousand of Origen's books were lost. Jerome retorted that he should not be held responsible to provide the complete historical records of Pythagoras. In fact, he doubts that any books by Pythagoras survive. Despite this, he argues that testimonies of Pythagoras' dogmas are still extant in the writings of Aristotle, Cicero, Brutus, Seneca, and Virgil as well as on inscriptions of public monuments throughout Magna Graecia.

Moreover, Jerome attests to his knowledge of the fact that Iamblichus, following the model of Pythagoras' own disciples, wrote an important commentary on Pythagoras' *Symbola*. He offers thirteen examples of these esoteric Pythagorean sayings or precepts.[56] For instance, he interprets the symbols, "do not stab at fire with a sword" as "do not harass someone who is angry and filled with pride with abusive words;" or "a heart should not be eaten" as "grief should be cast out of one's mind."[57] Although he only quotes thirteen of them, Jerome was probably the most important source for the Pythagorean *Symbola* in the Latin Middle Ages. The largest group of the *Symbola*, which originally date as far back as *ca* 400 BCE, are preserved by Iamblichus in his *De secta Pythagorica*, which also gives philosophical guidelines for their interpretation. Ficino was an important but not the sole propagator of Renaissance interests in the Pythagorean *Symbola*. After having circulated the *Symbola* in manuscript copies, he printed them in 1497 along with the *Aurea Verba* in a volume published by Aldus Manutius (*ca* 1449–1515) that also contained his periphrastic translation of Iamblichus' *De mysteriis*, as well as a number of other largely Platonic works.[58] Legions of other humanists – including the likes of Antonio degli Agli (†1477), Leon Battista Alberti (1404–72), Filippo Beroaldo the Elder (1453–1505), Erasmus (1466–1536), Lilio Gregorio Giraldi (1479–1552), and Giovanni Nesi (1456–1506) – also commented on some of the Pythagorean *Symbola*. Their attraction to the *Symbola* comes principally from the fact that

56 On the Pythagorean *Symbola*: Burkert 1972, 166–192, 219.

57 Jerome, *Apol. Ad. Ruf.* 39.

58 Martin Sicherl published spearheading research on Ficino's manuscripts of Iamblichus in Sicherl 1957, but Ficino's immense debt to Iamblichus has only begun to be recognized in recent years: Celenza 2002, 71–91; Copenhaver 1987, 441–455; Giglioni 2012, 3–36; Robichaud 2017, 2016 , 2018, and 2020. Saffrey and Segonds 2006, 117–124; Toussaint 2006, I–XVII; id. 2014a; Saffrey and Stefani 2018.

the *Symbola* purport, like transcriptions of an oracle, to transcribe Pythagoras' oral teachings.[59]

Scaliger, however, does not share the same interests in the *Symbola* as the previous humanists, since in the quotation that he transcribes in his copy of Iamblichus he cuts the passage short, just before Jerome lists and interprets them (figure 13.4):

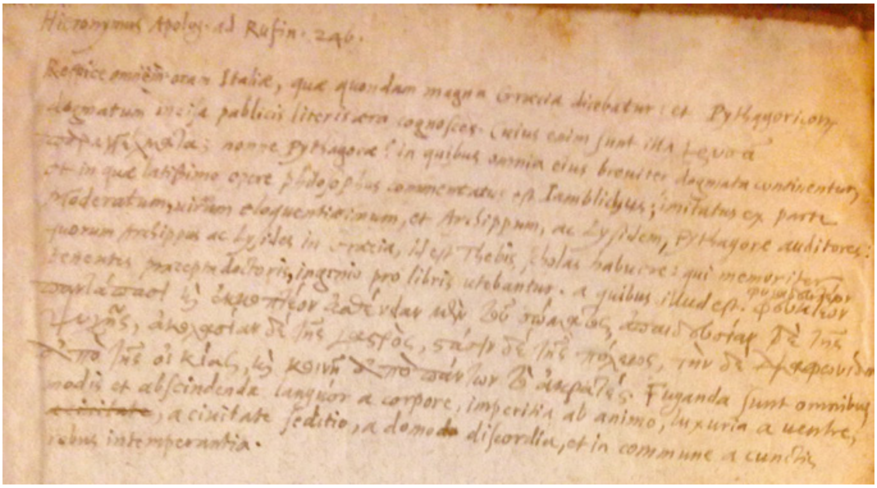
Hieronymus Apolog. ad Rufin. 246.

FIGURE 13.4 Scaliger's first annotation in the flyleaf of his copy of Iamblichus

> Respice omnem oram Italiae, quae quondam magna Graecia dicebatur: et Pythagoricorum dogmatum incisa publicis litteris aera cognosces. Cuius enim sunt illa χρυσᾶ παραγγέλματα; nonne Pythagorae? In quibus omnia eius breviter dogmata continentur, et in quae latissimo opere philosophus commentatus est Iamblichus; imitatus ex parte Moderatum, virum eloquentissimum, et Archippum, ac Lysidem, Pythagorae auditores: quorum Archippus ac Lysides in Graecia, id est Thebis scholas habuere, qui memoriter tenentes praecepta doctoris, ingenio pro libris utebantur, a quibus illud est φευκτέον φυγαδευτέον παντάπασι καὶ ἐκκοπτέον ἀσθένειαν μὲν τοῦ σώματος, ἀπαιδευσίαν δὲ τῆς ψυχῆς, ἀκολασίαν δὲ τῆς γαστρὸς, στάσιν δὲ τῆς πόλεως, τὴν δὲ διαφωνίαν ἀπὸ τῆς οἰκίας, καὶ κοινῇ ἀπὸ πάντων τὸ ἀκρατὲς. Fuganda sunt omnibus modis et abscindenda, languor a corpore,

59 On the circulation of the *Symbola* in the Middle Ages and the Renaissance, see Celenza 2001. For a list with bibliographical references of humanists who commented on the *Symbola*, see Ruiu in Pythagoras 2018, 33–38.

> imperitia ab animo, luxuria a ventre, ~~a civitate~~, a civitate seditio, a domo discordia, et in commune a cunctis rebus intemperantia.[60]

Nor is Scaliger interested in other topics addressed by Jerome: Pythagoras' discovery of the immortality of the soul and Virgil's adherence to this Pythagorean belief; Pythagoras' various reincarnations; Plato's agreement with Pythagoras; Origen's use of their teachings; and whether or not the teachings of Pythagoras, Plato, and Empedocles are found in the writings of the Apostles. Scaliger only seems interested in Jerome's opinion that later testimonies, and especially Iamblichus, preserve original fragments of Pythagoras' teachings.[61]

Scaliger's scholarly approach to Iamblichus' *On the Pythagorean Life* is even sharper in his second note (figure 13.5):

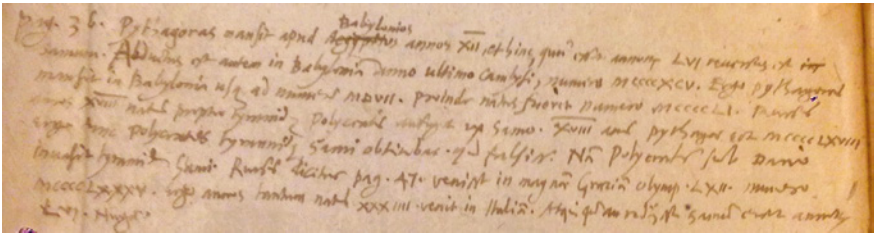

FIGURE 13.5 Scaliger's second annotation in the flyleaf of his copy of Iamblichus

> Pag. 36. Pythagoras mansit apud ~~Aegyptios~~ Babylonios annos xii, et hinc, quum esset annos LVI reversus est in samum. Abductus est autem in Babylonia anno ultimo Cambysi, numero MCCCCXCV. Ergo Pythagoras mansit in Babylonia usque ad numeros MDVII. Proinde natus fuerit numero MCCCCLI. Tametsi annos XVIII natus proprie Tyrannidem Polycratis aufugit ex Samo. XVIII annos Pythagoras est MCCCCLXVIII ergo tunc Polycrates Tyrannidem Sami obtinebat quod falsis. Nam Polycrates sub Dario invasit Tyrannidem Sami. Rursus, dicitur pag. 47 venisse in magnum Graeciam Olymp. LXII. Numero MCCCCLXXXV. Ergo annos tantum natus XXXIIII venit in Italiam. Atqui quum an redisse Samem eum annorum LVI nugat.

60 Jerome, *Apol. Ad. Ruf.* 39. Scaliger's copy of Iamblichus is: ΙΑΜΒΛΙΧΟΥ ΧΑΛΚΙΔΕΩΣ τῆς κοίλης Συρίας λόγοι δύο. Iamblichus 1598, Auct. S 2.17, Weston Stack, Bodleian Library, Oxford. The superscript and crossing out in my transcription reflect Scaliger's own emendations.

61 The first critical examination of Iamblichus' sources that one normally encounters is Rohde 1871–1872.

> Page 36: Pythagoras remained among the Babylonians for twelve years, and from there, he returned to Samos when he was 56. However, he was abducted to Babylonia in Cambyses' last year, in 1495. Therefore, Pythagoras remained in Babylon until 1507. Hence he was born in 1451. Although, he was precisely 18 when he fled out of Samos from the tyranny of Polycrates. Pythagoras was 18 years old in 1468, therefore Polycrates seized the tyranny of Samos at that time, which is false. Since Polycrates invaded the tyranny of Samos during the time of Darius. On the contrary, it is said on page 47 that Pythagoras came to Magna Graecia during the 62nd Olympiad, which is in 1485. Therefore, he was 34 years old when he came to Italy. And still when [Iamblichus] says that he was 56 years old when he returned to Samos he speaks nonsense.[62]

Scaliger wrote his notes in his copy of Arcerius' 1598 edition of Iamblichus' *On the Pythagorean Life* after he had already written his first major work of critical chronology, the *De emendatione temporum* (1583). The notes, therefore, date to the last years of his life and are related to his *Thesaurus temporum* (1606) in two ways. First, Scaliger's tacit use of Eusebius in the second note connects it directly to his *Thesaurus temporum*. As Anthony Grafton explains, the *Thesaurus temporum* began as an attempt to reconstruct Eusebius original lost Greek *Chronicle*. Specifically, Eusebius' *Canones*, book two of his *Chronicle*, was only known in the Latin translation by Jerome before Scaliger published the excerpts of the introduction and book one preserved in the Greek chronicle of George Syncellus.[63] Second, Scaliger's notes are significant because they are a critical analysis of the chronology of Pythagoras' travels in chapter four (§19, i.e., p. 36 in the 1598 edition) of Iamblichus' *On the Pythagorean Life*. Iamblichus there relates that after learning astronomy and geometry for twenty-two years among the Egyptian priests and after being initiated into their mysteries, Pythagoras was abducted by Cambyses' troops and brought to Babylon, where he studied their religion, astronomy, arithmetic, and other sciences for twelve years. Porphyry and others also elaborate that Pythagoras

62 Scaliger's diligent compilation of his conjectures and emendations of the Greek text of Iambl. *VP* are extant in three locations. In addition to his copy of Arcerius' 1598 edition, which is held at the Weston Library of the Bodleian Libraries, Oxford (Auct. S 2.17), a manuscript in Leiden contains further conjectures and emendations by him, MS. Leiden, Leiden University Library, Hemsterhusii 17, as does another copy of Arcerius' edition from the Spanheim library, now at the Staatsbibliothek in Berlin: see August Nauck's prolegomena to Iamblichus 1884, XXIX–XXXIII; Pistelli 1893, 34–37.

63 Grafton 1975, 170. On Scaliger, see also Grafton 1983–1993.

learned from the wisdom of Zoroaster, Zaratas, Zarathustra or other shadowy Chaldean mages and perhaps also from Jewish exiles at that time. Only afterwards did he travel to Italy and return to Samos at the age of fifty-six.

Iamblichus records a few historical events that occurred during Pythagoras' life – Cambyses' invasion of Egypt, Polycrates' tyranny in Samos, the sixty-second Olympiad – that help Scaliger date Pythagoras' life. But Scaliger also needs more information to triangulate Pythagoras' exact location in time. For this he turns to Eusebius.[64] In his *Canones*, Eusebius devises a way of resolving the relative chronologies of nineteen states by positioning their significant events in comparative tables, which he sets against a universal timeline that begins at the birth of Abraham and ends with Constantine's *vicennalia*. Scaliger uses Eusebius' universal Abrahamic calendar to cross-reference Iamblichus' chronology of Pythagoras' life.

Scaliger's reasoning can be reconstructed as follows. Iamblichus informs us that Pythagoras was taken by Cambyses from Egypt to Babylon for twelve years and that he returned to Samos when he was fifty-six years old. But Scaliger also calculates that Pythagoras was taken the year Cambyses died, which Jerome's translation of Eusebius' *Canones* records as occuring in 1495 according to Eusebius' dating system. From this information Scaliger ascertains the following:

Pythagoras' birth:	1451
Pythagoras, at 44, is captured by Cambyses in Egypt:	1495
Pythagoras, at 56, returns from Babylon to Samos:	1507

However, Iamblichus also supplies us with the information that Pythagoras originally fled Samos from the tyranny of Polycrates when he was eighteen, which Scaliger inaccurately lists as 1468 instead of 1469.

64 In order to combine the different calendars and systems of reckoning time in antiquity into a single linear and absolute timeline Scaliger conceived of the Julian Period, a cycle of 7980 years. He arrived at this figure by finding the common multiple of the three chief ancient calendar cycles: the solar cycle of twenty-eight years, the lunar cycle of nineteen years, and the political or indiction cycle of fifteen years. He would then coordinate each year in the Julian cycle with each of the three ancient systems by a series of three corresponding numbers. For details on his methods see Grafton in n. 63 above. Scaliger does not employ the Julian Period in his note to Iamblichus. The simpler universal Abrahamic calendar from Eusebius' *Canones* sufficed to examine Iamblichus' chronology for the life of Pythagoras.

Pythagoras' birth:	1451
Pythagoras, at 18, flees Polycrates and Samos:	1469 [*sic* 1468]
Pythagoras, at 44, is captured by Cambyses in Egypt:	1495
Pythagoras, at 56, returns from Babylon to Samos:	1507

Since this last piece of information contradicts the claim in Eusebius' *Canones* that the brothers Polycrates, Syloson, and Pantagnotus were tyrants in Samos in the year 1490, it must be incorrect according to Scaliger, which he indicates in his note by writing that Polycrates' tyranny of Samos occurred at the time of Darius. Scaliger must have been estimating as best as he could (or he is not completely accurate) since Eusebius records the reign of Cambyses under the entry for 1490, though Polycrates and Cambyses seem to have died around the same time, and only then did Samos fall under the control of Darius. Iamblichus also relates that Pythagoras came to Magna Graecia during the sixty-second Olympiad, which corresponds to Eusebius' 1485th year. This gives Scaliger new chronological data:

Pythagoras' birth:	1451
Pythagoras, at 18, flees Polycrates and Samos:	1469 [*sic* 1468] / *Euseb*.: 1490
Pythagoras, at 34, 62nd Olymp. arrives in Magna Graecia:	1485
Pythagoras, at 44, was captured by Cambyses in Egypt:	1495
Pythagoras, at 56, returns from Babylon to Samos:	1507

This final piece of evidence confirms to Scaliger that Iamblichus' chronology for Pythagoras' life is nonsensical and contradictory for two reasons. First, Iamblichus' dates for Polycrates' tyranny and Pythagoras' departure from Samos contradict the information from Eusebius. Second, based on Eusebius' date for the 62nd Olympiad (1485), Iamblichus' own chronology for Pythagoras' departure from his Babylonian captivity (1507) and his arrival in Magna Graecia (1485) is self-contradictory.[65] Scaliger's note, therefore, demolishes any pretense of chronological accuracy in Iamblichus' historical account of Pythagoras' travels.

65 All of the information above is found in the following locations: Iambl., *VP*, § 11, 19–28, 35, and 88; Scaliger 1606, 2: 128–129.

4 Conclusion

Dating lives and ancient historical events was often a tricky affair in the Renaissance. Although Scaliger's criticism of Iamblichus' account of Pythagoras' life is completely dependent on the accuracy of Eusebius' *Canones*, the ingenuity of Scaliger's simple analysis should not go unnoticed. Nonetheless, his use of Eusebius to criticize Iamblichus invites a more thorough comparison. Iamblichus' *On the Pythagorean Life* seeks to establish Pythagoreanism as an exemplary way to do philosophy, yet an important objective for Iamblichus' Pythagorean project is also the inclusion of multiple traditions into Pythagoreanism:

> In general, they say Pythagoras was a zealous admirer of Orpheus' style and rhetorical art, and honored the gods in a manner nearly like Orpheus', setting them up, indeed, in the bronze of statues, not bound down with our human appearances, but with those divine rites of gods who comprehend and take thought for all things, and who have a substance and form similar to the All. He proclaimed their purificatory rites and what are called "mystic initiations," and he had most accurate knowledge of these things. *Moreover, they say that he made a synthesis of divine philosophy and worship of the gods*, having learned some things from the Orphics, others from the Egyptian priests; some from the Chaldeans and the magi, others from the mystic rites in Eleusis, Imbros, Samothrace, and Lemnos, and whatever was to be learned from mystic associations [or from the *Etruscans*]; and some from the Celts and Iberians.[66]

Iamblichus' terminology here is far from random. For Iamblichus, Pythagoreanism is a synthesis (σύνθετον αὐτὸν ποιῆσαι) of multiple traditions that mirrors the unity of the multiplicity of beings in the noetic and cosmic order. In Platonic terms, Pythagoras applied the aphaeretic dialectical method to the traditions he encountered, analyzing each in order to synthesize their common traits, which he distilled into sayings, symbols, and rituals. Having learned from the wisest traditions, Pythagoras became an intermediary guide to help other philosophers in turn become godlike (ὁμοίωσις θεῷ), as well as unite (ἕνωσις) or come into contact (συναφή) with the One. Iamblichus also associates Pythagorean mathematics and its way of life to a specific Orphic religious tradition, even making the claim, which first Ficino, then Giovanni Pico would repeat in the fifteenth century, that Pythagoras was initiated into

66 Iamblichus 1991, 167 (§ 151). Emphasis and variant are my addition.

the Orphic mysteries by a certain Aglaophamus.[67] From the Orphic tradition, Iamblichus explains, Pythagoras learned that divine images (ἀγάλματα) are not simple anthropomorphic statues but divine symbols, it seems, either of mathematicals or of higher noetic forms. Pythagoreanism teaches that we should not make the Gods look like our physical bodies, but that we should strive to make our selves take the form of these higher intellectual realities, it seems by means of mathematical theurgy. In other words, Pythagoreanism teaches to follow the example of Pythagoras and form oneself through mathematics and philosophy as though "forming [one's own] statue."[68]

Even before Iamblichus, Plotinus too had written about the divine images in a similar manner: "the Egyptian sages did not use the forms of letters that signify words and propositions, and imitate the prescribed voices and utterances, but drawing images [ἀγάλματα], and inscribing a particular image [ἄγαλμα] for each singular thing onto their temples they displayed the nondiscursivity of the intelligibles, insofar as each particular image is a type of science and wisdom, a subject and a whole, and is neither discursive reasoning nor deliberation."[69] Porphyry and Iamblichus later wrote treatises on divine images, entitled *Περὶ ἀγαλμάτων*, and theories of divine symbolism are at the heart of their debate on religion, theurgy, and philosophy, which is now known by the title that Ficino assigns to Iamblichus' response, *De mysteriis*.[70] Theories of philosophical symbolism also left a deep impression on Ficino and Giovanni Pico's understanding of Pythagoreanism. The freedom of symbolic thinking facilitated the inclusion of different ancient philosophical traditions into their understanding of Christianity.[71]

Eusebius, by contrast, evinces an exclusive and triumphalist vision of Christianity, in part to respond to accusations in his day that Christianity was a new religion that could not compete with the religions of old.[72] In the *Chronicle* Eusebius employs comparative chronology to demonstrate the antiquity of Christianity (similarly to how Josephus argues against Apion). Universal history, he tells us, is a Christian linear history that follows a single common thread

67 Christian Lobeck (1781–1860) was the first to demonstrate how Iamblichus provides us with the first attested use of Aglaophamus as an Orphic initiator of Pythagoras: *Aglaophamus sive De theologiae mysticae graecorum causis* (Königsberg: Borntraeger, 1829). On Iamblichus and Aglaophamus, see also Brisson 2000, 237–253; id. 2016. On Ficino's revival of the figure of Aglaophamus and the Orphic connection of Pythagoreanism, see the works cited in n. 58 above.

68 Cf. Plotinus' reworking of the saying "never quit forming your own statue (ἄγαλμα)," from the *Phaedrus* 252d7 in *Enn.*, 1.6.9.13.

69 Plotinus 1959, 5.8.6.1–9.

70 Iamblichus 2013.

71 Robichaud 2017.

72 For example, Euseb., *Hist. eccl.*, 1.2.1

from the most ancient times of Abraham to Constantine. To this end, Eusebius does not just devise tables and dating systems to compare and critique relative chronologies against a singular and universal temporal matrix. Eusebius also conceives of his universal history symbolically, that is he imagines his theory of history in his *Ecclesiastical History* according to a figural exegesis of the Bible.[73] The figural correspondence between the age of Hebrew law and the age of true Christian revelation is transparent in Eusebius' account of how Jesus' divine name as "Christ" was revealed to Moses.

> Moses was himself the first to recognize how peculiar august and glorious is the name of Christ, when he delivered the tradition of the types and symbols of heavenly things, and the mysterious images (τύπους οὐρανίων καὶ σύμβολα μυστηριώδεις τε εἰκόνας), in accordance with the oracle which said to him, "See thou shalt make all things according to the type which was shown thee in the mount." [...] For this reason our Saviour has been called Christ and priest, on the authority of an oath, according to his order and not according to that of the others who received symbols and types (κατὰ τὴν αὐτοῦ τάξιν, ἀλλ᾽ οὐ κατὰ τὴν τῶν ἄλλων σύμβολα καὶ τύπους). For this reason, too, the narrative (ἱστορία) does not relate that he was anointed physically by the Jews or even that he was of the tribe of those who hold the priesthood, but that he received his being from God himself before the day-star, that is to say, before the construction of the world, and holds his priesthood to boundless eternity, ageless and immortal.[74]

Here too philosophical symbolism is operative in finding an infinite and eternal vanishing point for a doctrinal history. Only Christ, Eusebius claims, is fully and spiritually anointed, whereas the Jewish priests are merely figuratively anointed by material symbols, which are the visual representations of their invisible divine other half. Like the late ancient Platonists who establish their links with tradition by claiming the succession of leaders or *diadochoi*, Eusebius too establishes a continuity of tradition according to the principle of succession (*diadochê*), in his case an apostolic succession, which supplants the philosophical schools and the continuous genealogical succession of the Jewish priesthood.[75] As he argues in his apologetic *Praeparatio evangelica* and the *Demonstratio evangelica*, the wisdom of all "barbarian" and "pagan"

73 On the importance of Biblical exegesis for Eusebius' understanding of history see also Grafton and Williams 2006.

74 Eusebius 1926 (*Ecclesiastical History*), 1.3.1–2; 17–19.

75 On Eusebius' organization of his history according to the principle of succession, see Momigliano 1990, 140–141.

traditions should be unnecessary once the truth of Christianity is proven to be not only more ancient than them but also eternal. As one also observes in many Renaissance thinkers, scholarly criticism and religious prejudices merge in Eusebius' hermeneutics.[76]

Giovanni Pico studied Eusebius' writings for un-Eusebian purposes while preparing his *Oratio*, his 900 *Conclusiones*, and his *Heptaplus*.[77] For, like Eusebius, Giovanni Pico believes that Moses received the mysteries and divine symbols of the revealed names of God. But unlike Eusebius, he thinks that Pythagorean philosophy (as well as other ancient philosophical and theological traditions) also offer ways to reach God. Immediately after finishing his discussion on the Kabbalah near the end of his famous *Oratio*, Giovanni Pico professes his adherence to Iamblichus' vision:

> But to return to reviewing the theses of my disputation, I have also presented my own interpretation of the songs of Orpheus and Zoroaster. Orpheus is extant among the Greeks in an almost complete text; Zoroaster is lacunose among the Greeks but among the Chaldeans is read in a more complete text. Both are credited with being the fathers and authors of ancient wisdom. Indeed, to say nothing of Zoroaster, who is often mentioned by the Platonists and always with the utmost veneration, Iamblichus of Chalcis writes that Pythagoras regarded Orphic theology as the model on which he fashioned and formed his own philosophy. As a matter of fact, it is said for this reason alone that the Pythagorean maxims are thought to be sacred, for they derive from the Orphic teachings; from this first source flowed forth the secret doctrine of numbers and everything that is great and sublime in Greek philosophy. However, Orpheus (as was the practice among the ancient theologians) so well concealed the mysteries of his dogmas under the coverings of fables and hid them under the veil of poetry that, if one were to read his hymns, one would believe there is nothing more behind them than the most commonplace tales and trifles.[78]

Iamblichus' inclusive notion that Pythagoreanism is a philosophy built from multiple traditions stands behind Giovanni Pico's goals of religious and philosophical concord and his approach towards symbolic theology. Moreover, very much like Iamblichus, Giovanni Pico believes that Pythagoras can serve

76 For a recent and lucid account of Eusebius' apologetics, see Edwards 2015.

77 For Ficino's un-Eusebian use of Eusebius, see Monfasani 2009, 3–13. Robichaud 2018, 200–202, 228.

78 Giovanni Pico della Mirandola 2012, 273–275.

as a guide to help others convert to philosophy and assimilate to the divine. Comparing this conversion to Moses' ascent of Mount Sinai and Socrates' frenzies, Giovanni Pico proclaims in his *Oratio* that this self-transformation is not just a beatific vision, but a contact and transformation into God: "And at last, roused by ineffable love as if by a frenzy, and borne outside ourselves like ardent seraphim, filled with the godhead, we shall no longer be ourselves, but He Himself Who made us."[79] It is our ability for metamorphosis that rouses Giovanni Pico to make his well-known comparison of man to a chameleon and Proteus, and to praise the Pythagorean metamorphoses and Enoch's transformation into the angel Metatron in Hebrew theology in the epigraph from the *Oratio* quoted at the beginning of this chapter.[80]

In using various religious and philosophical traditions in his *On the Pythagorean Life*, Iamblichus is simply following what he believes to be Pythagoras' own way of doing philosophy. Whereas one might conclude that Iamblichus' authorship of Pythagorean works involves the compilation and preservation of a number of older traditional sources, Scaliger, Bentley, Lloyd, Brucker and other Early Modern scholars would rather dismiss Iamblichus as a *nugator*, a lying charlatan trying to make fabulous claims about another charlatan, Pythagoras, in order to compete with Jesus and Christianity.[81] Although one should still acknowledge the chronological inconsistencies and the pseudepigraphic nature of the Pythagorean, Orphic, and Chaldean works that find their way into Iamblichus' writings, one should be equally cautious about the religious motives and hermeneutical prejudices of Early Modern assessments of Iamblichus. Moreover, Iamblichus was certainly not naive when it came to the difficulties of using Pythagorean sources. He warns his readers at the very beginning of the first chapter of his *On the Pythagorean Life* that

79 Ibid., 169. For an interesting discussion on related topics, see Copenhaver 2019, which the author kindly shared with me in advance of its publication.

80 Ficino too thought of Pythagoreanism in terms of a philosophy of self-transformation, though without Giovanni Pico's strong emphasis on the Kabbalah. In fact, inspired by Iambl. *Protr.* (the second book of the *De sect. Pyth.*), Ficino composed his own commentary and arrangement of the first set of translations that he completed for Cosimo de' Medici in 1464 as a protreptic exhortation to convert (ἐπιστροφή) to philosophy and reach towards the One. On the importance of Iambl. *Protr.* for Ficino's ordering of and commentary on his first translations of Plato, see Robichaud 2018, 77–95. On the importance of conversion (ἐπιστροφή) for Ficino's thinking, see also Soranzo and Robichaud 2017, 135–166.

81 As was noted by John Wallis in a letter to Lloyd, this conclusion is not without its own contradiction, for if Iamblichus and Porphyry are truly making up stories about Pythagoras to compete with Christianity, perhaps modern scholars ought to soften their criticism of Pythagoras, who might not have been a charlatan after all. See Levitin's brief discussion of this passage in Levitin 2015, 228.

much of these obstacles come from the fact that his Pythagorean sources have been long neglected and “concealed by outlandish teachings and secret codes (*symbola*) obscured by numerous false and spurious treatises, and entangled in many other similar difficulties.”[82] Both Ficino and Pico knew Iamblichus’ report about the pseudepigraphic nature of much of these Pythagorean texts, as they also knew that Iamblichus claims (once more in the first paragraph of the *De mysteriis*) that the writings attributed to Hermes Trismegistus were in fact not written by a certain author named Hermes but the cumulative texts of priests ascribing their pseudepigraphic writings to Hermes as a way to honor the God who inspired them.[83] Iamblichus’ approach to the difficulties of studying Pythagorean pseudepigrapha also involves invoking the Gods for their help, instead of developing chronological and critical methods like Scaliger et al. Nevertheless, religious concerns do not disappear from the works of Early Modern scholars of Pythagoreanism. Regardless of how one assesses his scholarly merit, it is beyond doubt that Iamblichus did not share the same historical methods and goals as Early Modern students of chronology, since Iamblichus is less concerned to examine the inconsistencies among the traditional biographies from which he drew his material than he is in giving a full account of a philosophical way of life.[84]

What is more, Iamblichus did not write a biography of Pythagoras per se, but, as the title indicates, a book *On the Pythagorean life*. Put differently, for Iamblichus, the Pythagorean way of life does not begin and end with the life of Pythagoras, it is continuously reborn and reshaped in the lives of those who follow in his footsteps. Granted, this work can be seen as part of an ancient tradition of philosophical lives, like Diogenes Laertius’ *Life of Plato*, which often introduced the Thrasyllan ordering of Plato’s corpus, and Porphyry’s *Life of Plotinus*, which introduced Porphyry’s edition of Plotinus’ *Enneads*, but Iamblichus’ *On the Pythagorean Life* does not introduce Pythagoras’ works *senso strictu*, but rather Iamblichus’ own *De secta Pythagorica*. It should also be noted how *On the Pythagorean Life* serves the goals of the *De secta Pythagorica* as a whole. Detaching it from the larger work to treat it as though it were a

82 Iamblichus 1991, *VP* 31 (§ 2).

83 Iambl., *Myst.*, 1.1; Robichaud 2017, 73–74. See also Fowden 1993^2, 31–44. Ficino also knew from the scholion that introduced the text of the *Myst.* in his manuscripts that Iamblichus composed the *Myst.* under the pseudonym of the Egyptian priest Abamon in response to his disciple Anebon (pseudonym used by Porphyry in the letter addressed to him).

84 As Mark Edwards put it, “Iamblichus did not wish to be a scholar, for the business of the philosopher is not with facts, but lives.” Edwards 1993, 162. On the philosophical role of the Pythagorean life as the exemplary life of the philosopher see also Macris 2006, 287–329; id. 2009, 139–168. Two modern translations of Iamblichus’ text with useful introductions are: Iamblichus 1991 and Iamblichus 2011.

modern biography misconstrues it and risks missing how this text functions as part of a larger agenda. The *De secta Pythagorica* is itself a compendium of reworked Pythagorean literature and follows a specific pedagogical order, which it is reasonable to assume could have been used in Iamblichus' own school at Apamea. *On the Pythagorean Life* presents the student with a philosophical model, which the second book of the *De secta Pythagorica*, the *Protrepticus*, exhorts them to imitate. It is only once students have set themselves on the Pythagorean way of life, which Plato compares to the Homeric way of life (or the lack of a Homeric way of life, as the case may be) in the last book of the *Republic*, that is, it is only once they have purged themselves and prepared their mind, will they be ready to begin studying the common principles of all mathematics, followed by each individual branch of mathematics, arithmetic, geometry, music, and astronomy. Finally, studying mathematics leads to dialectic, which, according to Iamblichus, is the capstone and *mathesis universalis*, as it were, that binds them with common principles derived from the One. There are clear echoes of these Late Ancient goals in the Renaissance and reception of Pythagoras and Pythagoreanism.

Giovanni Pico was a philosophical voyager who like Pythagoras of lore, on the one hand, travelled far and wide seeking out the best teachers from the greatest variety of traditions at famous universities and centers for humanistic, Christian, Islamic, Jewish, Greek, Arabic, Chaldaean (or Aramaic), and Hebrew learning in Italy and France, and on the other hand, like Iamblichus, explored as many ancient sources for wisdom as he could in his ever-expanding monumental library. He is also an interesting figure in the history of Renaissance Pythagoreanism especially because he is so complex, and precisely because in debunking astrology's claims for being an ancient – or the most ancient – science in his *Disputationes adversus astrologiam divinatricem* (he argued, for example, that there is no evidence that any philosophers in Plato or Aristotle's age knew about astrology) he also became a pioneer of the kind of methods for critical and technical studies of ancient chronology that would eventually be used against his own philosophical writings.[85]

Giovanni Pico's approach towards Pythagoreanism, strongly inflected by Iamblichus and Kabbalah, turns mathematics into theology and promises that a philosophical or even theurgic way of life can become a route towards divinization. For the priest preaching against Giovanni Pico's Kabbalah in Santa Maria del Fiore, this was too much to bear since for him there could only

85 On Giovanni Pico's use of technical chronology to criticize astrology, see Grafton 1997. Giovanni's nephew, Gianfrancesco Pico would continue this approach not just arguing against astrology, but against all traditions of ancient philosophy, which he described as a threat to Christianity and a source for diabolical errors, heresies, and unbelief.

be one way to become like God, which could not be found in Platonic λόγοι, Pythagorean ἀριθμοί, Orphic μῦθοι, or Chaldean λόγια, but in Christianity's only true intermediary, Christ as λόγος incarnate. It is no surprise that the controversies over Pythagoreanism discussed above – Jerome and Rufinus; Giovanni Pico and the papacy; Benivieni and the priest of Santa Maria del Fiore; Reuchlin and Pfefferkorn; Charpentier and Ramus; as well as Scaliger, Bentley, Lloyd, Brucker et al. – involve religious polemics since they are all fundamentally about the nature of the ordering principle of the cosmos, as well as man's connection to this first principle. The Renaissance revival of Pythagoreanism gave scholars working on the history of philosophy and antiquity an opportunity to hone their critical skills and methods on its sources, but more than simply being about the technical details of ancient chronology, Early Modern scholarship on Pythagoreanism strikes at its theological claims. Behind Early Modern rejections of Pythagoreanism one periodically still hears the beating drum of Christian triumphalism, sometimes retuned and borrowed from late antiquity, seeking to drown out pluralistic voices of religious traditions.

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CHAPTER 14

Pythagorean Number Mysticism in the Renaissance. An Overview

Jean-Pierre Brach

Frequently considered, in earlier periods of Western culture, as a symbol or, at the very least, as an almost legendary individual,[1] Pythagoras is nevertheless perceived during the Renaissance as retaining more of an historical consistency, and the reality of his existence remains generally unquestioned, if somewhat imprecise. Among other exponents of the supposed *prisca theologia*,[2] he stands as a typical example of a single individual retaining the characteristics of a prophet, holy man, seeker of wisdom, political adviser, scientist, musician and philosopher alike, all qualities which play a central role in the culturally widened humanistic definitions of a "Pythagorean" philosophy. One must of course keep in mind that philosophy is here considered as being essentially of a revelatory nature, and much akin to a divine illumination, an empowerment which some of the Early Modern Humanists frequently bestowed on antique thinkers and religious figures,[3] endowing them with a prophetic status almost equal to that hitherto reserved by Christianity for the Jewish Scriptures.

As recently outlined by Christiane L. Joost-Gaugier, the influence of (neo) Pythagoreanism in the Renaissance is both manifold and widespread.[4] It covers most areas in the realms of science and the arts,[5] but is nevertheless

1 Something he already was, for the most part, in Plato's or Aristotle's time; Riedweg 2008, 42ff.; Macris 2018, 810–818 (with bibliography).

2 Gentile 2012.

3 They were probably also influenced by the "divine" character commonly attributed to Pythagoras by his Hellenistic bio/hagiographers. Macris 2003 and 2006.

4 Allen 2014; Joost-Gaugier 2009 (this study is, however, to be consulted with extreme caution).

5 Perillié 2005. In this respect one must also recall the important role played by mathematics (in general) in the encyclopedic reorganization of knowledge launched by some of the first Humanists, as exemplified – among others – by G. Valla's (1447–1500) *De expetendis et fugiendis rebus* (Venice 1501), which includes a section on arithmology: see Tucci 2008 (I am indebted to M. Ghione for this reference).

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more specifically associated with theories about the transmigration of the soul and the theo-cosmological meaning of numbers.[6]

"Pythagorean numbers" is one of the favorite and most frequent Early Modern designations of what is often termed nowadays "number mysticism,"[7] "arithmology,"[8] sometimes "arithmosophy." Widespread in the Middle Ages, in the wake of Augustine and many other Church Fathers,[9] the interest in the qualitative side of mathematics is at a relatively low ebb during the scholastic period, due to the then dominance of Aristotelian doctrines in Latin theology, since these deny number any sort of natural efficacy, besides being a mere reckoning tool.[10]

Given this situation, the Renaissance or Early Modern period (15th–17th centuries) inherits for the most part the practice of number symbolism as a hermeneutical tool, mainly concerned with the interpretation of Scripture and aiming at shedding light on the network of spiritual analogies and correspondences assumed to be linking both Testaments.[11]

However, the persistent influence of Boethius (*ca* 480–524), who conferred its definitive philosophical luster in the Latin West to the doctrine of the theological transpositions of number,[12] and that of Nicholas of Cusa (1401–1464), who established mathematics as constituting the most appropriate basis and symbols for the knowledge of the divine sphere,[13] must not be forgotten or understated, the more so since they are both heavily dependent on Neoplatonic (and Neopythagorean) sources.[14]

6 Riedweg 2008, 128–133, stressing the role played by Platonic reinterpretations in the interest displayed by later periods in "Pythagorean" philosophy.

7 The barbaric term "numerology" designates contemporary pseudo-divinatory techniques linking numbers to names, the study of which lies outside our topic. For an effective criticism of this trend, see Bell 1933.

8 Delatte 1915, 139–140; Zhmud 2013, 2016, 2019 and 2020.

9 Kalvesmaki 2013; Brach 2013.

10 Aristotle, *Metaphysics* M.6, 1080b 16, for example. Porro 2018.

11 Brach 2015.

12 Masi 1983. As is well known, Boethius' *De institutione arithmetica* is in fact an adaptation of (and philosophical commentary upon) a Neopythagorean textbook (end of 2nd century CE) by Nicomachus of Gerasa (see Nicomachus 1926).

13 Rusconi 2011.

14 Counet 2000.

1 Coming into One's Pythagorean Inheritance

Heavily indebted to his reading of Plato and of Neoplatonic philosophers, whose works he famously translated into Latin,[15] Marsilio Ficino's (1433–1499) Pythagoreanism is as much about religion as it is about philosophy.

In presenting it in this guise, Ficino is being faithful to his image of the Samian as a *priscus theologus*, as well as – very likely – to his own desire of projecting himself as the last-in-line heir to such ancient doctrines and, accordingly, as their rightful exponent in contemporary Florentine learned circles.[16] The main avenues of this so-called Pythagorean doctrine are numbers on the one hand, "psychology" on the other hand. Psychology revolves here around the questions of the transmigration of the soul, of its immortality and of its purification prior to its ascension towards the divine, bearing consequently on practical religion and ethics as well.[17]

Number is supposedly at the root of a science of universal realities, including immaterial ones, and acts as both a cosmogonic and an ontologically productive principle.[18] Accordingly, Ficino writes:

> There are two ways, especially, to understand the truth of divine things, namely, mathematics and the purity of the soul.[19]

On the whole, Ficino's interest in number symbolism hasn't yet been made the object of the overall study it certainly deserves,[20] perhaps on account of the fact that the relevant references on the topic, far from being systematically arranged, are mostly scattered through his lengthy and numerous treatises and commentaries.

It is worth noting from the outset that, paradoxically, neither Ficino[21] nor any of the very first authors who have contributed to the rehabilitation of

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15 Iamblichus figures prominently among these philosophers, as the author of one of Pythagoras' biographies alluded to above and of several other treatises on Pythagoreanism and mathematics; Iamblichus 2006; O'Meara 1989 and 2014; Bechtle and O'Meara 2000; Macris 2009.

16 Celenza 1999, 705–706; Vasoli 2005.

17 Important to the moral and mystical preparation or purification of the soul are the famous *symbola*, familiar to the Christian tradition since Antiquity and much read in the Renaissance; see Celenza 1999, 693; 2001a, 4–34 and 2002; Robinson 2013.

18 Celenza 1999, 702–705.

19 Quoted by Celenza 1999, 694, n. 88.

20 A partial exception to this is the book by Allen 1994.

21 Ficino has also shown a penchant towards geometrical symbolism: Allen 1999; Toussaint 2000.

number symbolism within early Renaissance culture wrote a treatise specifically dedicated to this topic. There are several reasons for this, the main one being probably that numbers and their qualitative aspects, both ontological and cognitive, were perceived as integral parts of a more general cosmo-theological worldview, based on correspondences, correlative thinking and philosophical syncretism.

As we shall see, a systematic treatment of the subject of number mysticism belongs to a relatively later phase of the evolution of Western "esoteric" literature.

2 "That Which Is Old and New": Number, Magic and Kabbalah

As early as 1486, in his famous, so-called *Oration on the Dignity of Man*,[22] Pico della Mirandola (1463–1494) claimed to have somehow rediscovered number symbolism as a "new way of philosophizing by numbers," and announced his intention to restore it as a speculative current *à part entière*, on a par with Neoplatonism, magic and kabbalah. Consequently, arithmology ceases to be mainly a hermeneutical tool for the interpretation of passages from the Bible, to become – to a certain extent – an almost autonomous "art of numbers" capable of addressing (and answering) problems of cosmology, morals, metaphysics or theology.[23] In his *Conclusiones* (or 900 *Theses*; 1486) and *Apology* (1487), Pico introduces the concept of "formal number," implicating that mathematical entities – in typical Neoplatonic fashion – possess a specific ontological status, corresponding to a particular (rational) cognitive mode. Pertaining to an intermediary level of reality, situated above the material plane, the "formal numbers" are endowed with the faculty of bestowing "a power and an efficacy" on natural things which belong to the – less "formal," therefore less "actual" – physical degree of existence. In this manner, Pico relies much less on hackneyed numerical analogies than on the close interaction of the cognitive and ontological properties of arithmetic. Number is thus assimilated to a "formal," secondary cause, active on the material world.[24]

The idea according to which the more "formal" is endowed with a higher, more powerful degree of being than the material, is of course Pico's version of a classic Neoplatonic tenet which holds that immaterial entities are actually

22 Copenhaver 2016.

23 See his 85 "mathematical conclusions" in Farmer 1998, 466–485 (especially the fourth one, pp. 466–467).

24 Brach 2016b, 412ff.

situated higher up the ontological scale and are thus, paradoxically, correlated to a denser level of existence – and of intellectual capture – than that ordinarily occupied or entailed by physical realities.

Among other things, numbers are therefore perceived as capable of functioning on different planes, thus permitting their actual use in different disciplines such as natural philosophy, magic or kabbalah.

Since these disciplines, among a series of others, are presented by Pico (in his *Conclusiones*, essentially) as displayed according to an ascending hierarchical order, albeit closely articulated to one another,[25] it follows that material (or purely quantitative) number – as a cognitive tool in natural philosophy – must take a back seat to its formal (or Pythagorean) counterpart[26] and further, when dealing with Jewish theosophical literature, to its even higher kabbalistic analogue, the ten *sefirot*.[27] In the same way, kabbalah – as a supra-discursive practice – stands above magic and natural philosophy, and leads towards the union with the Active Intellect by transforming man into an angel – a theory which is at the root of the later, widespread assimilation of kabbalah with angelic magic, popularized by the likes of Jean Thenaud (*ca* 1480–1542) and Heinrich-Cornelius Agrippa (1486–1535).[28]

Even though he does not make use of the expression “formal number” as such, Johann Reuchlin (1455–1522) is familiar with Pico’s (and Ficino’s) theories.[29] He nevertheless takes the problem from a different angle, by first stating the identity of Pythagoreanism and of the kabbalah, and furthermore calling the former a derivation from the latter.[30] Although an apocryphal antiquity was frequently ascribed to Jewish kabbalah by its first Christian interpreters, Reuchlin’s historical twist is probably also intended to deflect an eventual charge of judaizing, coupled to a tendency to present kabbalistic materials as a “symbolic philosophy,” akin therefore to the method supposedly lying at the core of Pythagorean teachings.[31] As Pico before him, Reuchlin

25 Farmer 1998, 495 (“Magical Conclusion n° 4”) and 499 (“Magical Conclusion n° 15”); Pico della Mirandola 1572, V, 170; Fornaciari 2010.

26 Farmer 1998, 465 (“Mathematical Conclusion n° 5”) and 469 (“Mathematical Conclusion n° 6”). Pico later abandoned this Pythagorean outlook, as shown by Valcke 1985.

27 Wirszubski 1989 holds the opinion that “formal number” is for Pico merely synonymous with *sefira*, the respective occurrences of both expressions simply varying with the context, according to him; *contra*, see Buzzetta 2019, 94–104.

28 In book III of his *De occulta philosophia libri tres* (Heinrich-Cornelius Agrippa 1992 [ed. V. Perrone Compagni], 402ff.); Brach 2016a. On J. Thenaud, Fabre and Polizzi 2020.

29 Zika 1998, 171–176 (and *passim*); Leinkauf 1999.

30 These assertions are found in the introduction to his famous 1517 *De arte cabalistica* (Johann Reuchlin 1993); Zika 1998, 144–154.

31 Celenza 2001; Idel 2014; Schmidt-Biggemann 2016.

was conscious of the fact that Hebrew letters possessed numerical values, thus turning words and, above all, divine or angelic names into proper numbers.[32] An example of this is the importance assigned to the quaternary, considered in its expression either by the "divine *tetractys*"[33] of the Pythagoreans or by the *Tetragrammaton*, the most sacred Biblical name of God, composed of four letters.[34] In a similar way, the ten *sefirot* were also described by him as both divine aspects or attributes and intelligible numbers whose essence is derived from that of the divine intellect. Here again, numbers are presented as intermediary terms between natural and divine realities, simultaneously separate (like divine things) and inseparable (like natural objects) from matter and, thus, enabling the human intellect to transition smoothly from one plane to the other. From the conjunction of the primordial Unity and the infinite Dyad (or Binary), numbers "flow" according to a process of emanation and constitute, with their subsequent production of the *tetractys* and of the Decad, a progression which, through dots, lines, plane figures, solid figures and physical, three-dimensional bodies, finally gives birth to the whole cosmos, down to its basic natural constituents.

3 The French Connection: Pythagoras, from Natural Magic to Christian Mysticism

Initially influenced – like Reuchlin – by the Florentine authors[35] and obviously borne by the growth of the printing industry, a "French school" of arithmology followed suit and popularized the topic of both numerical and geometrical symbolism within learned circles, from the early 16th century onwards.

This led to the elaboration of various inquiries into mathematical analogies, as well as to a corresponding number of sub-genres within the relevant literature, represented by books which in some cases have later become famous. In

32 In this manner, numbers are also associated with the specific kind of Christian theurgy previously adressed by Reuchlin in his 1494 *De Verbo mirifico* (*On the Wonder-Working Word*); Roling 1999.

33 Riedweg 2008, 80–89; the *tetractys* originally designates a sacred oath formula including the first four numbers; the word as such means "fourthness."

34 Johann Reuchlin 1993, *passim*.

35 J. Lefèvre d'Etaples traveled to Italy for the first time during the Winter of 1491–1492 and met, among others, Ficino and Pico. Like Pico, he distanced himself rapidly enough from Ficino's involvement with magic and from the use of kabbalah, to turn towards Christian mysticism, under the combined influences of N. of Cusa and [Ps.-]Denys the Areopagite; see J.-M. Mandosio in Lefèvre d'Etaples 2018, VII–LVII.

this regard, one must be aware that the qualitative aspects of number are considered, by our set of writers, as essential to the study of its arithmetical properties, in proper "Pythagorean" (and Neoplatonic) fashion. This underlines that the philosophical or religious meanings ascribed to numerical entities were actually believed to be the result of certain formal characteristics, which represent the transposition of their respective quantitative values on subtler or immaterial planes of reality, thereby illustrating the indissoluble link between the two faces of number: contemplative and scientific.

It is an essentially speculative approach to mathematics that was initiated by Jacques Lefèvre d'Etaples (?1460–1536). Lefèvre himself dedicated Book II of his early and hitherto unpublished *De magia naturali* (written in 1494) to the practical role supposedly played by numbers in natural magic.[36] As underlined by J.-M. Mandosio, his magic is essentially of an astral kind,[37] and numbers are at the root of celestial influences on the terrestrial world.

Assimilated by Lefèvre to "ideas," numbers constitute the source of the living "chains" linking the different levels of existence and manifesting the network of correspondences between sublunar and celestial worlds (and creatures). Here again, numerical entities exhibit both an ontological and a cognitive aspect, since they allow us an intellectual grasp of (and supposed magical mastery over) the interplay of influences which maintain the life of the whole universe, while at the same time being (*qua* "ideas") the true *rationale* behind those influences and their transmission, from beyond the heavenly spheres. To this downward progression corresponds analogically (and in typical Neoplatonic fashion) an upward one, the *ascensio mentis* along the *scala numerorum*, eventually identified with the celestial spheres and angelic hierarchies, a testimonial, for the author, to the affinities between "ideas" and angels, that is, between Lefèvre's "Pythagorean philosophy" and Christian theology.[38] With the implication, moreover, of numbers as they appear in the constitution of divine names,[39] Pythagoreanism and theology are henceforth conflated, along with magic, in a peculiar form of Christian kabbalah underlining the power of divine names (and, therefore, of numbers) in the production of wonders and miracles – natural and otherwise –, ultimately culminating in the paramount name of Jesus, in accordance with Reuchlin's (and Cusa's) early views on the "wonder-working Name."[40]

36 Mandosio 2013, 37–79.

37 Ibid., 43; Lefèvre d'Etaples 2018, LIX–XCIX.

38 Mandosio 2013, 72–76.

39 Again, through the numerical values of Hebrew letters, as with Pico or Reuchlin.

40 See n. 32 above; Wilkinson 2015, 313ff.

Lefèvre's parallel interest in ordinary mathematics, culminating in the publication of several textbooks (1496) and in an edition of both Boethius' *De arithmetica* (1503)[41] and Euclid's *Elements* (1517), exerted a major influence on several French members of his erudite circle who, between 1510/11 and 1521, in Paris, published a few treatises on number symbolism.

Charles de Bovelles (1479–1567) offers a highly philosophical exposition of the ontology of number and of the correlated *elevatio mentis* towards the supreme Unity, supported by geometrical considerations which frequently owe much to Cardinal Nicholas of Cusa's (1401–1464) *Docta ignorantia* (*On Learned Ignorance*, 1440).[42] Besides that of Cusa, other recognizable influences on Bovelles' metaphysics are those of R. Lull, G. Pico and M. Ficino.[43]

A true pioneer in the study of geometry,[44] his interest and proficiency in this discipline were famous among his contemporaries. Many of his scientific treatises actually exhibit a strong tendency to dwell on the practical applications of geometry, as well as on the formal analogies between mathematical figures or bodies and natural objects, in true Pythagorean fashion.

Bovelles' combined penchant for the mechanistic aspects of science was part of a general didactic purpose, and of a plan to reform the contemporary cursus of studies in the field. Both endeavours can possibly be conceived of as parallel or complementary to Lefèvre's own efforts at the Cardinal Lemoine College in Paris (where Bovelles was his pupil, from 1495, and later also taught), as well as to his slightly later involvement in the (religious, this time) formation of laymen and of the clergy, alongside the bishop and reformer Guillaume Briçonnet (1470–1534) and his "Groupe de Meaux" (1521–1525).

In contradiction, however, with N. of Cusa's or L. Pacioli's (see *infra*) views in this respect, number symbolism, in Bovelles' work, is linked to the philosophical precedence ascribed to arithmetic over geometry. A Platonic commonplace, this superiority of number is linked to its greater "abstraction" and supposed capacity to lift up the mind from physical realities to the intellectual ones.[45] Being "immaterial," numerical entities appear to Bovelles better suited

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41 See n. 12 above. This interest does not keep him, Bovelles or Clichtove from pointing – from the earliest years of the 16th century – to the opportunities offered by mathematical studies for the purification of the soul and its contemplative ascent, as shown by several texts published in Rice Jr 1972, *passim*; Oosterhoff 2018.

42 See Nicholas of Cusa 2013.

43 Victor 1978, 45–46 who also points to his lesser involvement with religious reform, compared to that of his other three colleagues.

44 He was actually the first to publish a manual of geometry in French (Paris: H. Estienne, 1511).

45 As the author puts it: "Numbers are more elevated than magnitudes and more concealed, since they are posited in the soul, whereas magnitudes are more accessible and belong to corporeal reality" (Charles de Bovelles 1512, fol. 69v°; my translation).

than geometry to support the anagogical movement of the spirit towards the divine.[46] In this sense, geometry[47] can serve as a preliminary introduction to both the science of arithmetic as such, and to its mystical interpretation. This last aspect is discussed in the *Book of the Twelve Numbers*,[48] dedicated to the mystical analogies of the first twelve integers, considered separately and in their natural order. Relatively important aspects of its contents are the interrelation maintained between the quantitative and qualitative characteristics of certain numbers (a point already stressed above), as well as the continuation of epistolary discussions on the topic of number mysticism held in earlier years between Bovelles and others, such as Germain de Ganay (14?–1520),[49] letters which were published in the volume alongside the text itself. Scripture and Christian theological considerations are remarkably absent (apart from the theme of the Heavenly Jerusalem *a propos* the number 12) from the *Liber de duodecim numeris*, which insists on the scalar properties which, through their correspondences and analogies on the various planes of being, constitute numbers as ontological intermediaries in the intellectual ascension towards Unity.

Accompanying certain chapters, several charts and diagrams attempt, by ordering and classifying them, to synthesize the manifold numerical analogies associated with a given number (such as 4 and 6). These charts may have exerted an influence on those presented, somewhat later, in Book II of Agrippa's *Occult Philosophy*,[50] in chapters which are essentially concerned with arithmology and its companionship with magic.

46 Such a movement is, for the author, the principal function of human reason, as developed in the *De Sapiente* (1510; Charles de Bovelles 1982). See Gonzàlez-Garcia (forthcoming).

47 Bovelles actually dedicates the last book of his *Physicorum elementorum libri decem* to the symbolical correspondences of geometrical bodies. For him, however, knowledge of the Platonic polyhedra, in particular, tends towards a joint apprehension of both sensible and intelligible substances and can even lead to the contemplation of the mystery of the Trinity. Here, he may possibly be influenced by L. Pacioli (although he apparently never mentions him, unlike Lefèvre, who did so in the preface to his 1514 edition of N. of Cusa's works), as recognized by Sanders 1990, 110–112; 171–173, who also hypothesized (103, n. 37) an influence on Bovelles of Proclus' *Commentary on the First Book of Euclid's Elements* (Proclus 1970).

48 *Charles de* Bovelles 1510, which is part of a collection of tracts on different mathematical topics.

49 Vasoli 2001. To G. de Ganay were dedicated Lefèvre's *De magia naturali* and Clichtove's *Opusculum* (see *infra*). Letters on the relation between arithmology and the "three principles of natural magic" (dating back 1503–1505, to Ganay and several others) are also featured in Abbot Johann Trithemius' (1462–1516) published correspondence; Brann 1999, 117–133.

50 The use of charts is already present in Lefèvre's *De magia naturali* (Jacques Lefèvre d'Étaples 2018, 58–62) as well as in other philosophical books by Bovelles (*De Nihilo*, 1511;

Another friend of both Lefèvre and Bovelles, and their colleague at the Cardinal Lemoine College (where the study of the Liberal Arts was very much in favour), is Josse Clichtove (1472–1543), an important Humanist, ecclesiastical reformer and Catholic theologian. His only arithmological publication is his *Small Treatise on the Mystical Meaning of Numbers* (*contained*) *in the Bible* (1513).[51] If Bovelles sometimes made use of certain medieval sources,[52] Clichtove – anticipating in this regard the rather conservative theological stance he was later to adopt – took number symbolism all the way back to its medieval understanding, as a tool for the interpretation of numerical passages in Scripture. The main goal of this learned booklet is thus exegetical in nature, aiming as it does to illuminate the reading and meditation of the Bible by the mystical resources offered by number symbolism. Unlike in Bovelles' case, scriptural and Christian references are central to the author's purpose, as he solely references texts emanating from the Church fathers, medieval theologians or ecclesiastical writers in general.[53] Yet, as a result of this attitude, and in a manner akin to that of Bovelles' *Liber de duodecim numeris*, esoteric speculations about the "occult" disciplines (such as magic, kabbalah, Hermeticism, etc) are remarkably absent from Clichtove's concise dissertation, which examines numbers according to their natural order of succession but pushes it (in conformity with the contents of Scripture) much further than the duodenary to which Bovelles actually limits himself.

Another (and last) author, a member of the "Groupe de Meaux," who was familiar with Lefèvre's circle and who benefited also, at the end of his life, from the protection of Marguerite d'Angoulême (1492–1549),[54] is bishop Gérard Roussel (1500 ?–1555 ?), who published in 1521 a learned arithmological commentary on Boethius' *De arithmetica* (using Lefèvre's earlier edition of the Latin text).[55] Roussel's *mystica numerorum applicatio* is a chapter-by-chapter commentary, frequently longer than the original text, exploring all manners of

Charles de Bovelles 1983) and – extensively – in Gérard Roussel's commentary on Boethius (see *infra*). It seems to be a pedagogical device typical of treatises involving mathematics and emanated from this French circle; Oosterhoff (forthcoming).

51 Josse Clichtove 1513. This text actually happens to be the first purely arithmological tract to have been printed separately – the one by Bovelles being part of a larger collection of texts, as already stated above: Massaut 1968.

52 Céard 1982.

53 Even though he does mention Pythagoras and Aristotle in his "Preface" to Germain de Ganay.

54 Her spiritual director, between 1521 and 1524, was none other than Guillaume Briçonnet himself; see Martineau, Vaissière and Heller 1975–1979.

55 Printed by S. de Colines: Paris 1521. A copy of this work, annotated by the French Christian kabbalist Guillaume Postel (1510–1581), is now in the collections of the Bibliotheca

analogical transpositions of numbers and displaying a vast amount of erudition in school mathematics and music, philosophy, theology and arithmology. Here again, and by necessity, the link between qualitative and quantitative number comes to the fore, as do the didactic preoccupations, manifested by a great number of diagrams and charts inserted in the text. In writing his commentary, Roussel has somehow authored a second *De arithmetica*, of a number symbolical kind this time, a textbook celebrating the manifold deployment of basic numerical properties (examined by Boethius) in the realms of Biblical exegesis, philosophy, musical science or theology, with the help of a vast array of references ranging from Antiquity to his own time, from scientific writers (such as Euclid or Jordanus) to Christian thinkers.

In true Pythagorean fashion, geometrical considerations are developed on the basis of the study of figured numbers. Besides that of his sources, Roussel's doctrinal eclecticism – religious and philosophical – is likely a corollary of his preoccupation with instruction in general, and the spiritual edification in his readers – a preoccupation which comes as no surprise from yet another member of the evangelical "Groupe de Meaux." Number, quantitative measurement, proportion and their mutual relations serve here as rational unifying factors under the tutelage of Scripture and of Christian tenets, in a new, humanistic version of the Augustinian theological paradigm consisting of Holy Writ, Reason and Tradition.

4 Allocating Beauty: Proportion, Art, Architecture

Equally removed from preoccupations with "esotericism," magic and/or the kabbalah, Luca Pacioli's (*ca* 1447–1517) *De divina proportione*[56] is nevertheless a curious book made famous by its woodcuts (designed after Leonardo da Vinci's (1452–1519) drawings) illustrating all sorts of geometrical bodies. Juxtaposing theoretical and more technically oriented parts, with an essentially pedagogical aim,[57] the work's main argument is about the importance of mathematics in general for the understanding of nature, and concerns

Philosophica Hermetica in Amsterdam; Secret 1977, 115–132 ("II. Annotations de G. Postel à une *Arithmetica* de Boèce, commentée par Gérard Roussel").

56 Luca Pacioli 1509 (composed in Milan *ca* 1496–98).

57 Despite its Latin title, the whole work is actually written in Italian, which obviously contributed to its success. A translator of Euclid in Latin (1509) and, possibly, in Italian as well, Pacioli privately taught mathematics and accountancy throughout Italy for most of his career; Bucciarelli and Zorzetto 2018.

specifically the so-called "Golden Mean" (or "Golden Section")[58] and its application to architecture and the arts. Culled from both Euclid's *Elements* and Plato's *Timaeus* (55a ff.), with passing references to Pythagoras, the mathematical and symbolic properties of this geometrical division in mean and extreme ratio, as well as those of the five regular polyhedra, express the perfection of five fundamental divine attributes[59] and how such a perfection actually mirrors itself, according to Pacioli, in the fabric of the universe: the shape of "the All" (or "quintessence") is – according to Plato – the dodecahedron (a solid with twelve pentagonal faces), for the construction of which Euclid makes use of the "divine proportion," thus indicating the analogy between this ratio and the supernal force at work behind the creation. Seen from this perspective, geometry is more intrinsically concerned – compared with number itself – with material and corporeal measures, quantities and dimensions, insofar as these reflect, in the author's view, the divine perfections and the manner in which they intervene in the pattern of creation.

The second part of the work – an architectural treatise – examines the proportions of the human body, as compared to those found by Vitruvius in classical architecture.[60]

5 Pythagorean Syntheses

Wholly diverse in nature and scope from the preceding works, although very much "Pythagorean" (as well as kabbalistic) in spirit, is the bulky *De harmonia mundi* published in Venice by the Franciscan monk, Christian kabbalist and theologian Francesco Zorzi (1466–1540).[61] This encyclopedic and influential tome, replete with the most eclectic philosophical erudition, is concerned with the fabric and constitution of the universe, understood in terms of musical harmony, based in turn on mathematical analogies and proportions.[62] Divided into three main "canticles" (further subdivided in eight "tones" each[63]) respectively correlated to God (Creation), Christ (Redemption) and Man (Reintegration), and also to the classic threefold division of nature (angelic,

58 A particular case of the geometric proportional mean (of the type $ac = b^2$). Neveux 1995.
59 Unicity, trinity, transcendence, immutability and creative wisdom.
60 Giusti and Maccagni 1994.
61 Francesco Zorzi 2010.
62 Chaignet 1874, t. 2, 330ff.
63 In imitation of the musical octave.

heavenly and terrestrial),[64] the *De harmonia mundi* offers a vast synthetical *exposé* of the mysteries of creation and of their symbolical relation to the divine Unity. Under the influence of Pico, Ficino and Reuchlin (among so many other sources), Zorzi attempts to explain, via the symbolism of the Pythagorean *tetractys* (and denary),[65] allied to that of the ten *sefirot* or divine aspects mirroring each other within the universe, the relations between Unity and multiplicity, in marked Neoplatonic fashion. Central to his approach is the concept of the reducibility of creation to a symbolical language, ultimately identified with numbers. Manifesting themselves in the constitution of names/words, musical rhythm and proportions, numbers illustrate the harmony which supposedly describes and encrypts the ontological kinship between Unity and its hidden expressions, which make up the multiple universe. Centered on Man, understood as created in God's image and, therefore, as a perfect synthesis of the macrocosm, the third "canticle" evokes the eschatological harmony foreshadowed in man's structure. The relations of the human soul to the body are expressed in terms of musical proportions (borrowed mainly from a famous passage in Plato's *Timaeus* 35a ff.), as is the entire hierarchy of beings in the universe, which is actually based on the number 27 (another feature from the *Timaeus*). 27 represents the "perfect cube" of 3, considered here as the principal number of reference and the root of the entire harmony of creation,[66] whose ultimate, perfect development is precisely symbolized by 27. The *De harmonia mundi*, therefore, presents itself as an architectural model of reality, a model whose origin and final achievement coincide in God and are anticipated in Man and in his mystical reintegration. Such themes are described according to an immense network of numerical proportions and analogies, which supposedly account for the actual layout of creation as well as for the scheme of its future reintegration, conditioned by Man's spiritual evolution and expressed in recurring musical harmonies, in a *pars pro toto* scalar ontological ascension.

It is worth noting that, on at least one occasion, Zorzi even had the opportunity of applying his musical, kabbalistic and arithmological principles to concrete architectural planning and realizations. This took place during his

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64 Such a division is also meant to recall the tripartite structure of Dante's poem, on which Zorzi left a commentary; Francesco Giorgio Veneto 1991.

65 Although Zorzi's focus here is mostly on number 9 and its multiples, he nevertheless explicitly mentions both the Pythagoreans and Plato *a propos* the denary. On these two different understandings of the Decad, Deretić and Knežević 2020. (I owe this reference, as well as several others regarding Antiquity, to the kind generosity of my colleague Constantinos Macris.)

66 The creation itself is organized by Zorzi according to a scale of 9 (3^2), number of the angelic orders according to [Ps.-] Denys the Areopagite.

intervention in the reconstruction works undertaken at the church of the Franciscan convent of San Francesco della Vigna in Venice, for which he wrote a memoir (1535) dealing with the proportional analogies between the human body and the proposed monument that he felt (in agreement with the principal builder, Jacopo Sansovino) should be respected in the erection of the new building.[67]

An avid reader – and sometimes almost self-declared plagiarist – of Pico, Ficino, Reuchlin, Zorzi and their likes, and a friend and visitor of Trithemius,[68] the already mentioned H.-C. Agrippa is the author of a well-known and very influential treatise on magic, entitled *De occulta philosophia*.[69] In this work he devotes no less than fifteen chapters[70] to the numbers from 1 to 12 and beyond. Each of these chapters usually fills up between 1 and 3 pages of the book (the longest being devoted to the septenary), and is followed by a chart simultaneously summarizing and expanding the text by presenting the manifold correspondences and analogies governed by the relevant number, displayed according to a six-fold division.[71] Most especially concerned in this part of the book with the role played by mathematics in the three different types of magic he famously correlates with the natural, celestial and "intellectual" (i.e., angelic) worlds, Agrippa devotes as many brief introductory chapters to the nature and importance of number as such. Against the backdrop of a significant array of classical, medieval and contemporary authorities, he posits a Christian Neoplatonic understanding of number as the archetype of created things in the divine intellect, possessing a real kinship with the "ideas" and responsible for the cohesion of the whole universe, both on the functional and ontological levels. Distributed and active on all planes of existence, number is thus the origin of the secrets and mysteries of creation, in the natural, celestial and "divine" (angelic) realms.[72] Needless to say, Agrippa is here exclusively concerned with what he calls "formal and rational" mathematics, as opposed to their "material" counterpart. In a way which is very much inspired by Pico, he insists on the *formalis ratio* present in "natural" number, which is the

67 Foscari and Tafuri 1983.

68 The abbot was the actual dedicatee of the first version of Agrippa's *Occult Philosophy*, which has been preserved in manuscript.

69 See n. 28 above. I am quoting from this critical edition of the text.

70 Heinrich-Cornelius Agrippa 1992, 249–299.

71 The division in question is composed of elements belonging successively to the divine or "archetypal" world, to the three-fold partition of the cosmos, to the "microcosmos" (Man) and to the infernal world. The chapter devoted to the number 12 has 2 charts, respectively concerned with kabbalistic and "Orphic" doctrines – an obvious echo of Pico's syncretism; Lehrich 2003.

72 Heinrich-Cornelius Agrippa 1992, book II, chap. 2, pp. 252–253.

rationale behind the mysteries of God and of Nature compounded in "abstract" mathematical entities. One must be capable of effecting the conjunction and consonance of those numbers with the divine ones, in order to understand and operate the wonders that can be brought about via the magical use of mathematics. Still following Pico's wake, Agrippa holds that numbers pertaining to the first decad actually refer to divine realities and that those belonging to the tens, hundreds and thousands indicate the celestial, terrestrial and eschatological planes respectively. He then proceeds to correlate the three mathematical means[73] to the relations between the parts of the soul, of the body and of the whole living being.[74]

Although he was also a versatile, yet leading scientist of the Elisabethan Renaissance, and consequently much more personnally interested in the varied practical applications of mathematics than Agrippa ever was, it is another magician,[75] John Dee (1527–1608), who took it upon himself to write an important preface to the first English translation of Euclid's *Elements*, published in 1570.[76] In this seminal text, Dee articulated a spirited defence of the usefulness of applied mathematics in general (of which he was himself a skilled practioner[77]) to an *encomium* of "formal number" as the superior level of mathematical entities and of their understanding.[78] Dee shares with Pacioli (albeit in the course of a single text instead of several successive treatises, and despite considerable differences in tone and intent), a willingness to write in the vernacular and to attempt bridging a theological discourse on divine and cosmic harmony with certain elements belonging to scientific and practical knowledge.

Ultimately deriving from the supreme Monad, number as such is present as a pattern in the mind of the Creator, in angelic and human intellects, as well as in the natural realm. Representing the essence of reality, mathematics are thus endowed with a magical power, as they encompass all levels of creation and mediate – once again – between the divine and material spheres. Since they operate on all these planes, they are essential in implementing an ambitious synthesis of science, magic, kabbalah, and natural philosophy. Notwithstanding, Dee was also part of a long line of humanist thinkers who were not satisfied with scholarly sources and an exclusively bookish approach

73 In other words, the so-called arithmetical, geometrical and harmonic proportional means (cf. n. 58).

74 Heinrich-Cornelius Agrippa 1992, book II, ch. 3, pp. 254–255.

75 Harkness 1999; Clucas 2006.

76 London: J. Daye, 1570; reprint John Dee 1975.

77 And of which he distinguished no less than 19 different branches or *arts methodicall*.

78 Mandosio 2012.

to knowledge but who also maintained, like Pacioli or Bovelles, an interest in underlining or even developing the technical applications of algebra and geometry. In so doing, they intended to put contemporary mathematical discoveries at the immediate disposal of craftsmen and artisans alike, in order to provide them with the scientific background necessary to shed theoretical light on empirical procedures.[79]

6 One of a Kind: One Number Says It All

Having secured for itself, by the last quarter of the 16th century, an epistemological legitimacy in both the scientific and philosophical fields, Pythagorean number mysticism, before it produced some encyclopedic treatments, developed a peculiar kind of essay: the study of a given symbolic or religious theme through the lenses of a monography on a single number of the Decad. Already attested in Antiquity,[80] this specific sub-genre of arithmological literature focuses most frequently on the Septenary, although some of the treatises deal with other numbers, mainly the Ternary.[81]

Among the most meaningful examples of such endeavours is Alessandro Farra's *Settenario dell'humana ridutione* (*Septenary of the Human Conversion*).[82] A jurist and civil administrator, and a young member[83] of the newly-founded literary Academy of the *Affidati* in the city of Pavia, Farra composed his most conspicuous *opus* according to a seven-fold division. Under the strong (and avowed) influence of Pico and Ficino, his *Settenario* is a spiritual and philosophical discourse mixing up Pythagorean and kabbalistic themes about the seven steps of the mystical itinerary of the soul. Number symbolism is particularly influential in the seventh and final section of the book, the one devoted to the *filosofia simbolica ovvero le imprese* (the "symbolic philosophy or the images"),[84] which represents the final stage of the *ridutione* (conversion) mentioned in the title, in other words the acquisition of wisdom. Such wisdom resides, for Farra, in the contemplation of the intelligible principles regulating the cosmic proportions, accessed through numerical and geometrical symbols

79 Brach 2015, 115–117.
80 Especially, but not exclusively, for the Septenary. Cf. L. Zhmud's works (cited above, n. 8).
81 A list of these (to which may be added Croci's *Breve discorso della perfezione del numero ternario* [see Antonio Croci 1623]) can be found in Brach 1994, 76, 84, 90, 92, 94.
82 Alessandro Farra 1571/1594.
83 Born some time during the 1540's, Farra was elected in 1562. The Academy typically specialized in works of rhetorics and eloquence.
84 For a more detailed analysis, see Brach 1994, 73–75; Maggi 1998, 23–45.

manifesting the network of correspondences at work behind the harmony of creation.

Shortly afterwards, Fabio Paolini (*ca* 1550–1604) also published his *Hebdomades sive Septem de Septenario libri* (*Hebdomads, or Seven Books on the Septenary*),[85] which constitute a learned commentary on a single verse of Virgil's *Aeneid*.[86] A trained Humanist and physician, and a teacher of both the Greek and Latin languages and literature in several local institutions, Paolini is also a founding member of the second Venetian Academy, that of the *Uranici* (1587–1593). Certainly his most famous work, the *Hebdomades* represent an encyclopedic treatment of rhetorics and poetry, combined with speculative philosophy, astrology and symbolism, under the joint influences of Ficino, Pico, and his friend, the famous musician Gioseffo Zarlino (1517–1590).[87] The mere existence of treatises like Farra's or Paolini's (as well as of many others, dedicated to a single number) actually testifies to the widespread interest for the art of memory within the learned circles and Academies of Northern Italy. In such a context, the art of memory is combined with number symbolism, Ficinian astral magic, Christian kabbalah and musical theories as practical means to enhance the perception of both cosmic and esthetic harmony. Paolini develops a syncretic theory, which closely associates Orpheus and Pythagoras, who symbolically stand for the poetic/emotional and mathematical aspects of music, respectively. At the top of the septenary scale of knowledge, the mysteries of "natural magic" and "theology" (chap. 6 and 7) are conjoined with those of Pythagorean (or "ideal") numbers (chap. 5) in order to activate and vivify the three types of music originally distinguished by Boethius,[88] which are combined in turn with poetic discourse to actually produce the most perfect kind of harmony, supposedly capable of achieving all wonders, spiritual and otherwise,[89] and of which Orpheus is the symbol.

Quite similar speculations about the cosmological use of numbers, associating rhetoric, literature, poetical declamation and music, but somewhat less preoccupied with purely magical background or goals, are found in the works of the 17th-century author Teodato Osio (1605–1673). This little known writer published a spate of books in Milan, between 1637 and 1668, about the application of mathematics and music to the rhythm of Latin and Italian poetry and

85 Fabio Paolini 1589.

86 *Aen.* VI, 646: *Obloquitur numeris septem discrimina vocum* ([Orpheus] "accompanies their voices with the seven-note scale").

87 Vasoli 1998, 193–210, part. 204–207; McDonald 2012, 222–248.

88 *Musica humana, intrumentalis, mundana* (vocal, instrumental and "cosmic" music).

89 Radaelli 1999.

prose,[90] along with another work on architecture and land surveying.[91] Osio develops some classic musical considerations about "Pythagorean" (mainly Platonic) musical theory, whose intervals and proportions he applies to the prosody and tonal accents of the languages referred to above. Assimilating the continuous and discrete modes of quantity to the "Same" and "Other" of Plato's cosmogony in the *Timaeus*, he basically compares the components and fundamental structure of language – spoken as well as written – to the genesis of the geometric bodies, insisting on the cosmological role of proportion and numbers and bringing together the doctrine of world harmony and that of language constitution (syllables, tones, letters, word composition and etymology).[92] In his later productions, Osio insists less on language than on philosophical considerations about discrete and continuous quantity as tools of divine creation and fundamental elements of the universe, associated with the three main proportional means (see above) and to the terms of the famous quotation of the *Book of Wisdom*,[93] drawing analogies between the divine operations and those of human craftsmen. Dealing occasionally with magic and divination (using a method combining astrology, mathematics and music), such a cosmological discourse constitutes a very curious and understudied instance of Platonic speculations about nature and mathematical harmony in Northern Italy around the middle of the 17th century.

7 Pythagorean Number as a Philosophical Abstraction

The quite complex and much-studied natural philosophy and cosmology of Giordano Bruno (1548–1600) harbour a very personal conception of number and of "mathematical magic" which – despite the well-known influence of Agrippa on the *De monade, numero et figura*[94] – has in fact little to do with Pythagorean number mysticism as understood in the present article.[95] The Neoplatonic scheme of the *scala entis* and the correlative distribution of mathematical entities along its ontological hierarchy, let alone their "magical" power over physical realities, have no place in Bruno's thought. Although the *De monade* appears outwardly as a *compendium* of the significations attached to each number of the primary Decad, presented in their natural order, the

90 Teodato Osio 1637, 1653 and 1668.
91 Id. 1639.
92 Brach 1994, 90–92; Wuidar 2008, *passim*; Gaspari 2011 (who mentions several extant unpublished manuscripts on number symbolism and Pythagorean lore).
93 *Book of Wisdom*, XI, 21 ("You [*sc.* God] ordered all things in measure, number and weight").
94 Printed with two other important Latin philosophical poems in Giordano Bruno 1591.
95 Bonnet 2002; Giovannozzi 2012.

accent is in reality on the relations between physics and mathematics, and the focus of the book on the notions of order, form, figure and quality. As the internal components of reality, numbers and geometrical figures are essentially distinguished by their qualitative structural differences, bestowed upon them by the multiplicity of forms. Their main role is to order natural realities, receiving from them the qualitative determinations of which mere abstract quantity is devoid in itself. Abstract quantity as such belongs to the logical plane, intrinsically superior to the imagination but unable, nevertheless, to activate it. However, since all forms are dependent on the unique universal substance, mathematical abstraction, accounting for their common characters, allows us to grasp the relations of these multiple forms to the unity of their *substratum*, and thus to apprehend the underlying substantial constitution of being. The science of numbers leads us therefore towards the understanding of the unity of being and, conversely, geometry sheds light on the process by which plurality, intrinsically immanent to this unity, is actually deployed in the outward existence of forms and figures. By these considerations,[96] Bruno shows the ancillary character of both arithmetic and geometry towards natural philosophy and metaphysics; as well as their common abstract character, intermediary between the essence of reality and its material expansion.[97]

In his magical writings (drafted between 1588 and 1592[98]), whose contents are rooted in the natural philosophy elaborated in the earlier Italian dialogues (such as *De la causa, principio e uno*, *Cena delle ceneri*, and others), Bruno actually criticizes Agrippa's conception of the effective relation between language and being. Natural magic, for Bruno, is really inseparable from the physical properties of things and beings, and not subject to occult virtues.

8 Pythagoreanism as the Key to Universal Knowledge

An encyclopedic dimension is conferred to Pythagorean number symbolism by the publication of Pietro Bongo's (?-1601) enormously erudite *Numerorum Mysteria*.[99] Written by a member of an ancient and noble family from Bergamo,

96 Naturally somewhat oversimplified here; for a detailed study of the general status of mathematical disciplines in G. Bruno's works, Bönker-Vallon 1995.

97 Such a hierarchy is only valid on the cognitive plane, and is not to be understood as implying an ontological counterpart. The scalar nature of being is entirely deconstructed in Bruno's works, from the so-called "Italian Dialogues" onwards.

98 They include texts such as the *De magia mathematica* (*ca* 1590; Giordano Bruno 1999) and *Theses de magia* (in Id. 2000).

99 Definitive and most complete impression Bergamo: Ventura 1599 (reprint Pietro Bongo 1983); cf. Ernst 1983; Piccinini 1984.

Canon of the local Sant'Alessandro cathedral, this thick *quarto* tome is entirely dedicated to the tradition of "Pythagoreanism," and draws from every conceivable source available at the time, including magical, Hermetic, kabbalistic and esoteric writings (Lull, Cusa, Pico, Ficino, Lefèvre d'Etaples, Bovelles, Zorzi, Dee, etc). The book was frequently criticized for its motley and untidy character, which often degenerates in a mere mosaic of quotations.[100] Bongo nevertheless remained aware in it of certain theoretical issues at stake in his days. For instance, the growing opposition between traditional thinking (based on analogy, correspondences and the powers hidden in the essence of things) and a more modern trend of thought, which attempted to read the world through the lenses of experience and of a scientific *mathesis*, was not lost on him. Bongo contrasted the Book of Nature with that of Scripture, and exhibited an understanding of *pythagorica disciplina* as based on Tradition and authority, and as essentially concerned with scrutinizing the divine mysteries. Such a perspective naturally retained a strong theological flavor, insisting on the supposed *consensus* between Pythagoreanism (as Bongo understood it) and post-Tridentine Catholic perspectives. The symbolical use of numbers is here viewed not just as a general key to the harmony of reality and of spiritual life but also as an ontological principle at work behind the layout of creation.

Through its several editions and the echo it found in other contemporary publications, Bongo's treatise was instrumental in establishing number mysticism as a specific and relevant humanistic topic, of which it became one of the foremost textbooks, thus strongly contributing to the acceptance of such speculations into the body of mainstream scholarly culture.[101]

Thirty years later, the then archbishop of Milano Federico Borromeo (1564–1631) had an extremely limited number of copies of his *De pythagoricis numeris*[102] printed by the press established in the archbishopric by his distant cousin and predecessor on the Milan seat, the more famous Carlo Borromeo (1538–1584).[103] Here again, Pythagoreanism is considered as standing in the midst between religion and science, and viewed from a dual, contemplative and scientific, perspective on nature, typical of the early 17th century, a period sometimes as wary of esoteric speculations (both Borromeos were champions of the Catholic Counter-Reform in Italy) as it was defiant of certain

100 Seen from the opposite perspective, this feature helps making the book a useful dictionary of the allegorical meanings of numbers, which almost all artistic or intellectual trades could indifferently tap into.

101 Baroni 2012.

102 Federico Borromeo 2016.

103 Burgio and Ceriotti 2002.

requirements of scientific methods. The book examines the nature and properties of numbers, as well as their usefulness in the rational study of natural mechanisms, while leaving aside or criticizing their "occult," magical, divinatory or kabbalistic aspects, for which the author had entertained a strong interest in his youth but which he now scorned.[104] As an aside, we may note that, following a collaboration between the two men which apparently started in late 1610 or early 1611, F. Borromeo became the dedicatee of the *Taumatologia*[105] written by Giovan Battista Della Porta (1535–1615). Parts of this work deal with the natural powers attributed to numbers and music, a topic certainly reminiscent of Pico della Mirandola and consonant with Della Porta's famous interest in natural magic.[106] The *De pythagoricis numeris* enjoyed almost no circulation at all and was therefore virtually unknown; it was, however, rediscovered and quoted in recent times by the Florentine mathematician and specialist of Pythagoreanism Arturo Reghini (1878–1946).[107]

Another contemporary encyclopedic presentation of Pythagorean number philosophy[108] – albeit of a very different kind – is that of the famous English theosopher Robert Fludd (1574–1637).[109] Chapters on this topic open the second volume of his *Utriusque cosmi … historia*,[110] dedicated to the "microcosm," according to the leading theme and structural principle of the entire *opus*, which is the fundamental analogy between the two corresponding worlds, the universe and man. The analogy is in fact tripartite, according to Fludd, and includes the divine sphere, of which both the macro- and microcosm are supposedly perfect images. Following a classical (since Augustine) interpretation of this theory, the pattern of creation is essentially modelled on the ternary, as the expression of the three-fold operation of the Trinity. Beginning with "divine numbers," which he also calls "supersubstantial numbers," naturally

104 In the same year and place, F. Borromeo also published his *De cabbalisticis inventis libri duo* (Federico Borromeo 1978); see Campanini 2002. The famous Neoplatonic philosopher Francesco Patrizi (1529–1597) has left a hitherto unpublished manuscript *De numerorum mysteriis* (*On the Mysteries of Numbers*, 1594) dedicated to F. Borromeo, who was his pupil at the time (a critical edition of the text was announced a long time ago by Maria Muccillo).

105 It is possible that this work remained uncompleted and unpublished; extant fragments in Giovan Battista Della Porta 2013.

106 Bertolini 2017 (my thanks to M. Ghione for this reference); Verardi 2018, 138–140.

107 Reghini 2004, 46ff.

108 Akin, to a certain extent, to Zorzi's *Harmonia mundi*, although quite different of course in many respects.

109 Huffman 1988 and 2001; Janacek 2011, 43–74.

110 Robert Fludd 1617–1619.

extolling the importance of the Monad and of its created counterpart, the geometrical point, Fludd goes on to examine the mystical Dyad, and then the Ternary. He expands on the relation of the Trinity to the Monad, and on the manifold triadic correspondences within both worlds. Numbers and musical/harmonic proportions rule the general layout of the cosmic structure as well as the ratios between its different tiers, the whole argument being superbly illustrated by many engravings which have vastly contributed to making Fludd's work famous and helped initiate (along with Khunrath and a few others) the trend of "cosmo-theosophical" illustrations in 17th-century alchemical and Rosicrucian books.[111]

The author does make use of the expression "formal numbers" and, in his quest for universal knowledge,[112] imitates Agrippa in applying the primary Decad to the Godhead and the first, essential principles, the tens to the angelic spheres, the hundreds to the celestial world and the thousands to the sublunar universe and its basic elements.

The same tripartite blueprint necessarily applies to Man as microcosm, organized according to the ternary of intellect, soul and body and their hierarchy ruled by musical proportions and harmony. Divine influences on both the lesser and greater worlds are mediated by the celestial forces and this universal vitalism reminds us that the network of natural correspondences is to Fludd a basic component of reality, embedded into the *machina mundi* and essential to both its life and operation. The same goes for Pythagorean numbers which, for him, exert an action which is far from being merely symbolical or abstract; they constitute, on the contrary, an integral part of the intimate nature of the universe and of its workings, as well as an essential tool for their understanding.

Although more limited in scope and bulk than Bongo's or Fludd's books, Jan van Meurs the Elder's (1579–1639) *Denarius Pythagoricus* (*Pythagorean Decad*)[113] and Athanasius Kircher's (1602–1680) *Arithmologia*[114] represent typical encyclopedic treatments of number symbolism.

The former work is essentially a systematic inventory of classical sources concerning Pythagorean number theory. It consists mostly of an array of quotations about the nature and definition of number, its basic constitution (odd and even, male and female, perfect and imperfect, etc.) and the characteristics

111 Szulakowska 2011; Forshaw 2016.

112 Akin to Kircher's in this respect but of course quite different in tone, spirit and intentions.

113 Johannes Meursius 1631.

114 Athanasius Kircher 1665 (the full title in English reads: *Arithmology, or the Hidden Mysteries of Numbers*). See Leinkauf 1993, 192–235.

and mythological correspondences attached to the components of the Decad, examined according to their natural order. Unsurprisingly, the references are chiefly taken from pagan sources (literary, poetical, philosophical, historical) and a good number of Greek and Latin Church Fathers, who have frequently expressed theological views about scriptural numbers. By contrast, the majority of medieval authorities – with the exception of Boethius and a few other Western and Byzantine writers – are shunned by this Protestant historian, famous for his erudition but lacking in philosophical creativity. Speculation, moreover, is not the goal of the present *opus*, which is mainly descriptive, and aims at a didactic presentation of the topic, enhanced by several useful reference indexes.

One of the greatest and most famous polymaths of the 17th century, the Jesuit A. Kircher has dedicated a specific treatise to number symbolism. Although not one of his best-known books (probably because it is also of much less value to book-lovers than some of his other, richly illustrated productions[115]), the *Arithmologia* nevertheless reveals its author's interest for "occult" speculations – an interest Kircher had to rein in prudently, given the context of post-Tridentine Catholicism.

The book is divided in six parts, and deals with all sorts of topics linked to number symbolism: digits or numerical characters and their symbolism; the so-called "magic squares" and their astrological use in Antiquity, as well as in Judaic and Arabic religious cultures; divination by numbers, using magical seals and number combinations; amulets, angelic seals and magical alphabets based on numerals and geometrical figures. The last chapter, entitled "The mystical signification of numbers," considers the Decad and its multiple analogies and correspondences in Nature.

In an age thirsting for univeral knowledge, the *Arithmologia* intends to be an encyclopedic presentation of its topic, synthesizing the different doctrinal, magical and astrological aspects of number mysticism, as Kircher understands it. Apart from its vast erudition, the book clearly betrays its author's fascination for the kind of unorthodox speculations he is examining. Kircher justifies his interest in number magic and symbolism by assuming a dual necessity: first, to bring back to light this material by extracting it from its ancient, more or less reputable pagan or Christian sources; second, to salvage it from "heretical" contexts and criticize its eventual denaturation. Finally, his ultimate goal is to reintegrate these views within the scope of contemporary knowledge, and

115 Such as the *Mundus subterraneus*, published in the same year, or his *Oedipus Aegyptiacus* (1652–1654).

present them henceforth as acceptable and legitimate from the points of view of theology, science and natural philosophy.[116]

9 Pythagoras: Stranger in a Strange Land

From what we have seen above, pythagorean tenets about number and/or geometry, as understood during the Renaissance, are generally reinterpreted through neoplatonic lenses, which root the study of natural philosophy and cosmology in an ontological and theological viewpoint. Central, here, is the christianized idea of numbers being essentially a divine model, the essential principles of things, present in God's intellect and which serve as a pattern for the organization of the cosmos.

Conversely, numbers and geometrical figures are equally perceived as a scalar support for the ascension of the human mind from earthly matters towards the unitive contemplation of the immaterial being (*theosis, deificatio*), thus blending an ontological *and* a cognitive approach to the nature of reality and of the Godhead. Such a stance, in which theological considerations actually govern scientific knowledge, paves the way for doctrines which treat the qualitative aspects of number as superior to and ruling over the quantitative and material ones, yet maintaining a strict relationship between both: physical and metaphysical numbers are never very far removed, let alone independent, from one another.

Thus, Pythagorean arithmology is linked to a conception of number as "efficient cause," as well as to the doctrines of the unicity of creation and of the analogies and correspondences linking its different planes, of which man himself is a mirror, on an obviously smaller scale.

The progressive decline of such an organic worldview within European culture, from the late 16th and 17th century onwards, inevitably entailed the increasing scientific irrelevance of the currents of thought which depended on it, including of course that of arithmology and of other esoteric tenets which frequently resorted to it. Gradually, number found itself reduced to the status of a mere logical operator, devoid of any reference to a living, inner essence of things, thus rendering meaningless its previous role as a "factor of enchantment" within creation, mediating between material and spiritual realities. Harmony and proportions are no longer construed as key elements in the language of Nature – a language which is henceforth treated as both physically

116 Fletcher 2011, 53, 161–170, 174.

and metaphysically autonomous *vis-à-vis* the theological discourse. While analyzing the geometrical and quantitative aspects of space and of natural order, mathematicians develop a new *mathesis universalis* which aims at renovating the understanding of the relationship between knowledge and being. The components of the universe are therefore viewed as immanent, mechanical constructs submitted to quantitative laws, *in lieu* of elements of a pattern in the mind of God; accordingly, their study may no longer be conducted in terms of a quest for the mystic harmonies of numbers and of geometrical proportions, nor expressed cogently as the unfolding of a providential design, already inscribed in the nature of things by way of their mathematical structure.

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